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MODELLING AND SIMULATION OF TRANSPORT MACHINES WORKING CONDITIONS BY USING OF AUTOREGRESSIVE MODELS

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Key words : *Time Serie, Modelling, Simulation, Real Stationary and Non-stationary Processes, Autoregressive Moving Average Models, Working Conditions, Transport machines.*
Abstract: *Paper is dealing with a new method of modelling and simulation of stationary and non-stationary processes. The method is based on description of processes by **A**uto**r**egressive **M**oving **A**verage (ARMA) Models and their adaptive modifications. This paper presents possible application of ARMA models for modelling and simulation the real working process of typical working conditions of a truck.*

INTRODUCTION

It is necessary for complex determination of typical working conditions of any mechanical structure to get a survey about large amount of functioning factors – characteristics of working and to determine appropriate relationships among them. Therefore one need an adequate description of working conditions and basic themes of tested structure working based on simplified models.

One of available arts of computer modelling and simulation of working stochastic processes is well known theory of time series and its apparatus of autoregressive modelling.

1. AUTOREGRESSIVE MODELS

First logical technique for identification is to derive the mathematical model from system behaviour and directly from nature of its physical characteristics or as the dependence on input factors influencing this behaviour.

Mathematical representations of such relations are often stated in form of differential equations systems. Solutions of such systems whether numerical or exact is very time-consuming therefore not usable for the real time control.

An another problem with this possible way of identification is: some of the influencing factors are not well known and some can not be quantified. Another possible technique for system identification is to derive the mathematical model from internal interdependencies of given process only.

Clear Autoregressive (AR) model is suitable mathematical model for discrete non-stationary signal (Fig.1) that is considered to be visible representation of system dynamics [1].

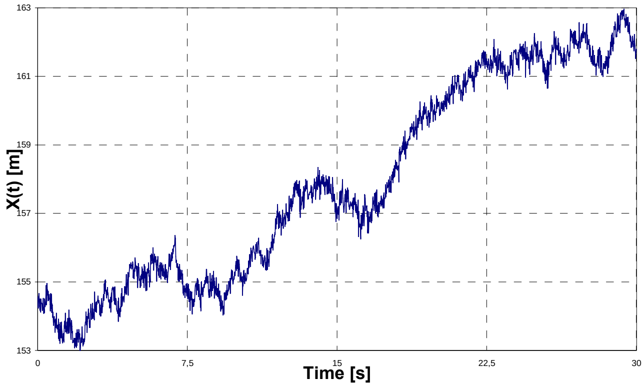


Fig.1 Example of real non-stationary process

Dependence of X (t) values on previous (in time) values X (t-1) is pictured in Fig. 2.

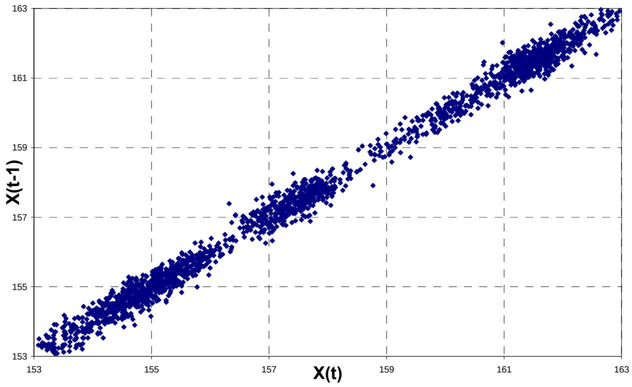


Fig.2 Interdependence X (t)=f {X (t-1)}

Linear regression can be clearly seen. This relation can be described by simple 1st-order Autoregressive model (1) in form [1]

(1) $$X(t) = a_1.X(t-1) + \varepsilon_t$$

where t is the time, $X(t)$ is discrete signal value in time t, $X(t-1)$ is discrete signal value in *time (t-1)*, a_1 is coefficient of Autoregressive model and ε_t is error resulting from model imperfections. This simplest model can be extended to universal n-th order Autoregressive model AR (n)

(2) $$X(t) = a_1.X(t-1) + a_2.X(t-2) + \ldots + a_n.X(t-n) + \varepsilon_t$$

where n is the model order, $X(t), X(t-1), X(t-2) \ldots, X(t-n)$ are discrete signal values in time $t, t-1, t-2 \ldots t-n$, $a_1, a_2, \ldots, a_n$ are coefficients of Autoregressive model and ε_t is error resulting from model imperfections [1,2,3]. If $X(t)$ depends not only on preceding values $X(t-1), X(t-2) \ldots X(t-n)$ but also on the values of preceding errors ε_{t-}

$_1$, ε_{t-2}... simple Autoregressive models are extended to Autoregressive moving average models (ARMA). The ARMA models can cover more complex character of process interdependencies and their coefficients relate closely to physical principles of observed process.

