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KRONECKER ALGEBRA AS A FRAME FOR OPTIMISATION OF RAILWAY OPERATION

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Key words: *Kronecker Algebra, Travel Time Analysis, Energy Analysis, Deadlock Analysis*

Abstract: *Kronecker algebra consists of Kronecker product and Kronecker sum. It can be used to model systems consisting of several actors and a number of limited resources. In particular, it can be used to model railway systems consisting of trains, their routes in the system, and track sections building up the railway infrastructure. In this paper we will show several applications of Kronecker algebra in the railway domain. In particular, we consider: deadlock analysis [1], travel time analysis [2], and energy analysis. Integrating all three types of analysis within one single type of Kronecker-based analysis is rather simple and can be done very efficiently. Our implementation is very efficient both in time and space. Kronecker algebra operations can easily be parallelized and thus our implementation can fully take advantage of today's multi-core computer architecture. In addition, our implementation shows that adding constraints (connections, overtaking ...) to the problem improves execution time. In fact, a harder problem is easier to solve.*

INTRODUCTION

Kronecker algebra and its applications in railway systems have been introduced in some previous scientific papers. In [1] it is shown how to avoid deadlocks within a railway network with several trains. In [2] it is explained how to calculate the travel time of trains within a railway system. Blocking among trains occurs due to sharing of track sections, connections and overtaking. Blocking time is incorporated into the calculated travel time.

In contrast to some preliminary papers which deal with the theoretical background, we show some practical examples in the following sections, which have been introduced in [7].

A SIMPLE EXAMPLE

In this section we give a small example on how deadlocks can be avoided by the Kronecker algebra based approach.

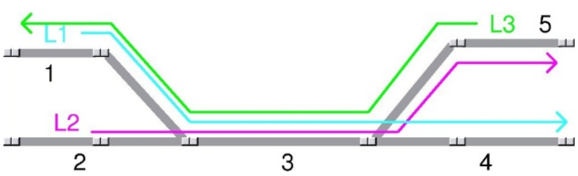

Figure 1. A simple example: Railway system

Figure 1 illustrates a typical scenario that may lead to a deadlock. The routes and the calculated travel time of the three involved trains and the corresponding travel time values for each track section are given in Table 1. The travel time for each train is calculated by summarizing the travel time of each track section of its route including the blocking time. Blocking occurs due to the shared track section 3.

If train L_3 enters track section 3 before the other trains move, a deadlock is unavoidable. The same problem will occur when train L_2 enters track section 3. Thus, there exists only one possibility to avoid a deadlock, namely L_1 has to enter and leave track section 3 first.

Trains	Routes	Travel Time
L1	p_3 (0), v_1 (5), p_4 (0), v_3 (3), v_4 (4)	12
L2	p_3 (0), v_2 (4), p_5 (0), v_3 (4), v_5 (5)	28
L3	p_3 (0), v_5 (5), p_1 (0), v_3 (3), v_1 (5)	21

Table 1. A simple example: Routes and travel time

After applying Kronecker algebra to this example, we get a graph (Figure 2 left), which consists of 42 nodes. The edges are labelled by the number of the train and its operation within the railway system, where *"p"* denotes *"enter (reserve) a track section"* and *"v"* denotes *"leave (release) a track section"*.

To increase readability three different node types are used in the graph:

- Red nodes denote deadlocks[1] or nodes from which only deadlocks can be reached.
- Green nodes denote safe states. A state is safe if all trains can perform their actions without having to take into account the moves of the other trains in the system, provided that the track section which they are to enter is not occupied by another train[2].
- From orange nodes both red and green nodes can be reached.

As we are interested in avoiding deadlocks the graph can be reduced, in particular the red nodes can be removed. Additionally all safe states can be eliminated, which have at least one safe state as predecessor. The resulting graph after reduction is illustrated on the right in Figure 2.

