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IDENTIFICATION AND MODAL ANALYSIS OF TRANSPORT MACHINES DYNAMIC SYSTEMS VIBRATIONS BY AUTOREGRESSIVE MODELLING

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Abstract: *In this paper, a method is developed for the modal analysis of vibrating structures whose properties may vary with time. It is based on an autoregressive model in a short- time scheme, and is called Short-Time Auto Regressive (STAR). This new method allows the successful modelling and identification of an output-only modal analysis of non-stationary systems. The originality of the proposed method lies in its specific handling of non-stationary vibrations, which allows the tracking of modal parameter changes in time. There is presented an update of the model used with respect to order as well as a new criterion for the selection of an optimal model order, based on the noise-to-signal ratio. It is shown that this criterion works stably with respect to block data length. As for the block size itself, a length equal to four times the period of the lowest natural frequency has been found to be efficient. To validate the proposed method, a system with three degrees of freedom is first numerically simulated under a random excitation.*

Key words: *Identification, Autoregressive model; Non-stationary vibration; operating modal analysis..*

INTRODUCTION

Operational modal analysis is a necessary method for modal parameters identification in structural testing and monitoring, especially when the structures operate under severe working conditions with a non linear behaviour. In modern modal analysis, it is important to monitor changes in the modal properties of the structures over time. Non-stationary vibration in systems with time-dependent properties may be analyzed through non-parametric and time-frequency methods such as the Short-Time Fourier transform (STFT). However, parametric methods offer a number of advantages, such as improved accuracy and resolution, which is why time domain methods are generally preferred. The last two decades have witnessed a trend toward the use of the time series model for non stationary modal analysis. Notable industrial applications include traffic-excited bridge vibration of structures, seismic vibration, robotic devices, wind turbines, fluid-structure interaction etc. In non-stationary cases, most of these methods involve a nonlinear model.

VECTOR AUTOREGRESSIVE MODELING IN MODAL ANALYSIS PROCESS

Assuming a random environment, the excitation may be ignored, and since the modal analysis required multiple measurement locations, a vector autoregressive model, with *d* sensors, can be expressed as follows [1]:

(1) $\{y(t)\}+[a_1].\{y(t-1)\}+[a_2].\{y(t-2)\}+...+[a_p].\{y(t-p)\}=\{e(t)\}$

The model, as rewritten here below, is a convenient form of a dimension d vector at order p [ ].

(2) $\{y(t)\}_{d\times 1}=[A]_{d\times dp}.\{\phi(t)\}_{dp\times 1}+\{e(t)\}_{d\times 1}$,

where $[A]_{d\times dp}=\left[-[a_1]\ -[a_2]\ \dots\ -[a_i]\ \dots\ -[a_p]\right]$ is the model parameter matrix, $[a_i]_{d\times d}$ is the matrix of autoregressive parameters relating the output $\{y(t-i)\}$ to $\{y(t)\}$, $\{\varphi(t)\}_{dp\times 1}=\left[\{y(t-1)\};\{y(t-2)\};\dots;\{y(t-p)\}\right]$ is the regressor for the output vector $\{y(t)\}$, $\{y(t-i)\}_{d\times 1}$ for $(i=1:p)$ is the output vector with delay time *i x T*, T is the sampling period (s) and $\{e(t)\}_{d\times 1}$ is the residual vector of all output channels, and considered as the error of the model.

If the data are assumed to be measured in a white noise environment, the least squares estimation is applicable. If N successive output vectors of the responses from $\{y(t)\}$ to $\{y(t+N-1)\}$ are considered *($N \geq dp+d$)*, then the model parameters matrix $[A]_{d\times dp}$ and the estimated covariance matrices of the un-noised part $\left[\hat{D}\right]_{d\times d'}$ and of the error part $\left[\hat{E}\right]_{d\times d'}$ can be estimated as follows [1]:

$$[A]=(\left[R_{12}^T\right][R_{11}]).(\left[R_{11}^T\right][R_{11}])^{-1}=(\left[R_{11}^{-1}\right][R_{12}])^T$$

(3) $$\left[\hat{D}\right]=\frac{1}{N}\left[R_{12}^T.R_{12}\right]$$

$$\left[\hat{E}\right]=\frac{1}{N}\left[R_{22}^T.R_{22}\right]$$

In these formulas, $[R_{11}]$, $[R_{12}]$ and $[R_{22}]$ are sub-matrices of the upper triangular factor $[R]$ derived from the QR factorization of the data matrix as follows [4]:

(4) $[K]=[Q].[R]$, where $[Q]_{N\times N}$ is an orthogonal matrix ($[Q].\left[Q^T\right]=[I]$, and $[R]$ has the form

(5) $$[R]_{N\times(dp+d)}=\begin{bmatrix}[R_{11}]_{dp\times dp} & [R_{12}]_{dp\times d}\\ 0 & [R_{22}]_{d\times d}\\ 0 & 0\end{bmatrix}$$ and data matrix $[K]$ is constructed from N samples in form:

(6) $$[K]_{N\times(dp+d)}=\begin{bmatrix}\{\varphi(t)\}_{dp}^T & \{y(t)\}_d^T\\ \{\varphi(t+1)\}_{dp}^T & \{y(t+1)\}_d^T\\ \dots & \dots\\ \{\varphi(t+N-1)\}_{dp}^T & \{y(t+N-1)\}_d^T\end{bmatrix}.$$

Once the model parameters matrix has been estimated, modal parameters can be directly identified from the eigen-decomposition of the state matrix $[\Pi]$ [4].

(7) $$[\Pi]_{dp\times dp}=\begin{bmatrix}-[a_1]_{d\times d} & -[a_2]_{d\times d} & -[a_i]_{d\times d} & \dots & -[a_p]_{d\times d}\\ [I] & 0 & 0 & 0 & 0\\ 0 & [I] & 0 & 0 & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & [I] & 0\end{bmatrix}.$$

THE STAR METHOD AND IDENTIFICATION WORKING PROCEDURE

In operational modal analysis, the dynamic parameters of the system are unknown, and thus, a priori knowledge about the model order is not available. From the above modelling, it is found that the number of samples in each block N must satisfy $N \geq dp+d$, where p is the computing model order, and

thus can be variable. It is also clear that the block size must be long enough to allow an exhibition of the vibratory features of the system and to cover the largest period in the signal. The block length of the sliding window must be adjusted from the greatest period. The optimal model must be selected from the order 2 to the maximum available order fitting data of all block sizes. Because it is time-consuming to repeat the computation for each order value, this procedure should be avoided. We present an algorithm allowing an effective updating of the solution by model order.

I. Order updating and new criterion for optimal model order selection

The data matrix $K^{(p)}$ at order p can be rewritten as:

$$(8)\quad [K]^{(p)}_{N\times(dp+d)} = \begin{bmatrix} \varphi^T(k) & y^T(k) \\ \varphi^T(k+1) & y^T(k+1) \\ \cdots & \cdots \\ \varphi^T(k+N-1) & y^T(k+N-1) \end{bmatrix} = \left[{K_1^{(p)}}_{N\times dp} \,.\, {K_2}_{N\times d} \right].$$

If the model order is updated to p+1, the data matrix has the form:

$$(9)\quad [K]^{(p+1)}_{N\times(d(p+1)+d)} = \left[{K_1^{(p)}}_{N\times dp} \,.\, K^o_{N\times d} \,.\, {K_2}_{N\times d} \right], \text{ where } K^o_{N\times d} \text{ are the added d columns in form}$$

$$K^o_{N\times d} = \begin{bmatrix} y^T(k-(p+1)) \\ y^T(k+1-(p+1)) \\ \cdots \\ y^T(k+N-1-(p+1)) \end{bmatrix}.$$

We can then compute the following matrix:

$$(10)\quad Q^{(p)T}.K^{(p+1)} = \left[Q^{(p)T}.{K_1^{(p)}}_{N\times dp} \quad Q^{(p)T}.{K^0}_{N\times d'} \quad Q^{(p)T}.{K_2}_{N\times d} \right] = \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & T_2 & R_{22}^{(p)} \end{bmatrix}, \text{ where } {T_1}_{dp\times d} \text{ and}$$

${T_2}_{(N-dp)\times d}$ are extracted from $Q^{(p)T}.K^0 = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$.