Universal ARMA model of n-th order in Autoregressive part and (n-1)-th order in moving average part is given by equation (3).

$$
\begin{aligned}
X(t) &= a_1.X(t-1) + a_2.X(t-2) + \ldots + a_n.X(t-n) + \\
&+ b_1.\varepsilon_{t-1} + b_2.\varepsilon_{t-2} + \ldots + b_{n-1}.\varepsilon_{t-n+1} + \varepsilon_t
\end{aligned} \tag{3}
$$

where $X(t), X(t-1), X(t-2)$ ... $X(t-n)$ are the discrete signal value in time $t, t-1, t-2$ ... $t-n$, values ε_t, ε_{t-1}, ε_{t-2} .. ε_{t-n+1} is error in the time $t, t-1, t-2$... $t-n+1$, values a_1, a_2 ... a_n are coefficients of Autoregressive part of model, b_1, b_2 ... b_n are coefficients of moving average part of model.

Main problem in identification, modelling and simulation by Autoregressive models is finding coefficients of AR and MA parts and determination of adequate order of the model. Coefficients of AR model can be simply found using least square method (LSM). For universal ARMA models non-linear LSM must be used. Both methods use matrix calculations for finding needed coefficients, which are very time-consuming and therefore not usable for on-line process, control or identification and they also can not be used for modelling of non-stationary time-varying process [2, 3, 4].

2. ALGORITHM OF ADAPTIVE AR MODELS

Because of the above stated reasons, procedure based on theory of adaptive and self-learning systems is used for describing system behaviour in real time. Algorithm for adaptive modelling [2] is based on a gradient method (steepest descent method) and can also be used for non-stationary processes. Model is able to adapt itself to the changes in process character.

It is supposed that n-th order Autoregressive model (2) is at any given time defined by the vector of its coefficients:

$$a(k) = [a_1(k), a_2(k) \ldots a_n(k)]^T \tag{4}$$

Using the steepest descent method, point of least squares $\Sigma\, \varepsilon_t^2$ is searched for. Search begins with an initial guess as to where the minimum point of $\Sigma\, \varepsilon_t^2$ may be. Minimal sum of squares S is in the point where

$$\frac{\partial S}{\partial a_k} = \frac{\partial}{\partial a_k}\left(\sum \varepsilon_t^{\,2}\right) = 0 \tag{5}$$

The updated values of AR model coefficients are obtained from [6,8]

$$\mathbf{a}(k+1) = \mathbf{a}(k) + \eta.\frac{\partial S}{\partial a} \tag{6}$$

where

$$\frac{\partial S}{\partial a} = -2\left[\varepsilon_{ft}.\mathbf{X}^T(k-1)\right] \tag{7}$$

is the gradient direction and positive value of η in equation (6) scales the amount of readjustment of the model coefficients in one time step. Then, the iterative corrections of coefficients are

(8) $$\mathbf{a}(k+1) = \mathbf{a}(k) + \mu.\left[e_t.\mathbf{X}^T(k-1)\right].$$

3. ADAPTIVE ARMA MODELS

In [2] adaptive AR models were extended to include also MA part to *Adaptive Autoregressive models with moving average*. To achieve this, vector of moving average coefficients must be considered

(9) $$\mathbf{b}(k) = [b_1(k), b_2(k)\ldots b_n(k)]^T$$

Same procedure as for vector of AR coefficients was used to derive formula (10) for iterative corrections' calculations of MA part coefficients

(10) $$\mathbf{b}(k+1) = \mathbf{b}(k) + \mu.\left[\varepsilon_t.\varepsilon^T(k-1)\right]$$

where ε_t is error from the last iterative step and $\varepsilon\,(k\text{-}1)$ is vector of preceding errors

(11) $$\boldsymbol{\varepsilon}(k-1) = \left[\varepsilon_{k-1}, \varepsilon_{k-2}, \ldots \varepsilon_{k-n+1}\right]^T$$

