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BENDING MOMENTS AND SHEAR FORCES CALCULATION ACCORDING TO EN 1991-2 LOAD MODEL 1

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Key words: *bending moments, shear forces, EN 1991-2, traffic load model LM1.*
Abstract: *In the present paper, fast calculation method for determination the effects of EN 1991-2 LM1 is presented. The influence lines method was used for the calculations. Both bending moments and shear forces at typical section can be easy calculated by using tables.*

ROAD TRAFFIC ACTIONS – VERTICAL LOADS (EXCEPT FATIGUE), DEFINED IN EN 1991-2 [1]

1.Load Model 1 (LM1) is represented as concentrated and uniformly distributed loads, which cover most of the effects of the traffic of lorries and cars. This model should be used for general and local verifications and is the main load model for bridges with spans more than 7 m;

2.Load Model 2 (LM2) can be predominant in the range of loaded lengths up to 3 m to 7 m and normally is used for load effects calculation for short structural members;

3.Load Model 3 (LM3) is representing special vehicles (e.g. for industrial transport) which can travel on routes permitted for abnormal loads;

4.Load Model 4 (LM4): A crowd loading should be used only for some transient design situations.

Load models defined in EN 1991-2 should be used for the design of road bridges with loaded lengths less than 200 m. The length of 200 m corresponds to the maximum length taken into account for the calibration of LM1.

Loads due to the road traffic, consisting of cars, lorries and special vehicles (e.g. for industrial transport), give rise to vertical and horizontal, static and dynamic forces. The defined load models do not describe actual vehicles. They have been selected and calibrated so that their effects (with dynamic amplification included where indicated) represent the effects of the actual traffic in European countries.

Because of LM1 is the main load model for bridge design (except fatigue load models), and mention in the paragraph above, gives the author a reason to search a fast calculation method for TS that will give the same load effects for simply supported bridges. The presented method below allows designers to use a table for their calculations.

LOAD MODEL 1 DEFINITION [1]

Load Model 1 consists of two partial systems, as follows:

1. Double-axle concentrated loads (tandem system: TS), each axle having the following weight:

(1) $\alpha_Q Q_k$

2. Uniformly distributed loads (UDL system), having the following weight per square metre of notional lane:

(2) $\alpha_q q_k$

Load Model 1 should be applied on each notional lane and on the remaining areas.

On notional lane Number *i*, the load magnitudes are referred to:

(3) $\alpha_{Qi} Q_{ik}$

(4) $\alpha_{qi} q_{ik}$

On the remaining areas, the load magnitude is referred to:

(5) $\alpha_{qr} q_{rk}$

Where: α_{Qi}, α_{qi}, and α_{qr} are adjustment factors that should be selected depending on the expected traffic and possibly on different classes of routes. The values should be accepted in the national annexes.

The values of Q_{ik}, q_{ik} and q_{rk} are shown in Table 1 [1].

Table 1 - Load model 1: characteristic values

Location	Tandem system *TS*	*UDL* system
	Axle loads Q_{ik} (kN)	q_{ik} or q_{rk} (kN/m^2)
Lane Number 1	300	9
Lane Number 2	200	2,5
Lane Number 3	100	2,5
Other lanes	0	2,5
Remaining area (q_{rk})	0	2,5

The details of Load Model 1 are illustrated in Fig. 1.