We must now triangularize the right term matrix in equation 13. If we decompose only the small sub-matrix T_2, it easily yields:

$$(11)\quad T_2 = Q_T.\begin{bmatrix} R_T \\ 0 \end{bmatrix}.$$

Where ${R_T}_{d\times d}$ is an upper diagonal matrix and ${Q_T}_{(N-dp)\times(N-dp)}$ is the product of the Householder transformations (or Givens rotations). Equation (10) then becomes:

$$(12)\quad Q^{(p)T}.K^{(p+1)} = \begin{bmatrix} I_{dp\times d'p} & 0 \\ 0 & Q_T \end{bmatrix} = \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & R_T & 0 \\ 0 & 0 & Q_T^T\left[R_{22}^{(p)}\right] \end{bmatrix} \Rightarrow \begin{bmatrix} I_{dp\times d'p} & 0 \\ 0 & Q_T \end{bmatrix} Q^{(p)T}K^{(p+1)} = \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & R_T & R'_{22} \\ 0 & 0 & R''_{22} \end{bmatrix},$$

where ${R'_{22}}_{d\times d'}$ and ${R'_{22}}_{(N-dp-d)\times d}$ are obtained from multiplication $\begin{bmatrix} R'_{22} \\ R''_{22} \end{bmatrix} = Q_T^T.R_{22}$.

It can be seen that the first d x p rows of the right hand side in equation (16) are not affected by the above transformations, and the factor matrix $R^{(p+1)}$ at order p+1 is thus updated:

$$(13)\quad R_{11}^{(p+1)} = \begin{bmatrix} R_{11}^{(p)} & T_1 \\ 0 & R_T \end{bmatrix};\; R_{12}^{(p+1)} = \begin{bmatrix} R_{12}^{(p)} \\ R'_{22} \end{bmatrix};\; R_{22}^{(p+1)} = R''_{22} \text{ as is the Q matrix in form:}$$

$$(14)\quad Q^{(p+1)} = Q^{(p)}.\begin{bmatrix} I_{dp\times dp} & 0 \\ 0 & Q_T \end{bmatrix}.$$

As well as the two covariances matrices from (4) and (5) in form:

$$(15)\quad \hat{D}^{(p+1)} = \left[R_{12}^{(p+1)}\right]^T .R_{12}^{(p+1)} = \left[R_{12}^{(p)}\right]^T .R_{12}^{(p)} + \left[R'_{22}\right]^T .R'_{22} = \hat{D}^{(p)} + \left[R'_{22}\right]^T .R'_{22}$$

$$(16)\quad \hat{E}^{(p+1)} = \left[R_{22}^{(p+1)}\right]^T .R_{22}^{(p+1)} = \left[R''_{22}\right]^T .R''_{22} = \left[R_{22}^{(p)}\right]^T .R_{22}^{(p)} - \left[R'_{22}\right]^T .R'_{22} = \hat{E}^{(p)} - \left[R'_{22}\right]^T .\left[R'_{22}\right].$$

It is seen from equations (15) and (16) that as the model order increases, the norm of the deterministic covariance matrix increases as well, while one of the error parts decreases by the same amount. The global noise-to-signal ratio (NSR) is:

$$(17)\quad \hat{NSR} = \frac{\left\|\hat{e}(t)\right\|}{\left\|\hat{y}(t)\right\|} = \frac{Tr(\hat{E})}{Tr(\hat{D})}$$

It is therefore monotonically decreased in terms of the model order. The Noise-rate Order Factor (NOF) defines the change in the NSR within two successive order values:

$$(18)\quad NOF^{(p)} = \hat{NSR}^{(p)} - \hat{NSR}^{(p+1)}$$

It is seen that the NOF is always positive, falling quickly at low orders and converging at high orders, which is why it serves as a criterion for selecting the model order. The optimal order should therefore be chosen after a significant change in the NOF. The NOF converges to a minimum value after the optimum order is reached. Since the solution is effectively updated, the selection of optimal order can be applied for time-varying systems.

II. Working procedure

Since there is no leakage present in parametric model-based modal analyses, it is therefore not necessary to use a window, and so we propose that the data be processed in combination with a progressive search for the model order, as follows. Firstly, the above VAR model is initially applied to a block of data with a reasonable low order value. The length of the first block size could be specified if the smallest natural frequency of interest of the structure is known. Modal parameters are identified and the natural frequencies are estimated by using the signal to noise ratio of each eigen-value (MSN) [4] in order to find the smallest frequency to use to specify the length of the next block. Once the block size is chosen, the optimal model order is selected by the NOF and an order equal to or higher than this minimum value is used to get the modal parameters. The overlapping process can also be employed by changing the sliding step, which can vary from only one sample to the whole of the block window.

a. Simulation on mechanical system

A numerical simulation of the proposed method was applied on 3 degrees of freedom (D.O.F.) system, (Fig. 1) under a random force excitation. The mechanical properties of the system are first considered constant, and present three natural frequencies at 6.4 Hz, 12.6 Hz and 25 Hz, and damping rates at 2%, 4% and 7.8%, respectively.