Another problem arises when deciding value of convergence constant. It influences converging speed of algorithm and also its sensitivity to random or systematic changes in process environment character. Procedure for calculating μ constant, based on experimental work [3, 5] was presented for use in area of adaptive control

(12) $$\mu_k = \varphi + \beta.C_k$$
$$C_k = \left(1 - \frac{1}{\alpha}\right)C_{k-1} + \frac{1}{\alpha}.\varepsilon_t^{\,2}$$

where α is constant describing system memory, it influences model sensitivity to random process changes, β is constant characterising system dynamics and φ is constant for correction of numeric calculation errors. Actual values of these constants can be chosen so the model sensitivity to stochastic events and response speed to process character changes are as required.

4. COMPUTER MODELLING AND SIMULATION OF STOCHASTIC PROCESSES

Shown theoretical algorithms was used by developing of program ARMA-FIND, this was developed on authors department [3]. This software is a 32–bit application made in developing surroundings of DELPHI and working in the operating system Windows XP or NT.

It contains users menu, which apart from basic functions with file, configurations, work with windows and help functions contains two submenus – submenu of "*Simulation*" and submenu of "*Identification*" (Fig.3).

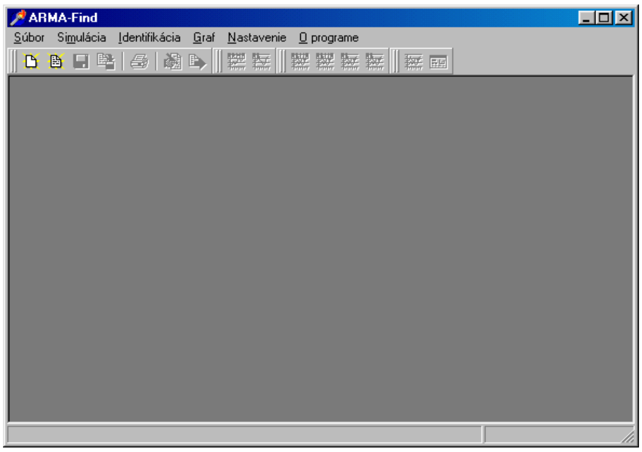


Fig.3 The environment of ARMA-FIND

Item "*Simulation*" enables adjustment and conversion of incompatible input files of time series to compatible ones and simulation (generation) of time series basing on given AR or ARMA models order and parameters with possibilities of mean and dispersion selection of simulated series.

The heart of the program is submenu "*Identification*", by means, which is possible to make selection of the identification method and way of chosen time series, whereupon it is possible to use either adaptive algorithm of time series identification or make identification using non-linear least squares method. Identification by means of higher presented non-linear (respectively for AR models – linear) least square method is available in item *Identification* and its sub-menu *NLINLS*. Here are four options. First two- *Model AR – after orders* and *Model AR- complete calculation* give as results of identification AR model, described by (2).

Next item – *Model ARMA- after orders* gives coefficients of beforehand selected order of ARMA (n, n-1) models determination. It means, that we it is necessary beforehand to determine required order (known number of coefficients) of autoregressive part - a_k and moving average part - b_k which principally determine number of former values the calculated value depends on.

The initial guess is of coefficients of ARMA (n, n-1) model. To coefficients of moving average part are assigned value of zero and coefficients of autoregressive part obtained by application of linear least square method. Simultaneously the sum of squares of deviation value expressing deviation of theoretical model from real model is calculated. Then the proper iterative calculation follows, which outputs are the coefficients of model.

Fig.4 Results of an optimal ARMA model determination

The last important item is *Model ARMA- complete calculation*, which aim is to find an optimal ARMA (n, n-1) model. This model is the best describe of stochastic system, which output is a time series of a real processes.

Because, in most cases we don't know optimal order of model, beforehand it is necessary to determine by an iterative procedure an optimal order of model for description of given system (Fig.4). An algorithm of optimum autoregressive model determination was published in [2, 3, 5].

