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THEORETICAL BACKGROUND AND SOFTWARE SUPPORT OF THE METHOD FOR PARAMETRIC IDENTIFICATION OF MECHANICAL STRUCTURES DYNAMIC SYSTEMS

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Abstract: *The paper deals with one of possible ways of an identification of stochastically loaded mechanically structures. The purpose of this approach is to find an algorithm of a forecasting control of their working in real working conditions. It deals with a proposal of an application of vector time series moving average models (VARMA). The paper contains a theoretical principle of problems solved and a description of a real testing methodize.*
Key words: *Mechanically structure, stochastic loads, identification, vector autoregressive models VARMA, virtual model of crane jib, software support - ArmaGet, ARMASA Package.*

INTRODUCTION

It is well known that working of majority of machines is significantly influenced by different kinds of stochastic loads. There is possible to respect the tendency a limitation of energetically and material consumption to oversize their dimensions. But it is necessary to look for some more ingenious methods to deal with this problem's. Some of them are the ways to control (influence) the working of a mechanical system in respect to their proposed behaviour. But it needs to follow of the system behaviour in the real time and to make some necessary controlling interventions.

MATHEMATICAL FORMULATION OF PROBLEM AND ITS BACKGROUND

By the experimental identification by means of digital Fourier analyzer there are two essential steps involved in the determination of the modal parameters and mode shapes. The starting point of the analysis technique relies on the estimation of spectra and subsequent determination of corresponding transfer functions. In the second step the measured transfer functions are used to extract the necessary modal information. Anticipating mostly the stochastic nature of mechanical dynamic system's excitation and response, time series methods and Data Dependent Systems (DDS) [8] approach seems to be very suitable and effective in the area of dynamic modelling too. There are two theoretical areas concerning the above mentioned problems. The first is a classic approach to the vibrations of mechanical dynamic systems. It is well known, that in case of n-degree of freedom systems they are represented by system of ordinary differential equations of second order in matrix form as:

$$(1) \quad \mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t)$$

where **M**, **B** and **K** are n x n mass, damping and stiffness matrices, $\ddot{\mathbf{x}}, \dot{\mathbf{x}}, \mathbf{x}$ and $\mathbf{F}(t)$ are n-dimensional column vectors of accelerations, velocities, displacements and force respectively. For the type of solution of this system are essential the eigen-values of left-side of matrix equation, which are usually, obtained as complex numbers. The real parts of them have physical meaning as damping ratios and imaginary parts are natural frequencies.

The second theoretical area of theme is theory of stochastic processes. It was shown in [1], that any continuous system could be represented (in a scalar case) as differential equation in form

(2) $$\left(D^n + \alpha_{n-1} D^{n-1} + \alpha_{n-2} D^{n-2} + \ldots + \alpha_1 D^1 + \alpha_0\right). X(t) = \\ = \left(\beta_m D^m + \beta_{m-1} D^{m-1} + \ldots + \beta_1 D^1\right). Z(t)$$

for which holds $E\left[Z(t)\right] = 0$ and $E\left[Z(t).Z(t-n)\right] = \delta(n).\sigma_Z$, where *n, m* indicate the order of the model, D = d / dt is the differential operator, Z(t) is white noise, E denotes the expectation operator, $\delta(n)$ is the Diracs delta function and α's and β's are model parameters.

When such system is sampled at uniform interval Δt, differential equation (2) becomes a difference equation [4]

(3) $$X_t - a_1.X_{t-1} - a_2.X_{t-2} - \ldots - a_n.X_{t-n} = \varepsilon_t - b_1.\varepsilon_{t-1} - \ldots - b_{n-1}.\varepsilon_{t-n+1}$$

for which holds $E(\varepsilon_t) = 0$ and $E(\varepsilon_t.\varepsilon_{t-n}) = \delta_k.\sigma_e^2$. Values X_t, X_{t-1}, X_{t-2}, … are values of process, *a* 's and *b* 's are parameters of the model, ε_t, ε_{t-1}, ε_{t-2}, ..., ε_{t-n+1} are residuals, E denotes the expectation operator and δ_k is Kronecker delta function,. Such a model is called Autoregressive Moving Average of order (n, n-1) – ARMA (n,n-1).