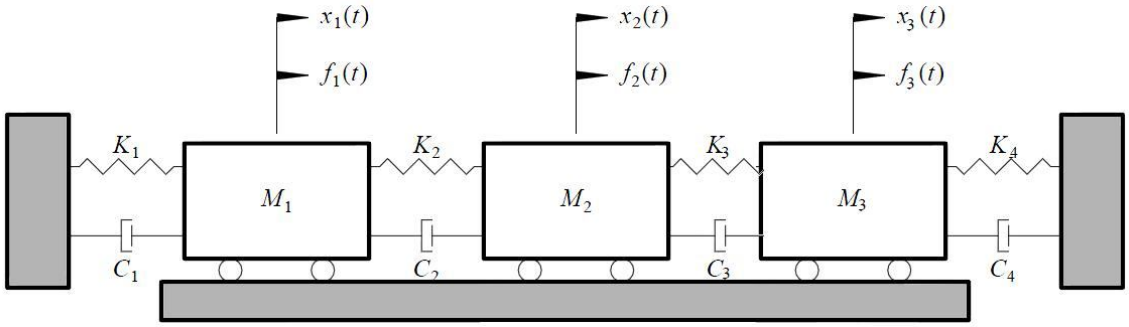

Fig. 1 Three D.O.F mechanical system

b. Discussion on block data length

Fig. 2 shows the frequencies and damping rates of the above structure identified by the VAR method, with varying data lengths.

We can observe that the block size must be larger than 3 times the longest period T_{max} in order to produce the smallest natural frequency value. For that reason, the block size was chosen to be equal to 4 times the period. The results found were obtained from multiple simulations. In conclusion, when a moderate damped system is subjected to a random excitation, its modal parameters can be monitored at block sizes equal to 4 times the period of the smallest natural frequency. Fig. 3 plots the NOF curves of the 3 D.O.F. system at various data length sizes.

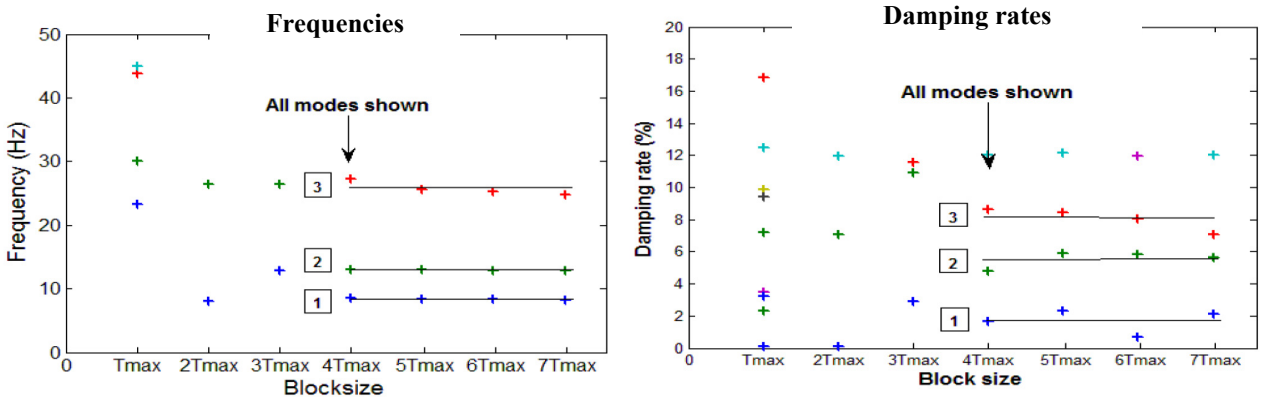

Fig. 2 Modal parameter identification with block size

It is seen that the structural model order is found accurately at 3 regardless the data length showing the stability of the NOF criterion with respect to data length. It also confirms that if the properties of the structure are subjected to change, the optimal order can still be tracked [2], and does not depend on the block size once this latter is long enough to show the smallest frequency.