CONNECTIONS AND OVERTAKING

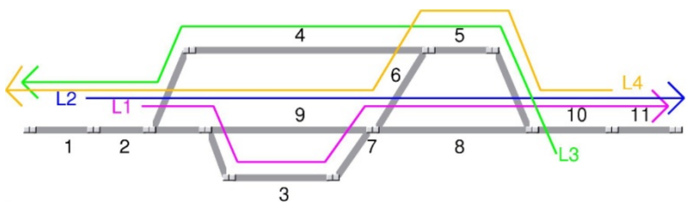

Figure 2. Connections and overtaking: Railway system

Now we will show a more elaborate example containing connections and overtaking. The system is depicted in Figure 2 and the routes are defined in Table 2. By applying Kronecker algebra 298,721,280 states are produced, where only 206 states of interest will remain after the reduction. The resulting graph is depicted in Figure 5. p_{12}, v_{12}, p_{13} and v_{13} are additional constraints. We assume that train L_2 overtakes L_1 and L_4 overtakes L_3 within the station, respectively. For this reason additional artificial track sections 12 and 13 are needed

[1] Deadlock analysis for railway systems via our approach is studied in [1].

[2] If a track section is occupied by another train, the movement of the train wanting to enter may be delayed (blocked) but no deadlock can occur.

for each train pair to ensure synchronization between the trains. The same strategy is used for connections.

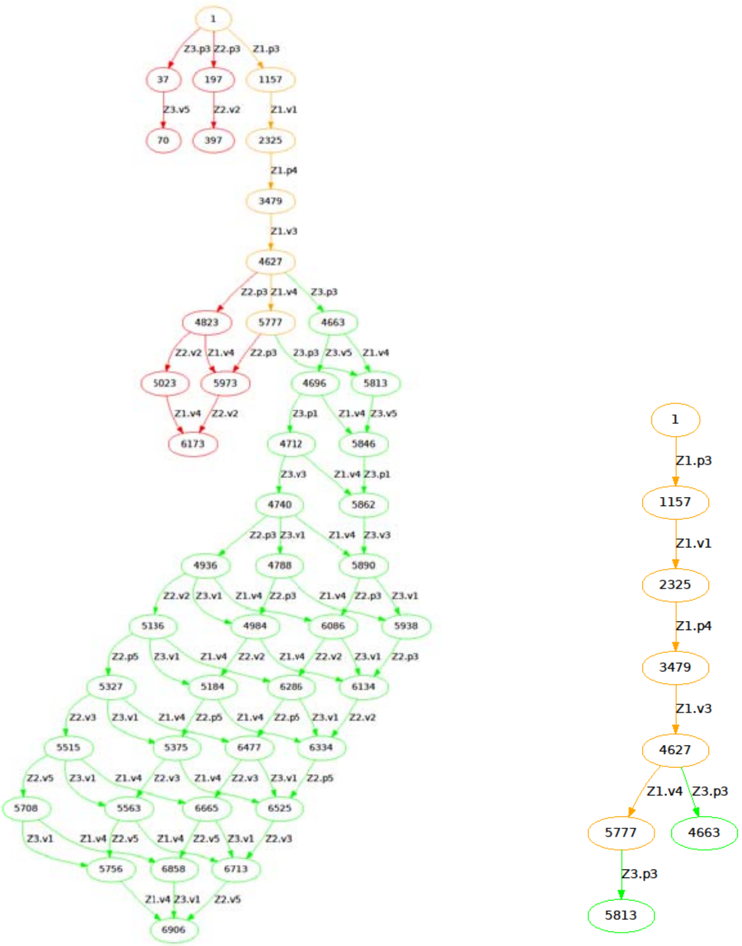

Figure 3. A simple example: Resulting graph and reduced graph

As a result L_1 will go to track section 3 and waits until L_2 has passed track section 7. L_3 will go to track section 4 and waits until L_4 has passed track section 2 but L_4 will have to wait until L_2 has passed section 7.

