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**RESEARCH OF THE CHANGE RATE OF UNCOMPENSATED  
CENTRIFUGAL ACCELERATION IN SPECIFIC POINTS OF SOME TYPES  
OF TRANSITION CURVES**

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***Abstract:** Research of the change rate of uncompensated centrifugal acceleration of some types of transition curves has been made. And for that matter linear approximations and calculus methods have been used. The results have been compared with the limit values in the Bulgarian Regulations.*

The uncompensated centrifugal acceleration, caused by the centrifugal force, has the biggest dynamic impact on safety, security and comfort of the passengers during the movement of train along the circle and transition curve. Because of that there is a need to reduce the effect of this acceleration. One way is to make cant - difference in elevation (height) between the inner and outer rail, but here appears new force – centripetal force, caused by the inclination of wagon to the center of the curve. This force in some cases has also unfavourable effect – in case of emergency if the train stops in the circle curve.

The researches of the impact of the uncompensated centrifugal acceleration on passengers show that values from  $\alpha_h=0,15 \text{ m/s}^2$  can hardly be felt. Up to this value of the acceleration the trains are moving tranquilly, the passengers are quite comfortable the goods are lying steadily and the rails are wearing out evenly. By increase of the lateral accelerations to  $\alpha_h=0,31 \text{ m/s}^2$  the passengers are beginning to feel some discomfort, but the movement of the train is still steady according to all indicators. When the lateral acceleration is reaching from  $0,6$  to  $0,7 \text{ m/s}^2$  the movement of the train becomes unsteady, the goods are beginning to come out of their places, the rails are being loaded unevenly and therefore they are being wearied out unevenly. The passengers are losing their feel for comfort and become nervous, although the safety movement of the train is not yet in danger. By lateral accelerations with measures  $1,3 - 1,4 \text{ m/s}^2$ , the movement of the train become critical, the goods are beginning to glide, the movement is extremely unstable, the wearing out of the rails reaches a magnitude, that the rails might get destroyed or other elements of the superstructure can be damaged or even it may comes to accidents with serious consequences.

According to the Bulgarian Regulation №55 for design and building of rail ways, stations, crossings and other elements of the rail road infrastructure, in Bulgaria is admitted  $0,65 \text{ m/s}^2$  as the maximal value of the centrifugal acceleration and the maximal centripetal acceleration by emergency train stop in curve between two stations or in station in curve

without pat form -  $0,98 \text{ m/s}^2$  by maximal admitted cant 150 mm (railroads first and second category) and  $1,05 \text{ m/s}^2$  by cant of 160 mm (speedways). These limitary values will be further used for comparison of the results.

As we know the uncompensated centrifugal acceleration in circular curve can be estimated with the following formula:

$$\alpha_n = \frac{V^2}{13.R} - \frac{h}{153}, (\text{m/s}^2) \quad (1)$$

where

V is the speed of the train in km/h;

R is the radius of the circular curve in m;

h is the cant (superelevation between the two rails) in mm;

In the transition curve the radius and the cant are changing constantly. According to Bulgarian Regulation №55, the ramp of the cants gradually change is a straight with a slope  $K=10.V_{\text{max}}$ . By new road design the beginning of the ramp is identical with the transition curve begin (TCB or HIIK, fig. 2) and the end of the ramp is identical with the transition curve end (TCE or KIIK). Therefore when the cant and the length of the transition curve are calculated in advance through linear interpolation, the value of the cant in random point of the transition curve can be estimated.

The radius in the transition curve is changing its value from  $\rho=\infty$  in TCB to  $\rho=R$  in TCE. In the interval from TCB to TCE  $\rho$  is changing from  $+\infty$  to  $R$  constantly, lightly and monotonously. The radius in one random point of the transition curve can be estimated through linear approximation and calculus method. After the estimation of the unknown values through the formula (1) the uncompensated centrifugal acceleration can be calculated for a random point of the transition curve.

In this comparative analysis it will be considered different types of transition curves: cubic parabola, clothoid, Bloss's curve and two variants of Schramm's curve in speed interval from 100 to 200 km/h. For the purpose to be shown the most unfavorable results, it will be worked here with the minimal admitted radiuses according to Bulgarian Regulation №55 for relevant speeds: for 100km/h the rate of the minimal radius R is 500 m, for 130 km/h  $R=800\text{m}$ , for 160 km/h  $R=1500\text{m}$  and for 200km/h  $R=2500\text{m}$ .