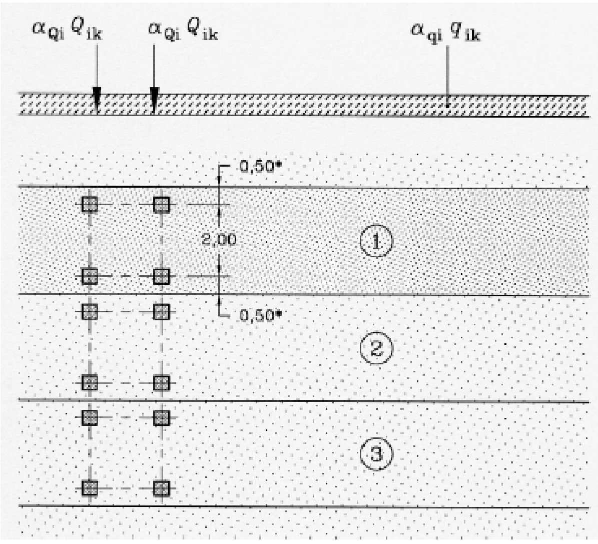

Fig.1 Load Model 1

PROPOSED METHOD

Influence lines for bending moment (M_y) and shear force (V_z), at section "s" at a distance "x" from the support, of a simply supported beam loaded with two point loads on a distance 1,2 m (similar to *LM1 TS* system in a longitudinal direction) are shown on Fig. 2.

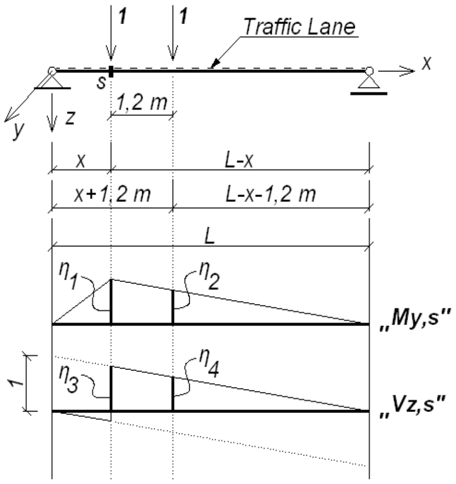

Fig.2 Influence lines for load effects at section "s"

The influence lines are drawn with their positive values above the reference line.

Load effects, according to the influence lines method are as follows:

(6) $$M_{y,s} = \eta_1 + \eta_2$$

(7) $$V_{z,s} = \eta_3 + \eta_4$$

Where η_1 to η_4 are the corresponding influence line ordinates under the point loads, as follows:

(8) $$\eta_1 = \frac{x\left(L-x\right)}{L}$$

(9) $$\eta_2 = \frac{x\left(L-x-1,2\right)}{L}$$

(10) $$\eta_3 = \frac{L-x}{L}$$

(11) $$\eta_4 = \frac{L-x-1,2}{L}$$

The maximum bending moment from such point loads is not at the middle of the span. Location of the section with maximum bending moment can be found from the equation:

(12) $$\frac{dM_{y,s}}{dx} = \frac{2L-4x-1,2}{L} = 0$$

The section with maximum bending moment from *LM1 TS* is at distance from the support:

(13) $$x_{M_{y,\max}} = \frac{L}{2} - 0,3 \text{ m}$$

LOAD EFFECTS CALCULATION FROM ANY VALUE OF *TS*

To simplify load effects calculation author propose a table which can be used by designers. Bending moments ($M_{y,s}$) and shear forces ($V_{z,s}$) can be calculated with formulas, as follows:

$$M_{y,s} = k_{My} \sum_{i=1}^{n_1} \alpha_{Qi} Q_{ik} \tag{14}$$

$$V_{z,s} = k_{Vz} \sum_{i=1}^{n_1} \alpha_{Qi} Q_{ik} \tag{15}$$

Where: n_1 is the number of notional lanes on a carriageway that takes tandem systems in accordance with the values, given in Table 1 (maximum 3).

Values for k_{My} and k_{Vz} at typical sections for simply supported beams can be found in Table 2.

When there is a grillage bridge superstructure system, the designer should calculate the load redistribution between beams first. There are several hand calculation methods and some details about such calculations, according to the transversal stiffness of the bridge superstructure with comments can be found in [3] and [5]. In present days there are plenty of software products (mainly based on the FEM method) that can be used for such precise calculations.

Comments for the influence of different load distribution in load effects calculation can be found in [4].