Trains	Routes
L_1	$p_3, v_2, p_{12}, p_7, p_8, v_3, v_7, p_{10}, v_8, p_{11}, v_{10}, v_{11}$
L_2	$p_2, v_1, p_9, v_2, p_7, p_8, v_9, v_7, p_{10}, v_8, p_{12}, p_{11}, v_{10}, v_{11}$
L_3	$p_5, v_{10}, p_4, v_5, v_{13}, p_2, v_4, p_1, v_2, v_1$
L_4	$p_{10}, v_{11}, p_5, v_{10}, p_6, v_5, p_7, p_9, v_6, v_7, p_2, v_9, p_{13}, p_1, v_2, v_1$

Table 2. Connections and overtaking: Routes

EXTRA-LONG TRAINS

Sometimes it happens that a train is much longer than a track section within a railway station. As a result such extra-long trains can’t be used for crossings. Figure 4 illustrates such an example.

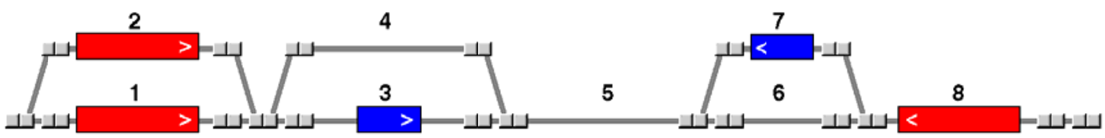

Figure 4. Extra-long trains: Railway system

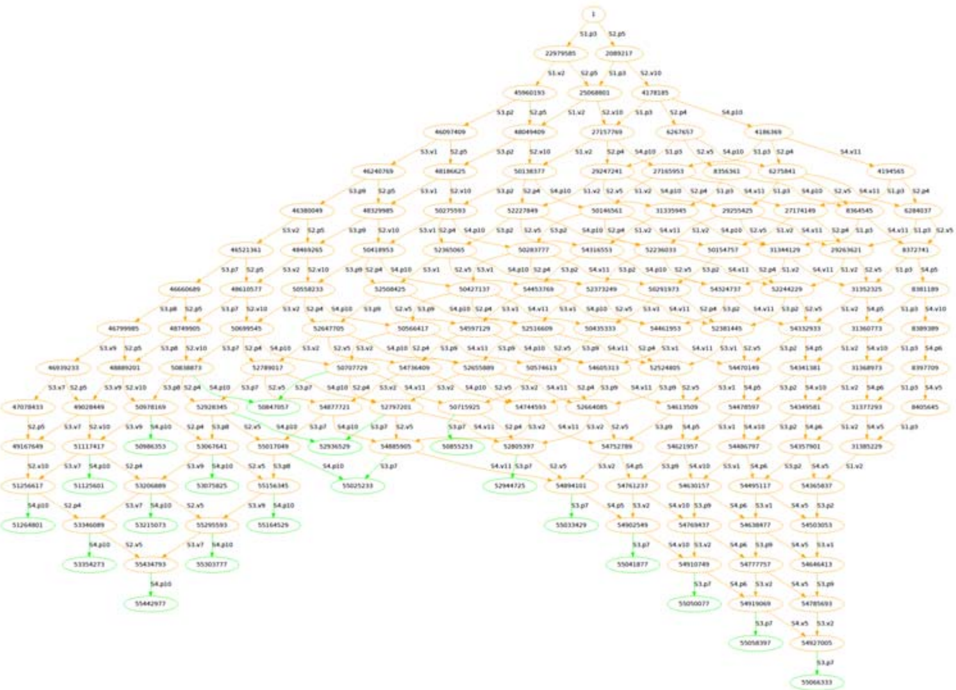

Figure 5. Connections and overtaking: Reduced graph

By applying Kronecker algebra 16,384,000 possible states are produced and after reduction only 93 nodes remain. The resulting graph depicts all possible sequences of train movements which will not result in a deadlock (Figure 6).

The first step to solve the situation above is that one of the short trains has to move first to the next section. Then the long trains can start their journey. A similar example with four trains und thus less complexity can be found in [7].