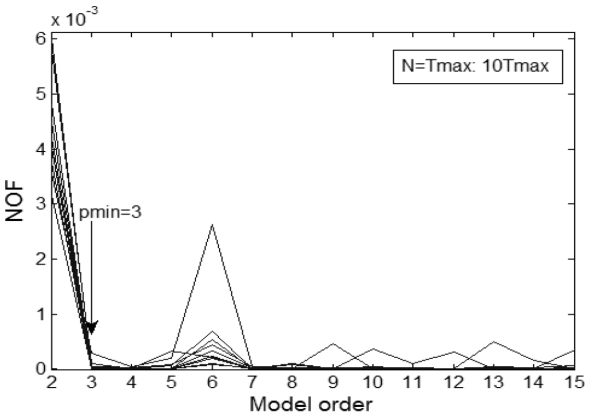

Fig. 3 Optimal model order at different block sizes

c. Simulation on mechanical system with time-dependent parameters

The 3 D.O.F. system above has been modified to vary its mechanical properties, and is always subjected to a random excitation. The mass M2 is now a time variant factor which changes following the function, as shown in Fig. 4.

Since the data is non-stationary, the optimal model and modal parameters can vary with time, and therefore, the block size should be changed. If the smallest frequency is available either through prior knowledge of the system or from the frequency range of interest, the initial block size is chosen to be 4 times its period. When more data are acquired, the block size is adjusted based on the smallest frequency identified in the previous step. Fig. 5 and Fig. 6 showing the optimal model and the computing block size to use to track the change in the system properties. It is shown that the optimal order is primarily monitored at 3, except for some outer values.

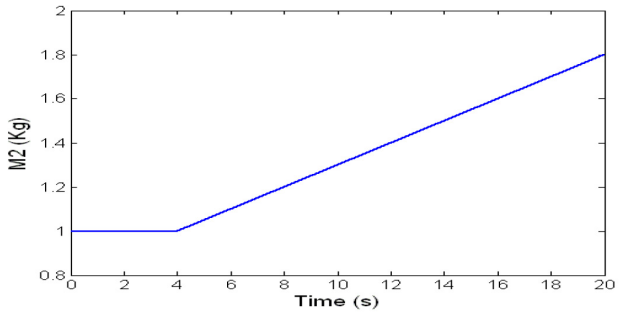

Fig. 4 Simulated time-varying mass

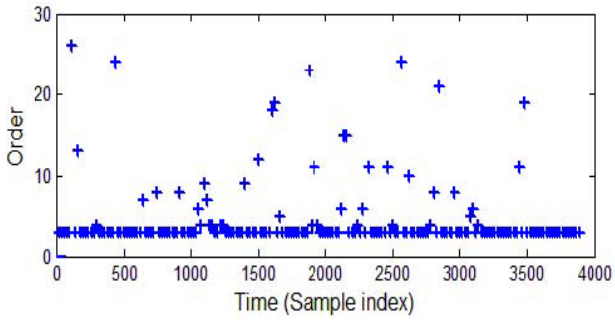

Fig. 5 Monitoring of optimal order on simulation

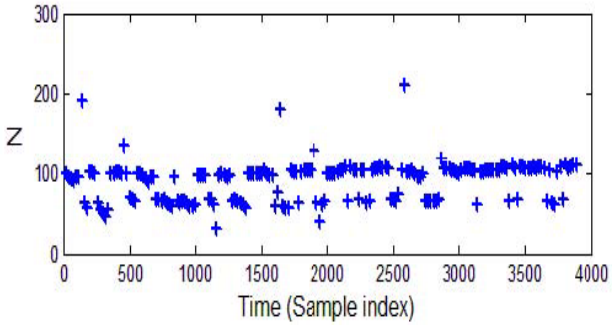

Fig. 6 Monitoring of block size on simulation

We can observe an adjustment on the block length when the change appears at the half end of the monitoring period. The changes in frequencies and damping rates are plotted on Figures 7 and 8. As is logically seen when the mass is increasing, all frequencies tend to decrease, and can be well tracked. On the other hand, we see a high variance for the damping rates. The damping identification only gives an approximate range of values.