It is obvious that parameters of continuous and discrete model of the same process must be functionally related. The simplest way to express this relationship is by using roots μ_i and λ_i of characteristic equations of formula (2) or (3). The relationships then takes form as

(4) $$\lambda_i = e^{\mu_i \Delta t} \quad \text{or} \quad \mu_i = \frac{1}{\Delta t} \ln \lambda_i$$

In case of multivariate systems (mechanical dynamic systems) the Vector Autoregressive Moving Average – VARMA model is obtained in form [4,5]:

(5) $$\mathbf{A_0}.\mathbf{X_t} - \mathbf{A_1}.\mathbf{X_{t-1}} - \ldots - \mathbf{A_n}.\mathbf{X_{t-n}} = \boldsymbol{\varepsilon}_\mathbf{t} - \mathbf{D_1}.\boldsymbol{\varepsilon}_\mathbf{t-1} - \ldots - \mathbf{D_{n-1}}.\boldsymbol{\varepsilon}_\mathbf{t-1},$$

respectively in form

(6) $$(\mathbf{A_0} - \mathbf{A_1}.\mathbf{B}^1 - \mathbf{A_2}.\mathbf{B}^2 - \ldots - \mathbf{A_n}.\mathbf{B}^n).\mathbf{X_t} = \\ = (\mathbf{1} - \mathbf{D_1}.\mathbf{B}^1 - \mathbf{D_2}.\mathbf{B}^2 - \ldots - \mathbf{D_{n-1}}.\mathbf{B}^{n-1}).\boldsymbol{\varepsilon}_\mathbf{t}$$

which can fully express the relationships in structure during its vibrations and where $\mathbf{X_t}$ and $\boldsymbol{\varepsilon}_\mathbf{t}$ are vectors of measurements and white noise series, $\mathbf{A_i}$ and $\mathbf{D_i}$ are matrices of system parameters, **B** is vector of back shift operators, $\boldsymbol{\sigma}_{\boldsymbol{\varepsilon}\mathbf{2}}$ is matrix of dispersion and reciprocal correlation's and δ_k is Kronecker delta function. If one analyses a mechanical dynamic system with a numerical technique and its vibrations and exciting forces measure in uniform sampling intervals Δt, it is possible to develop discrete models to describe the relationship between output (vibration) and input (exciting forces) after Fig. 1.

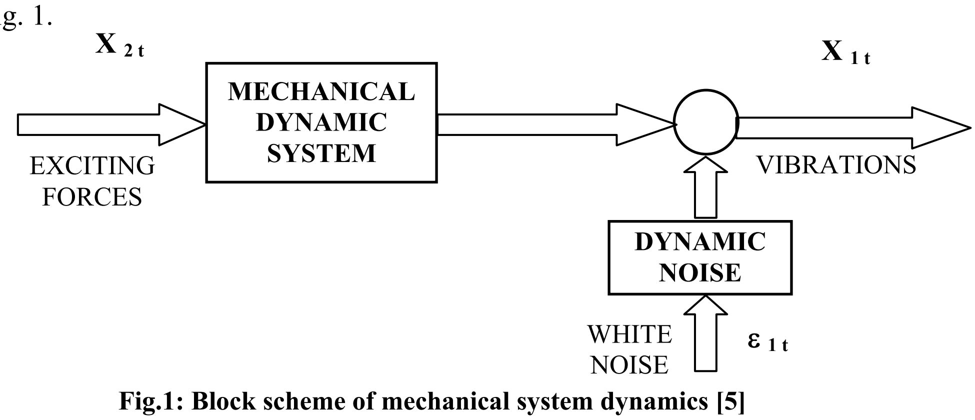

Fig.1: Block scheme of mechanical system dynamics [5]

Dynamics of the mechanical system and dynamics of noise is represented by a discrete transfer function. Supposing non-existence of feed back between vibrations of structure can be expressed in its excitation (which hold for structures tests) one gets a resulting model of structure dynamics in form [6]

$$(7)\quad \left(1 - a_{111}.B - a_{112}.B^2 - \ldots - a_{11n}.B^n\right).X_{1t} = \left(a_{120} + a_{121}.B + \ldots + a_{12n}.B^n\right). \\ .X_{2t} + \left(1 - b_{111}.B - b_{112}.B^2 - \ldots - b_{11(n-1)}.B^{(n-1)}\right).\varepsilon_{1t}$$

where attached assumptions shown in formula (5).

VARMA MODELS IN STOCHASTICALLY LOADED PARTS IDENTIFICATION PROCESS

There is necessary to identify such a system at first. It means to get its statistically adequate mathematical model. There is possible by using this model and by developing sufficient fast and correct machine control system and suitable software to forecast behaviour of system in the near future. We can get in such a way the possibility of making some controlling corrections before the system reaches an unstable region.

Vector Autoregressive Moving Average Models (VARMA)

It was found the as a suitable solution for a stochastically loaded mechanical structure identification can be used the ARMA (autoregressive moving average) models or their vector modification VARMA (Vector Autoregressive Moving Average) models [1], [7]. A stochastically loaded part of structure and its behaviour during time can be described by using of scalar autoregressive moving average model ARMA. Its identification (stochastically adequate model) but gives just an information about its own behaviour without a relationship to the whole structure during acting of different working regimes.