5. MODELLING AND SIMULATION OF REAL WORKING CONDITIONS OF TRANSPORT MACHINES

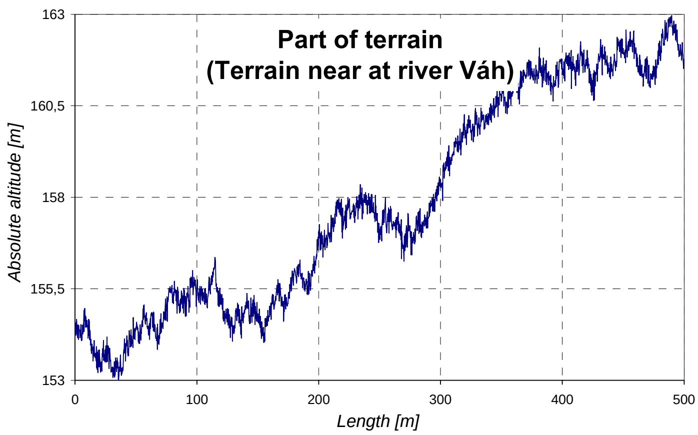
Because the chosen mathematical apparatus allows getting of models based of physical principles of researched processes which are developed from recorded courses of examined values it was necessary to obtain experimental data, which characterise typical working conditions [3,9].

Therefore a tot of experimental measurements was done which describe typical working conditions of trucks (unevenness of road surface, segmentation of terrain, typical working speed during different types of working modes etc.). Longitudinal and transverse unevenness had a dominant position as a typical of transport machines working conditions generally.

Therefore the experimental measurement was realised which consisted of following steps:

- finding of roads and their parts representing individual categories of roadways from point of view of quality of their coverage,
- selection of actual parts of chosen length,
- own measurement using suitable equipment,
- filtering of trends and rough unevenness with suitable software package (STATGRAPHICS) and
- evaluation of chosen parts by means of selected dynamic methods [2] and verification of their classification for relevant categories of quality.

As a result of former steps there were obtained courses of longitudinal unevenness together from 6 sectors (5 qualitative categories of road and one sector of terrain). Because of limited scope of this paper just causes of 5th category of road and the values of terrain are presented on Fig.5. The values of evaluating parameters "C" and "IRI" are presented in Tab.1.



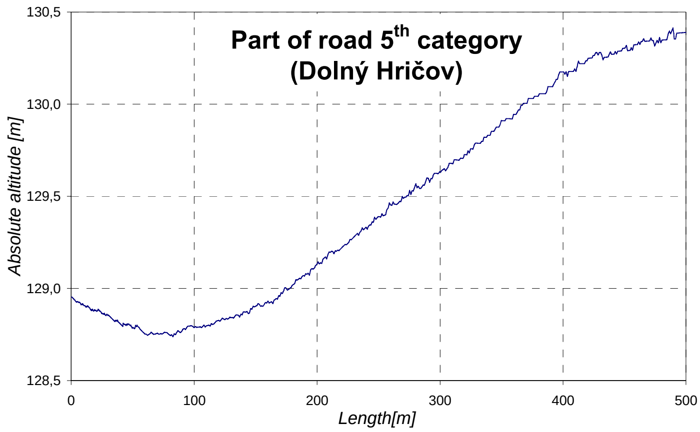


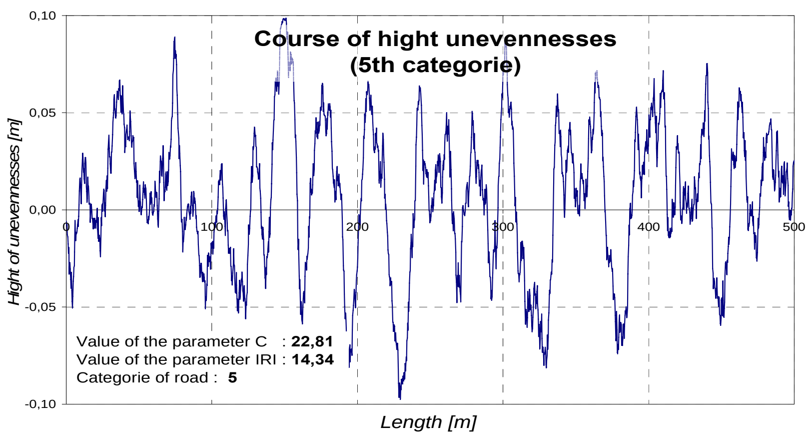
Fig.5 Courses of absolute high unevenness of 5th category road and terrain

If was determined that from obtained parameters of “C” and “IRI” chosen sections really are represented chosen qualitative categories and there for they can be judged as representative of qualitative different surfaces.