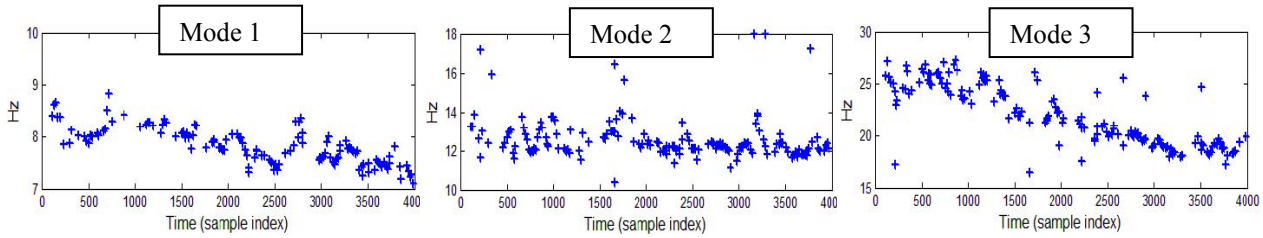

Fig. 7 Monitoring of frequencies on simulation

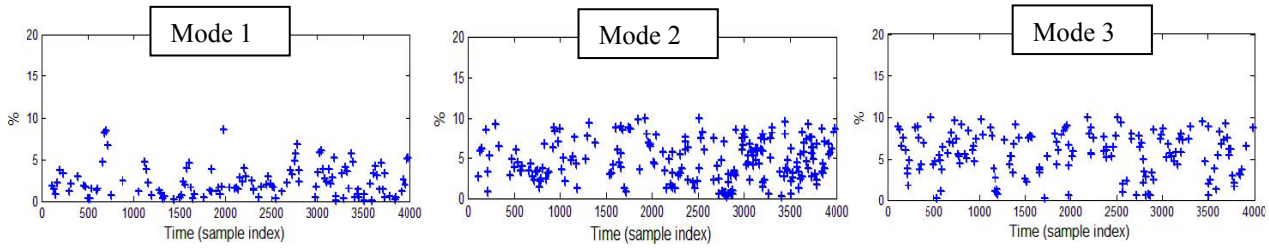

Fig. 8 Monitoring of damping rates on simulation

The method was experimentally applied on the real data measured on a steel plate emerging from water too, and compared to the conventional Short-Time Fourier Transform (STFT) method. It is shown that the proposed method outperforms in terms of frequency identification, whatever the non-stationary behaviour (either slow or abrupt change) due to the added mass effect of the fluid [3].

CONCLUSIONS

It was found that the minimum structural model order does not depend on the block size if this latter is long enough. The block size was minimally found to be equal to four times the wavelength of the first natural frequency, and consequently, block sizes vary with variations in the first natural frequency. Numerical simulations and experiments show that the proposed method can be used to track a slow change as well as a sudden change in the modal properties of the structure, and that it outperforms the STFT method. While the monitoring of natural frequencies has been successfully dealt with, those of damping rates are not enough precise. Research is still ongoing on damping identification of time-varying system as it is found in fluid-structure interaction. Unfortunately, the results on the damping rate identification were not significant, and research is still ongoing on this subject.

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ИДЕНТИФИЦИРАНЕ И МОДАЛЕН АНАЛИЗ НА ВИБРАЦИИ ОТ ДИНАМИЧНИ СИСТЕМИ НА ТРАНСПОРТНИ МАШИНИ ЧРЕЗ АВТОРЕГРЕСИВНО МОДЕЛИРАНЕ

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Ключови думи: *идентифициране, авторегресивен модел, нестационални вибрации, операционен модален анализ..*

Резюме: *В доклада е разработен метод за модален анализ на вибриращи конструкции, чиито свойства могат да варират във времето. Той се базира на авторегресивния модел в краткосрочна схема и се нарича Short-Time Auto Regressive (STAR). този нов метод позволява успешно моделиране и идентифициране на изходящ модален анализ на нестационарни системи.*

Оригиналността на предложения метод се крие в специфичната си обработка на нестационалните вибрации, което позволява проследяването на промените в модалните параметри във времето. Представена е актуализация на модела, използван по отношение на реда, както и като нов критерий за избор на оптимален модел за ред въз основа на съотношението шум-сигнал. Показано е, че този критерий работи стабилно по отношение на дължината на блок данните. Що се отнася до размера на самия блока, е установено, че е ефективна дължината, равна на четири пъти период от най-ниската естествена честота установено, да бъде ефективна. За да се валидира предложеният метод, най-напред е числено симулирана система с три степени на свобода при случайно възбуждане.