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APPROXIMATE METHOD FOR DETERMINING THE UNCOMPENSATED CENTRIFUGAL ACCELERATION IN A TRANSITION CURVE

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Abstract: *A method for approximately determining the uncompensated centrifugal acceleration in transition curves has been proposed. The formula for calculation of cant (superelevation) in circle curve has been used, in which the radius of curvature and cant in a given point of transition curve are unknown. Linear Approximations, Calculus Methods and interpolation have been used for finding the mentioned values. A computer tables for easiness of the calculations have been created.*

Introduction

Many different dynamic forces appear during the movement of train along the railway. Some of them are more important for the system “rolling stock-railroad” than others. Some of them are external for the train – their impact point is the upper area of the rail head, and others are internal for the train and they appear between the locomotive and wagons or between their moving and stationary parts. In railway design only the external forces are considered. They can be divided in two major groups: horizontal and vertical. The vertical forces have an impact on the sub- and superstructure’s bearing capacity. The horizontal forces are longitudinal and transversal. The longitudinal horizontal forces (temperature, brake, traction) are important for the longitudinal stiffness of the railroad. The transversal horizontal forces have influence on the security and safety of the rail transport. They also have huge impact on passengers and freights.

The Method

The effect of these forces appears in the curves. Basic is centrifugal force. It depends on the speed of train and the radius of the curve. The impact of these forces is getting greater with the increasing of the speed and decreasing of the radius. For reducing this effect in curves helps the cant - the difference in elevation (height) between the two rails. Also putting transition curve between the straight and the curve has favourable effect. In the whole length of the transition curve the ramp of cant’s gradually change must be built.

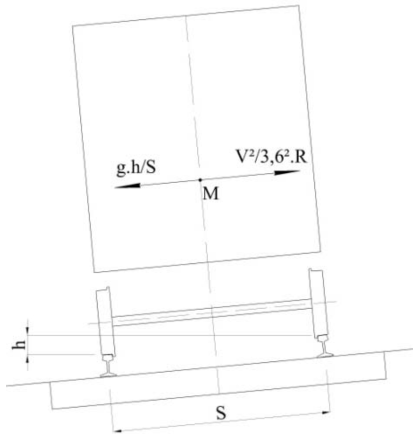

Figure 1. Transversal horizontal forces.

The centrifugal force causes transversal acceleration, pointing outward the center of the curve (Fig. 1). High levels of centrifugal acceleration can be dangerous and may lead to railhead wear or even derailment. Also they cause discomfort to the passengers.

The uncompensated centrifugal acceleration $\alpha_н$ is the difference between the acceleration $\alpha_{вн}$, which arises from the action of the centrifugal force and the acceleration $\alpha_{вт}$, caused by the centripetal force (it occurs due the cant) (Fig. 1):

$$\alpha_н = \alpha_{вн} - \alpha_{вт} = \frac{V^2}{3.6^2.R} - \frac{g.h}{S}, \text{ [m/s}^2\text{]}$$

Where V is the speed of train in [km/h]

R – radius of the curve in [m]

g – gravity of earth (9.81 m/s^2)

h - cant in [mm]

S – gauge in mm (in Bulgaria 1500 mm)

After the substitution and transformation it becomes:

$$\alpha_н = \frac{V^2}{13.R} - \frac{h}{153}, \text{ [m/s}^2\text{]} \qquad (1)$$

So this is the formula for determining the uncompensated centrifugal acceleration in circle curve.

However it can't be used to find the acceleration in a random point *i* of a transition curve. The transition curve has a variable radius of curvature and cant along its length. The value of the uncompensated centrifugal acceleration is different in a random point. In Eq. (1) there are three variables- the speed V_i, the radius R_i, and the cant h_i. The speed is the maximum speed between two stations and this speed is fixed and given. In the Bulgarian rail design standard the ramp of cants gradually change is a straight (Fig. 2) to alleviate the railroad building and maintenance. So the cant in a random point of the transition curve can be found by interpolation:

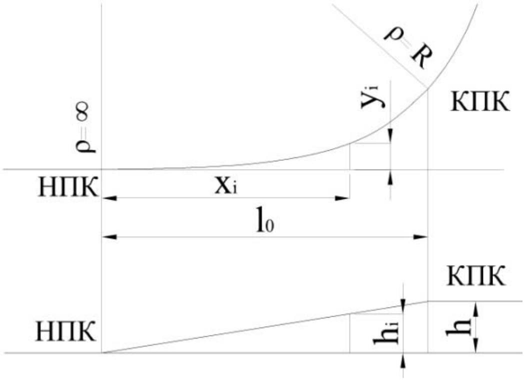

Figure 2. Transition curve and its ramp of cants gradually change.

$$\frac{h_i}{h} = \frac{x_i}{l_0} \Rightarrow h_i = \frac{x_i}{l_0}.h$$

More circumstantial is to find the radius ρ in this random point. The transition curve changes its radius from $\rho=\infty$ in НПК (transition curve begin) to $\rho=R$ in КПК (transition curve end) and in the interval between them changes its curvature constantly, lightly and monotonously. From the math it's known that the radius of curvature in a random point with coordinates x and y is:

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \qquad (2)$$

Main problem is determining of the first and second derivative of curvature. A relevant linear approximations and calculus methods can be used for that matter. To find the curvature in a random point i with coordinates $(x_i;y_i)$ of the transition curve (Fig. 3) the coordinates of two neighbour points $i-1(x_{i-1};y_{i-1})$ and $i+1$ $(x_{i+1};y_{i+1})$ must be known. Than an approximate calculation of the $\frac{dy}{dx}$ in the middle point i can be made in the following way:

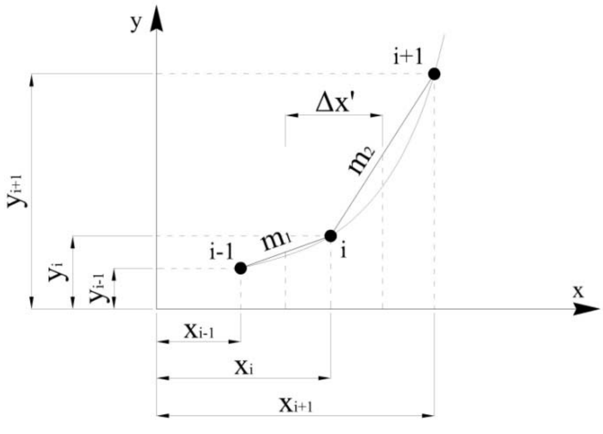

Figure 3. Three points coordinates.

The slope of the line joining $i-1$ and i is given by:

$$m_1 = \frac{\Delta y}{\Delta x} = \frac{y_i - y_{i-1}}{x_i - x_{i-1}}$$

The slope of the line joining *i* and *i+1* is given by:

$$m_2 = \frac{\Delta y}{\Delta x} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

The average value of these slopes gives a rough value for $\frac{dy}{dx}$:

$$\frac{dy}{dx} \approx \frac{m_1 + m_2}{2}$$

The partial (differential, specific) slope of the slope (i.e. the second derivative) will be the change in slope Δm divided by the change in x for the interval $\left(\frac{x_i - x_{i-1}}{2} + x_{i-1}\right);\left(\frac{x_{i+1} - x_i}{2} + x_i\right)$ (which are the mid-points of the 2 lines joining the 3 given points):

$$\Delta m = m_2 - m_1$$

and

$$\Delta x' = \left(\frac{x_{i+1} - x_i}{2} + x_i\right) - \left(\frac{x_i - x_{i-1}}{2} + x_{i-1}\right)$$

The approximate value of the second derivative will be:

$$\frac{d^2 y}{dx^2} \approx \frac{\Delta m}{\Delta x'}$$

These values must be substituted in the Eq. (2) to calculate the radius of curvature in the given point *i*. Now there are no more unknown variables and the uncompensated centrifugal acceleration in given point *i* of the transition curve can be found after substituting thus determined values in Eq. (1).

By repeating this method it's possible to calculate the uncompensated centrifugal acceleration in each specific point of the transition curve with known coordinates and to create a graphic of the change of the acceleration along the transition curve. This can be used to create an algorithm for computing the results in MS Excel or to develop a PC software utility.

Conclusion

This method can be applied by comparative analysis of types of transition curves in dynamics when a high precision is not necessary or in maintenance, repairing and reconstructing of the railroad transition curve and also to determine its condition. This method is practical; there is no need of specific math knowledge or use of complicated math software, which require additional training, and its use can be facilitated by developing an appropriate software algorithm.

References

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