Location	"C"	"IRI"	Category
Žilina	0,85	1,61	I
Brodno	2,31	2,62	II
Čičmany	3,92	4,28	III
Varín	13,85	8,54	IV
Dolný Hričov	22,81	14,37	V
Near of river Váh	42,05	22,74	Terrain

Tab.1 Calculated values of „C“ and „IRI“ parameters for chosen sections of road

It was necessary from of point of view of further practical application of experimentally obtained models to separate the random part of unevenness from its trend. It was used the software pack STATGRAPHICS which allows directly selection of trends and seasonally from recorded values. To demonstrate this are on Fig.6 shown courses of stochastic parts of profile unevenness for series from Fig.5.



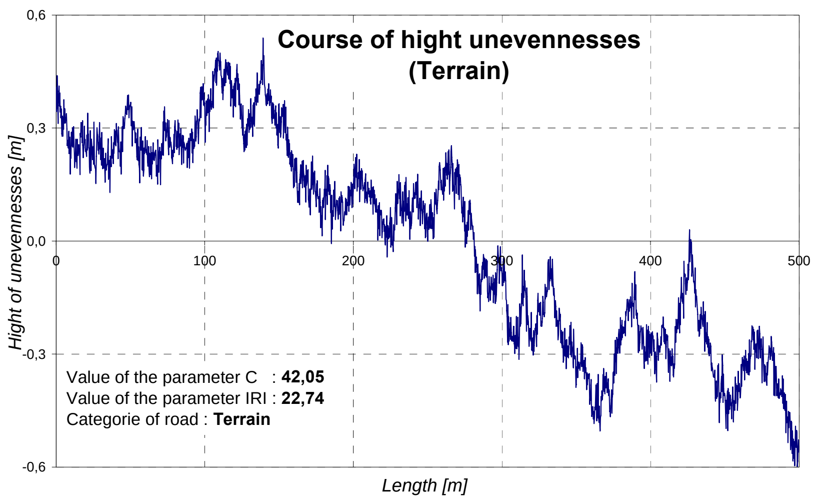


Fig.6 Random components courses of altitude of unevenness 5th category road and terrain

It was used the ARMA-FIND software to get real and simulated data. Practically it means optimal autoregressive model and its parameter determination (order and coefficients of model) for all inputs of discrete values of altitude unevenness to selected sectors of roads. Fig. 7 presents the window after activated menu item "*ARMA-Complete Calculation*". Finding parameters of adequate models for sector of 5^{th} category road and sector of terrain are presented in Tab. 2.

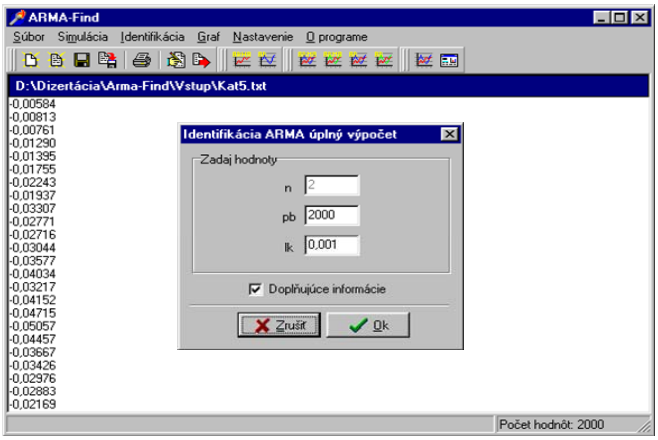


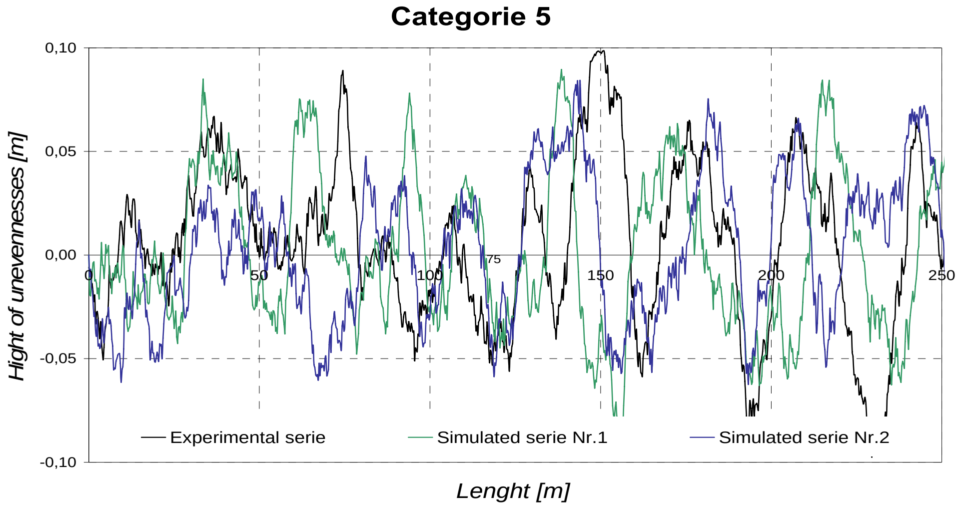
Fig. 7 Window of menu item „ARMA–Complete calculation"