Figure 6. Extra-long trains: Reduced graph

ENERGY ANALYSIS

By applying Kronecker algebra shared resources and their access can be modelled. In the previous section, some examples with track sections as shared resources are explained; with the restriction that only one train can enter a track section. The model can be extended to use a shared resource which can be used by more than one train simultaneously. In particular, so-called counting semaphores [3] can be used to model discrete power resources. For example a counting semaphore of size four allows four p-operations before it blocks.

If we quantise energy into standardised packages e.g. 1 MWh, we can model a power station or substation capable of producing e.g. 20 MW by a counting semaphore of size 20. Of course a more fine-grained approach is viable, too. So we may quantise energy into 100 kWh or even 10 kWh steps. We assume that it is known a priori how much energy each train needs for each track section. Now in our model, a train acquires amounts of power from the power station before it enters a certain track section by issuing exactly the number of p-operations that correspond to the amount of power it will need. On leaving the track section, the train will issue the same number of v-operations to release its power needs.

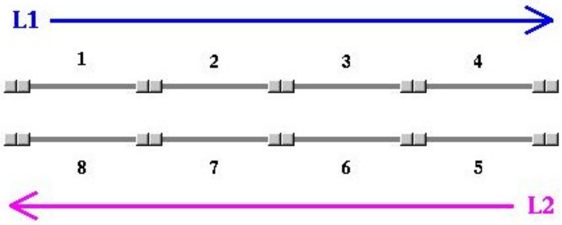

Figure 7. Energy analysis: Railway system

We give an example consisting of a simple railroad system with two trains which is illustrated in Figure 7, where train L_1 needs two energy units for track section 1 and 2 and one for 3 and 4. L_2 needs two units for track section 5 and 6 and one for 7 and 8. The routes including the time values of each track section for the two trains and the resulting travel time can be found in Table 3, where p_9 models the reservation of one single energy unit and v_9 its release. Figure 8 (left) illustrates the resulting graphs with 4 energy units available and Figure 8 (right) shows the result with 3 energy units available

Trains	Routes	Travel Time	
		3 units	4 units
L_1	p_9 (0), p_9 (0), p_2 (0), v_1 (4), p_3 (0), v_2 (6), v_9 (0), p_4 (0), v_3 (3), v_4 (4), v_9 (0)	24	26
L_2	p_9 (0), p_9 (0), p_6 (0), v_5 (3), p_7 (0), v_6 (4), v_9 (0), p_8 (0), v_7 (5), v_8 (4), v_9 (0)	17	16

Table 3. Energy analysis: Routes and travel time

CONCLUSION

We have presented some practical examples for the application of Kronecker algebra where we model movements of trains within a railway network and access to a shared resource (e.g. track sections, available energy capacity). This approach can be used to model complex railway systems including aspects of being deadlock-free, being conflict-free, and being minimal in terms of energy demand. The theoretical background of Kronecker algebra can be found in some preliminary papers [1, 2, and 6].