Table 2 - Values for k_i at typical sections for simply supported beams

Span [m]	x = 0	x = L/8		x = L/4		x = 3/8 L		x = L/2		x = L/2--0,3 m
	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}
6	18,0000	11,6250	15,5000	19,5000	13,0000	23,6250	10,5000	24,00	8,0000	24,5455
7	18,2857	13,8125	15,7857	23,2500	13,2857	28,3125	10,7857	29,00	8,2857	29,4737
8	18,5000	16,0000	16,0000	27,0000	13,5000	33,0000	11,0000	34,00	8,5000	34,4186
9	18,6667	18,1875	16,1667	30,7500	13,6667	37,6875	11,1667	39,00	8,6667	39,3750
10	18,8000	20,3750	16,3000	34,5000	13,8000	42,3750	11,3000	44,00	8,8000	44,3396
11	18,9091	22,5625	16,4091	38,2500	13,9091	47,0625	11,4091	49,00	8,9091	49,3103
12	19,0000	24,7500	16,5000	42,0000	14,0000	51,7500	11,5000	54,00	9,0000	54,2857
13	19,0769	26,9375	16,5769	45,7500	14,0769	56,4375	11,5769	59,00	9,0769	59,2647
14	19,1429	29,1250	16,6429	49,5000	14,1429	61,1250	11,6429	64,00	9,1429	64,2466
15	19,2000	31,3125	16,7000	53,2500	14,2000	65,8125	11,7000	69,00	9,2000	69,2308
16	19,2500	33,5000	16,7500	57,0000	14,2500	70,5000	11,7500	74,00	9,2500	74,2169
17	19,2941	35,6875	16,7941	60,7500	14,2941	75,1875	11,7941	79,00	9,2941	79,2045
18	19,3333	37,8750	16,8333	64,5000	14,3333	79,8750	11,8333	84,00	9,3333	84,1935
19	19,3684	40,0625	16,8684	68,2500	14,3684	84,5625	11,8684	89,00	9,3684	89,1837
20	19,4000	42,2500	16,9000	72,0000	14,4000	89,2500	11,9000	94,00	9,4000	94,1748
21	19,4286	44,4375	16,9286	75,7500	14,4286	93,9375	11,9286	99,00	9,4286	99,1667
22	19,4545	46,6250	16,9545	79,5000	14,4545	98,6250	11,9545	104,00	9,4545	104,1593
23	19,4783	48,8125	16,9783	83,2500	14,4783	103,3125	11,9783	109,00	9,4783	109,1525
24	19,5000	51,0000	17,0000	87,0000	14,5000	108,0000	12,0000	114,00	9,5000	114,1463
25	19,5200	53,1875	17,0200	90,7500	14,5200	112,6875	12,0200	119,00	9,5200	119,1406
26	19,5385	55,3750	17,0385	94,5000	14,5385	117,3750	12,0385	124,00	9,5385	124,1353
27	19,5556	57,5625	17,0556	98,2500	14,5556	122,0625	12,0556	129,00	9,5556	129,1304
28	19,5714	59,7500	17,0714	102,000	14,5714	126,7500	12,0714	134,00	9,5714	134,1259
29	19,5862	61,9375	17,0862	105,750	14,5862	131,4375	12,0862	139,00	9,5862	139,1216
30	19,6000	64,1250	17,1000	109,500	14,6000	136,1250	12,1000	144,00	9,6000	144,1176
40	19,7000	86,0000	17,2000	147,000	14,7000	183,0000	12,2000	194,00	9,7000	194,0887

Span [m]	x = 0	x = L/8		x = L/4		x = 3/8 L		x = L/2		x = L/2--0,3 m
	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}	k_{Vz}	k_{My}
50	19,7600	107,875	17,2600	184,500	14,7600	229,8750	12,2600	244,00	9,7600	244,0711
60	19,8000	129,750	17,3000	222,000	14,8000	276,7500	12,3000	294,00	9,8000	294,0594
70	19,8286	151,625	17,3286	259,500	14,8286	323,6250	12,3286	344,00	9,8286	344,0510
80	19,8500	173,500	17,3500	297,000	14,8500	370,5000	12,3500	394,00	9,8500	394,0447
90	19,8667	195,375	17,3667	334,500	14,8667	417,3750	12,3667	444,00	9,8667	444,0397
100	19,8800	217,250	17,3800	372,000	14,8800	464,2500	12,3800	494,00	9,8800	494,0358

Note: Values in Table 2 are 10 times scaled.