Cat.	Coefficients of optimal ARMA model		Sum of squares
5.	a(1) = 0,623808 a(2) = 0,095081 a(3) = 0,592155 a(4) = -0,050585 a(5) = 0,518931 a(6) = -0,797272	b(1) = -0,391528 b(2) = -0,335376 b(3) = 0,197765 b(4) = 0,196800 b(5) = 0,777805	SSC = 0,063145
Terrain	a(1) = 0,867707 a(2) = 0,186025 a(3) = -0,057025 a(4) = 0,002244	b(1) = 0,542767 b(2) = 0,046748 b(3) = -0,047127	SSC = 2,545889

Tab.2 Finding parameters of adequate models for sectors of 5^{th} category and terrain

For computer simulation of working condition characteristics available especially on computer stress analysis of critically parts of trucks, on influence analyses to selected working properties of transport machines and for automatic control of various load machines too was used the finding adequate models. These models were used for generation of new, statistically adequate, series of analysing process values.

For this purpose created program ARMA-FIND contains the item "*Simulation*" which was successfully applied by generation of new courses altitude of unevenness of reference sectors. For presented parts of 5th category and terrain there are two new simulated courses and initial experimental course presents on Fig.8.



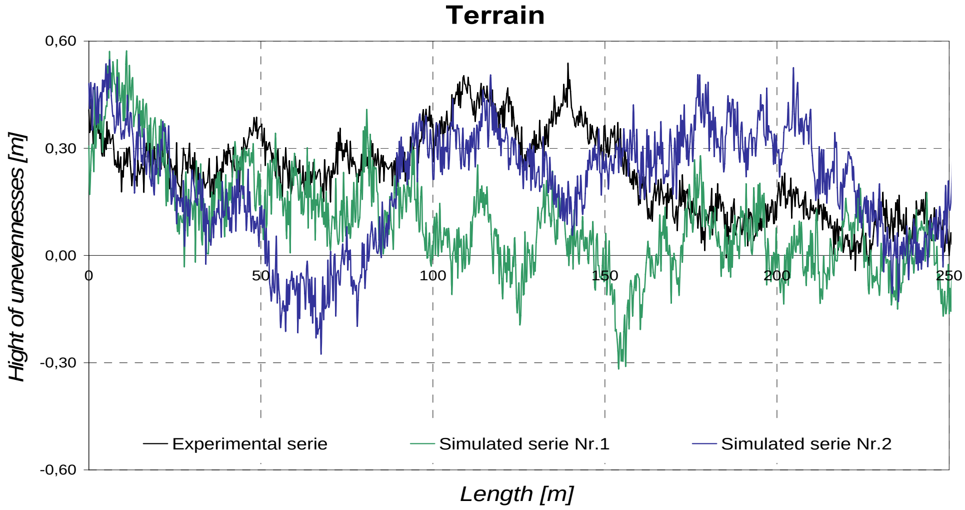


Fig.8 The courses of simulated series of analysing sectors of 5th category road and terrain

During verifying their physical adequacies were once more used dynamic methods of evaluation sectors of roads from point of view longitudinal unevenness.

It was found and verified that determined mathematical models very good describe both analysed characteristics. Determined differences were in both of tested factors neglect able and statistically non-significant.

CONCLUSIONS

It was not possible from point of view of determined scope of paper to show all of obtained outputs from realised calculations and experiment (mainly made on computers). Therefore they are presented just some of them in this paper. Based on above-mentioned results it is possible to state that chosen theoretical apparatus and methodology of autoregressive modelling (namely adaptive one). Is a suitable tool for modelling and simulation of different working condition factors.

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