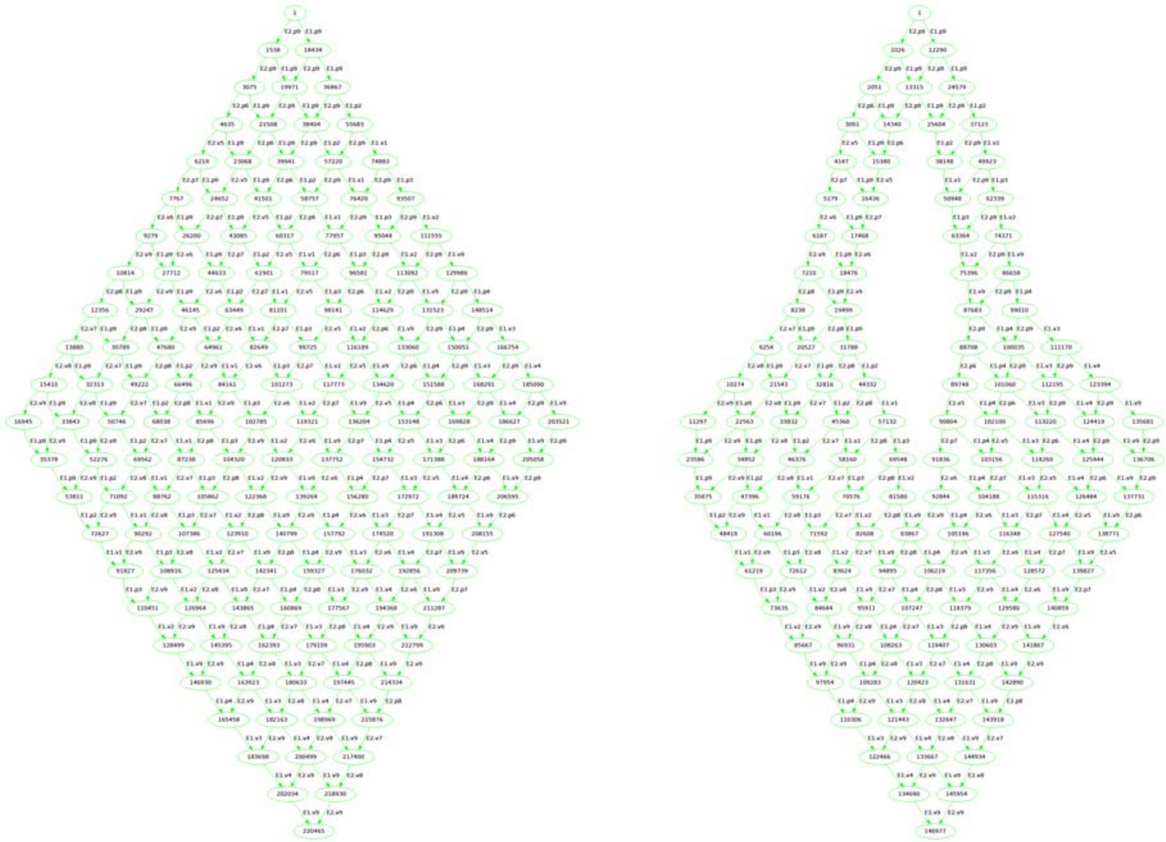

Figure 8. Energy analysis: Resulting graphs

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АЛГЕБРА НА КРОНЕКЕР КАТО РАМКА ЗА ОПТИМИЗИРАНЕ НА ЕКСПЛОАТАЦИЯТА НА ЖЕЛЕЗНИЦИТЕ

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Ключови думи: *Алгебра на Кронекер, анализ на времето за пътуване, енергиен анализ, анализ на безизходно положение*

Резюме: *Алгебрата на Кронекер се състои от произведение на Кронекер и сбор на Кронекер. Тя може да се използва за моделиране на системи, състоящи се от няколко участника и редица ограничени ресурси. В частност, тя може да се използва да моделира железопътна система, състояща се от влакове, техните маршрути в системата и коловозните секции, изграждащи железопътната инфраструктура. В тази статия ще покажем някои приложения на алгебрата на Кронекер в областта на железопътния транспорт. В частност, разглеждаме: анализ на безизходното положение [1], анализ на времето за пътуване [2] и енергиен анализ. Интегрирането на трите вида анализи в един единствен Кронекер-базиран анализ може да се реализира много ефективно. Реализирането на нашите алгоритми е ефективно, както по време, така и по отношение на необходимия обем памет. Операциите в алгебрата на Кронекер лесно могат да бъдат паралелизирани и така предложените алгоритми да се приложат върху многоядрена компютърна архитектура. В допълнение, нашето изследване показва, че добавянето на ограничения към задачата (връзки, изпреварване, ...) подобрява времето за изпълнение. В действителност по-трудна задача се решава по-лесно.*