A method that represents tandem system *TS* with equivalent uniform loads is presented in [2]. Calculated equivalent uniform load, according to [2], can be added to the UDL system and this allows designers to use only uniform loads for bending moment and shear force calculation for simply supported beams. It also gives a possibility for easy preliminary calculations of prestressing force needed to balance the equivalent uniform load. Some calculations and result comparison of load effects in bridge superstructure between common and proposed [2] methods were reported and commented in [4].

The equivalent uniform loads can be obtained, as follows:

$$q_{My,s}{}^{UL} = k_{My}{}^{UL} \sum_{i=1}^{n_1} \alpha_{Qi} Q_{ik} \tag{16}$$

$$q_{Vz,s}{}^{UL} = k_{Vz}{}^{UL} \sum_{i=1}^{n_1} \alpha_{Qi} Q_{ik} \tag{17}$$

Where: n_1 has the same meaning as in formulas (14) and (15).

Values of the coefficients for equivalent uniform loads for tandem system *TS* at typical sections for simply supported beams are given in Table 3.

Table 3 - Values for k_i^{UL} at typical sections for simply supported beams

Span [m]	x = 0	x = L/8		x = L/4		x = 3/8 L		x = L/2		x = L/2--0,3 m
	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}
6	60,000	59,048	68,889	57,778	86,667	56,000	140,000	53,333	106,667	54,5455
7	52,245	51,545	60,136	50,612	75,918	49,306	123,265	47,347	94,694	48,1203
8	46,250	45,714	53,333	45,000	67,500	44,000	110,000	42,500	85,000	43,0233
9	41,481	41,058	47,901	40,494	60,741	39,704	99,259	38,519	77,037	38,8889
10	37,600	37,257	43,467	36,800	55,200	36,160	90,400	35,200	70,400	35,4717
11	34,380	34,097	39,780	33,719	50,579	33,190	82,975	32,397	64,793	32,6019
12	31,667	31,429	36,667	31,111	46,667	30,667	76,667	30,000	60,000	30,1587
13	29,349	29,146	34,004	28,876	43,314	28,497	71,243	27,929	55,858	28,0543
14	27,347	27,172	31,701	26,939	40,408	26,612	66,531	26,122	52,245	26,2231
15	25,600	25,448	29,689	25,244	37,867	24,960	62,400	24,533	49,067	24,6154
16	24,063	23,929	27,917	23,750	35,625	23,500	58,750	23,125	46,250	23,1928
17	22,699	22,580	26,344	22,422	33,633	22,201	55,502	21,869	43,737	21,9251
18	21,481	21,376	24,938	21,235	31,852	21,037	52,593	20,741	41,481	20,7885
19	20,388	20,293	23,675	20,166	30,249	19,989	49,972	19,723	39,446	19,7637
20	19,400	19,314	22,533	19,200	28,800	19,040	47,600	18,800	37,600	18,8350
21	18,503	18,426	21,497	18,322	27,483	18,177	45,442	17,959	35,918	17,9894
22	17,686	17,615	20,551	17,521	26,281	17,388	43,471	17,190	34,380	17,2164
23	16,938	16,873	19,685	16,786	25,180	16,665	41,664	16,484	32,968	16,5070
24	16,250	16,190	18,889	16,111	24,167	16,000	40,000	15,833	31,667	15,8537
25	15,616	15,561	18,155	15,488	23,232	15,386	38,464	15,232	30,464	15,2500
26	15,030	14,979	17,475	14,911	22,367	14,817	37,041	14,675	29,349	14,6906