In the beginning and only for the needs of the comparative analysis it will be considered one more unfavorable case for the speed of 200km/h and  $R=2000\text{m}$  (in this case it becomes maximal length of the transition curve and maximal cant of 160mm). For the need of easiest calculating it was compounded an MS Office Excel algorithm and the results are shown in the next *table 1* below.

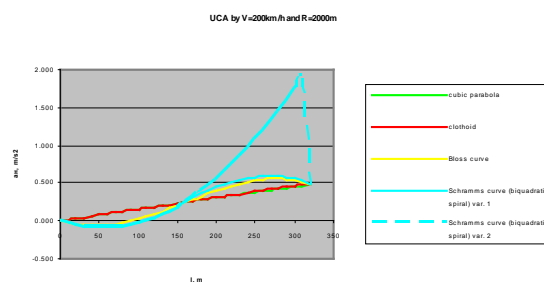


fig. 1

The table makes it visible, that the change of the lateral uncompensated centrifugal acceleration (UCA) by cubic parabola and clothoide is linear along the length of the transition curve and it reaches its maximal rate in the circular curve, which in this case is  $0,49 \text{ m/s}^2$ , while the graph data of these two types of transition curves are almost completely identical. Therefore for the Bulgarian conditions, even by this most unfavorable case (from the view point of the transition curve's length), the two types of transition curves have identical, even almost completely identical geometric as well as dynamic characteristics. In contrast to them, by the other two types of transition curves, the situation is different. As it is shown on the *table 1*, the Blos's curve and the first variant of the Schramm's curve have curvilinear graph data of change of the accelerations. And there are extremum points of the two curves, which positive values exceed the value of the acceleration in the circular curve. This is due to the fact, that the ramp of the cant is rectilinear. If the ramp of the cant were realized according to the theory, shown here in the fig. 2 below, with a type, similar to the lineament of the graph data of the two curves, then they would have a linear change of the increase of the acceleration or again a curvilinear change, but with much smaller extremum points.

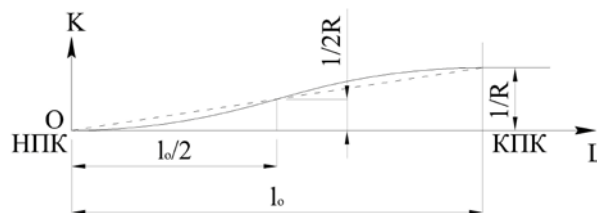


Fig. 2

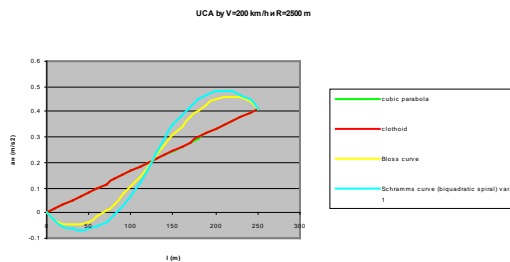
Also it is evident, that in the first section of the transition curve the centrifugal acceleration has negative values, which means that a centripetal acceleration appears. This is again due to the rectilinear lineament of the ramp of the cant, because the curvature of these two types of transition curves is changing slowly than the increase of the ramp of the cant (the more the equation of the transition curve is involving higher power, the more the transition curve in its first section is getting near to a straight line) and actually the acceleration by taking up the outside wheel in this section is higher than the centrifugal acceleration.

In geometrical way variant 2 of the Schramm's curve was estimated as the coordinates of detailed points were calculated with the formulas for the first half of the curve, but along the whole length of the transition curve. The result is a type of a curve, which has a first section with smaller curvature in comparison with a cubic parabola and clothoide, but in the transition curve end /TCE/ it has an equal offset -  $p$  with them (in contrast to the Blos's curve and variant 1 of the Schramm's curve, by which the movement aside is smaller.) The answer of the question, if such type of a curve is usable in the railways as a transition curve, gives us figure (1). There is clearly shown, that in the second section of the transition curve the lateral uncompensated centrifugal accelerations are increasing to such an extent, that exceed the admissible levels quite a lot ( $0,65 \text{ m/s}^2$ ), after that in the TCE follows a sudden dynamic hit in order to reach their values in the circular curve. This result presents, that in order to compensate the first straighter in geometrical sense section of the transition curve, in the end section the radius intensive decreases and reaches values smaller than the radius in the circular curve, which is presented below in the *table 1* with the calculations as follows:

**Table 1**

$\alpha_n$	h	R	$\Delta y$	$\Delta x$	dy/dx	$\Delta m$	$\Delta x'$	$d^2y/dx^2$
0.000	0 <sup>∞</sup>							
-0.03	5	877714.29	0.000	10.000	6.51042E-06	0.0000113932	10	1.13932E-06
-0.05	10	245760.00	0.000	10.000	3.25521E-05	4.06901E-05	10	4.06901E-06
-0.07	15	111709.09	0.001	10.000	9.76563E-05	8.95182E-05	10	8.95182E-06
-0.08	20	63340.20	0.001	10.000	0.000221354	0.000157878	10	1.57878E-05
-0.09	25	40688.74	0.003	10.000	0.000423177	0.000245768	10	2.45768E-05
-0.09	30	28313.36	0.005	10.000	0.000722656	0.00035319	10	3.5319E-05
-0.08	35	20827.10	0.009	10.000	0.001139324	0.000480144	10	4.80145E-05
-0.07	39.999	15958.42	0.014	10.000	0.001692711	0.00062663	10	6.26631E-05
-0.05	44.999	12615.98	0.020	10.000	0.002402351	0.00079265	10	7.92653E-05
-0.03	49.999	10222.90	0.028	10.000	0.00328778	0.000978206	9.99	9.78212E-05
0.005	54.999	8451.08	0.038	10.000	0.004368534	0.001183302	9.99	0.000118331
0.041	59.999	7102.77	0.050	10.000	0.005664158	0.001407946	9.99	0.000140797
0.083	64.999	6053.04	0.064	10.000	0.007194205	0.001652147	9.99	0.000165219
0.132	69.99	5219.83	0.080	10.000	0.00897824	0.001915923	9.99	0.0001916
0.186	74.999	4547.46	0.099	10.000	0.01103585	0.002199297	9.99	0.000219943
0.247	79.998	3997.03	0.121	9.999	0.013386651	0.002502304	9.99	0.000250253
0.313	84.998	3540.74	0.146	9.999	0.016050297	0.002824989	9.99	0.000282536
0.386	89.997	3158.28	0.175	9.998	0.019046502	0.00316742	9.99	0.0003168
0.465	94.996	2834.53	0.206	9.998	0.022395052	0.003529681	9.99	0.000353057
0.549	99.995	2558.05	0.242	9.997	0.026115837	0.003911888	9.99	0.000391323
0.64	104.99	2320.04	0.281	9.996	0.030228876	0.00431419	9.99	0.000431617
0.737	109.99	2113.68	0.324	9.995	0.034754361	0.004736778	9.99	0.000473965
0.84	114.98	1933.58	0.371	9.993	0.039712698	0.005179895	9.99	0.0005184
0.949	119.98	1775.44	0.423	9.991	0.045124569	0.005643847	9.98	0.000564962
1.064	124.97	1635.82	0.479	9.989	0.051010999	0.006129013	9.9	0.000613701
1.186	129.96	1511.92	0.540	9.985	0.057393436	0.006635862	9.98	0.000664682
1.313	134.96	1401.44	0.606	9.982	0.064293852	0.00716497	9.97	0.000717981
1.448	139.94	1302.49	0.677	9.977	0.071734855	0.007717036	9.97	0.000773692
1.588	144.93	1213.51	0.754	9.972	0.079739826	0.008292905	9.96	0.000831928
1.735	149.91	1133.17	0.836	9.965	0.088333075	0.008893593	9.96	0.000892827
1.889	154.89	1060.38	0.924	9.957	0.09754003	0.009520317	9.95	0.000956553
0.493	160	2000	1.018	9.948				

This is inadmissible and leads to dynamic break in the transition curve end (TCE), which is in breach of the theory for the transition curves and also in breach of the rules for fluent and safety movement of the trains. This type of transition curve can't be used in the railways and therefore it will not be considered further. Now the above described worst case scenario will be respectively elaborated. And the results are presented in fig. 3 for an admissible speed  $V = 200$  km/h and radius  $R=2500$  m:



**fig. 3**

Fig. 3 presents, that the values of the accelerations are in the admissible limits. The graph data for the change of the uncompensated centrifugal acceleration are analogous to the first case for the different types of transition curves.