Span [m]	x = 0	x = L/8		x = L/4		x = 3/8 L		x = L/2		x = L/2--0,3 m
	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}	k_{Vz}^{UL}	k_{My}^{UL}
27	14,486	14,439	16,845	14,376	21,564	14,288	35,720	14,156	28,313	14,1707
28	13,980	13,936	16,259	13,878	20,816	13,796	34,490	13,673	27,347	13,6863
29	13,508	13,467	15,711	13,413	20,119	13,337	33,341	13,222	26,445	13,2339
30	13,067	13,029	15,200	12,978	19,467	12,907	32,267	12,800	25,600	12,8105
40	9,850	9,829	11,467	9,800	14,700	9,760	24,400	9,700	19,400	9,7044
50	7,904	7,890	9,205	7,872	11,808	7,846	19,616	7,808	15,616	7,8103
60	6,600	6,590	7,689	6,578	9,867	6,560	16,400	6,533	13,067	6,5347
70	5,665	5,658	6,601	5,649	8,473	5,636	14,090	5,616	11,233	5,6172
80	4,963	4,957	5,783	4,950	7,425	4,940	12,350	4,925	9,850	4,9256
90	4,415	4,411	5,146	4,405	6,607	4,397	10,993	4,385	8,770	4,3856
100	3,976	3,973	4,635	3,968	5,952	3,962	9,904	3,952	7,904	3,9523

Note: Values in Table 3 are 100 times scaled.

Using Table 3, bending moments and shear forces can be calculated, according to the formulas bellow:

(18) $$V_{z,x=0} = q_{Vz,x=0}^{UL}\frac{1}{2}L$$

(19) $$M_{y,x=\frac{1}{8}L} = q_{My,x=\frac{1}{8}L}^{UL}\frac{7}{128}L^2$$

(20) $$V_{z,x=\frac{1}{8}L} = q_{Vz,x=\frac{1}{8}L}^{UL}\frac{3}{8}L$$

(21) $$M_{y,x=\frac{1}{4}L} = q_{My,x=\frac{1}{4}L}^{UL}\frac{3}{32}L^2$$

(22) $$V_{z,x=\frac{1}{4}L} = q_{Vz,x=\frac{1}{4}L}^{UL}\frac{1}{4}L$$

(23) $$M_{y,x=\frac{3}{8}L} = q_{My,x=\frac{3}{8}L}^{UL}\frac{15}{128}L^2$$

(24) $$V_{z,x=\frac{3L}{8}} = q_{Vz,x=\frac{3L}{8}}^{UL}\frac{1}{8}L$$

(25) $$M_{y,x=\frac{1}{2}L} = q_{My,x=\frac{1}{2}L}^{UL}\frac{1}{8}L^2$$

(26) $$V_{z,x=\frac{L}{2}} = q_{Vz,x=\frac{L}{2}}^{UL}\frac{1}{8}L$$

(27) $$M_{\max} = M_{y,x=\frac{1}{2}L-0,3\text{ m}} = q_{My,x=\frac{1}{2}L-0,3\text{ m}}^{UL}\frac{1}{8}L^2$$

Note: For shear force calculation, with equivalent load, at the middle span only half of the beam should be loaded with the equivalent uniform load $q_{Vz,x=1/2L}^{UL}$, or just to use formula (26) for a correct result.

CONCLUSION

Presented method for fast calculation of load effects from tandem system *TS* which is a part of the main load model LM1 simplify many hard calculations for the designers. It can be used not only for a preliminary calculations and obtaining balancing loads and also in many calculation tasks in the engineering practice in connection with the bridge design. This method also minimizes the risk of calculation mistakes in the load effects calculation and gives a possibility easy to calculate the maximum bending moment, which is not at the middle of the beam.

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ИЗЧИСЛЯВАНЕ НА ОГЪВАЩИТЕ МОМЕНТИ И НАПРЕЧНИТЕ СИЛИ ОТ ТОВАРЕН МОДЕЛ LM1 СПОРЕД EN 1991-2

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Ключови думи: *огъващи моменти, напречни сили, EN 1991-2, товарен модел LM1.*

Резюме: *В настоящият доклад е представен бърз метод за изчисляване на ефектите от товарния модел LM1 според EN 1991-2. За изчисленията е използван методът с използване на линии на влияние. Огъващите моменти и напречните сили за най-често изследваните сечения могат да бъдат лесно изчислявани с помощта на таблици.*