The results are presented in fig. 4 for an admissible speed  $V = 160 \text{ km/h}$  and a minimal admissible radius  $R=1500 \text{ m}$ :

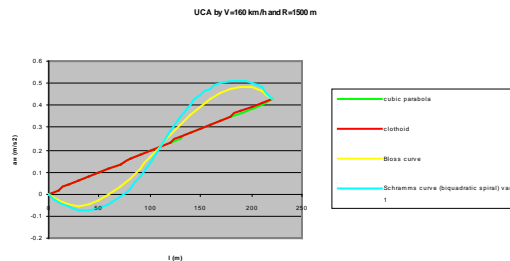


fig. 4

And also here it is presented, that the type of graph data are analogous to the above two cases and the difference is only in the size of the extremum values of the Bloss's curve and the Schramm's curve and also in the size of the acceleration in the transition curve. In analogous way the results for the two other cases are estimated –  $V= 100 \text{ km/h}$  and  $R=500 \text{ m}$  and  $V=130 \text{ km/h}$  and  $R=800 \text{ m}$ .

Fig. 5 presents the change of the maximum, middle and minimal values of the uncompensated centrifugal acceleration for the various minimal admissible values in the speed range 100 – 200 km/h.

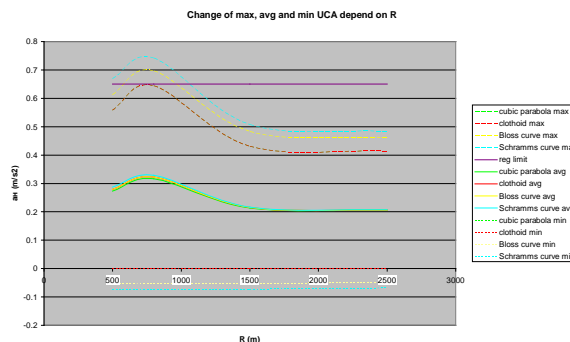


fig. 5

Fig. 5 shows, that the most unfavorable case is estimated by radius  $R=800\text{m}$  and speed  $V=130 \text{ km/h}$ . Here in the transition curve end (TCE)  $\alpha_H = 0,6446 \text{ m/s}^2$  whereas by the Bloss's curve and the Schramm's curve there is even an exceeding of the admissible value in the extremum point. The minimal values of the two types of transition curves are in the limit values of the admissible maximal centripetal acceleration. It will be more acceptable by a railway design with this project speed to avoid working with the minimal admissible radius.

The Schramm's curve has also such exceeding of the admissible value for radius  $R=500 \text{ m}$  and speed  $V=100\text{km/h}$ , while the Bloss's curve is situated in the limit. By higher speeds and radiuses the maximal values for each of the four types of transition curves are in the admissible limits.

From the estimated results it can be done the following conclusions:

In the speed range 100-200 km/h according to the rules of Bulgarian regulation №55, in dynamical and geometrical way the most appropriate for use are the cubic parabola and the clothoid as the differences with regard to the geometry and the arising dynamic forces of the cubic parabola and the clothoid (which increase linear till their maximal value in the transition curve end) are unsubstantially small. By the Blos's curve and the Schramm's curve the acceleration along the length of the transition curve are changing non-linear. Positive extremum values (higher values than these in the circular curve) and negative extremum values (centripetal acceleration) are estimated, which are due to the rectilinear lineament of the ramp of the cant. Variant 2 of the Schramm's curve, which has an offset equal with the cubic parabola and clothoid, is inapplicable for use in railway and in road design as a transition curve, because of violating the rules for smoothness and undisruptability of movement and there is an dynamic break in the transition curve end.

By project speeds up to 130 km/h it is recommended to avoid using the minimal admissible radius  $R=800$  m because of the fact that in this case it is reaching the limited value of the uncompensated centrifugal acceleration by clothoid and cubic parabola. And by Blos's curve and the Schramm's curve it is even over the limit whereby the purpose of the design of high speed railways is for the uncompensated centrifugal acceleration not to exceed  $0,5 \text{ m/s}^2$ .

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