We have found as one of possible ways the use of vector autoregressive moving average models VARMA to improve accuracy of stochastically loaded mechanical structures identification.

These models are suitable for stochastically loaded mechanical structures identification which outputs are reflections on stochastically loads in more number of points – vector time series (Fig.2).

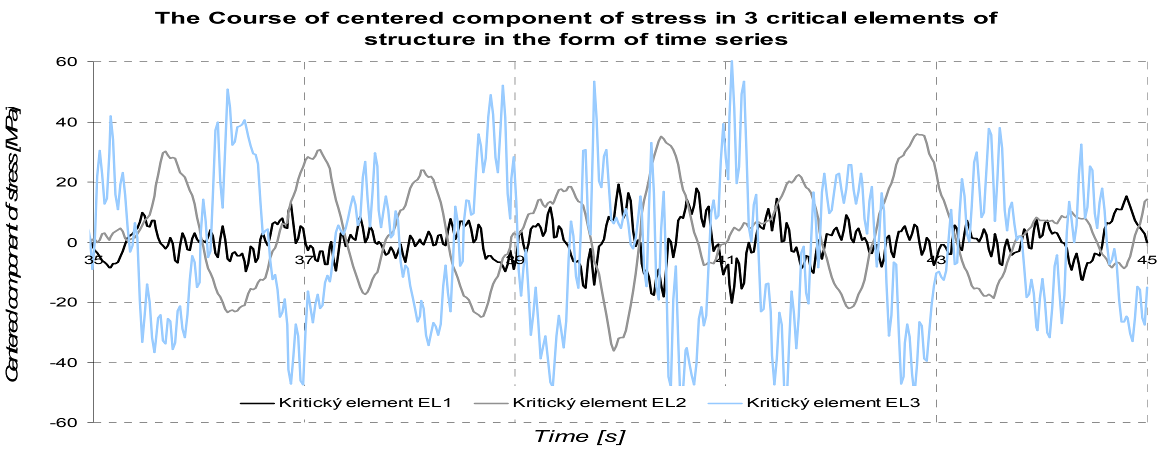

Fig. 2. A Vector Time Series of Stress in elements of structure.

A searched vector model VARMA (m,n) can be expressed as a matrix equation in form [1]

$$(8)\quad \mathbf{x_t} - \mathbf{A_1}.\mathbf{x_{t-1}} - \mathbf{A_2}.\mathbf{x_{t-2}} - \ldots - \mathbf{A_m}.\mathbf{x_{t-m}} = \boldsymbol{\varepsilon}_\mathbf{t} - \mathbf{B_1}.\boldsymbol{\varepsilon}_\mathbf{t-1} - \mathbf{B_2}.\boldsymbol{\varepsilon}_\mathbf{t-2} - \ldots - \mathbf{B_n}.\boldsymbol{\varepsilon}_\mathbf{t-n}$$

or in written out form [3]

$$(9)\quad \begin{bmatrix} x_{1t} \\ x_{2t} \\ \dots \\ x_{kt} \end{bmatrix} - \begin{bmatrix} a_{111} & a_{121} & \dots & a_{1k1} \\ a_{211} & a_{221} & \dots & a_{2k1} \\ \dots & & & \\ a_{k11} & a_{k21} & \dots & a_{kk1} \end{bmatrix} . \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \\ \dots \\ x_{kt-1} \end{bmatrix} - \dots - \begin{bmatrix} a_{11m} & a_{12m} & \dots & a_{1km} \\ a_{21m} & a_{22m} & \dots & a_{2km} \\ \dots & & & \\ a_{k1m} & a_{k2m} & \dots & a_{kkm} \end{bmatrix} . \begin{bmatrix} x_{1t-m} \\ x_{2t-m} \\ \dots \\ x_{kt-m} \end{bmatrix} =$$

$$= \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \dots \\ \varepsilon_{kt} \end{bmatrix} - \begin{bmatrix} b_{111} & 0 & \dots & 0 \\ 0 & b_{221} & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & b_{kk1} \end{bmatrix} . \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \dots \\ \varepsilon_{kt-1} \end{bmatrix} - \dots - \begin{bmatrix} b_{11n} & 0 & \dots & 0 \\ 0 & b_{22n} & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & b_{kkn} \end{bmatrix} . \begin{bmatrix} \varepsilon_{1t-n} \\ \varepsilon_{2t-n} \\ \dots \\ \varepsilon_{kt-n} \end{bmatrix}$$

this can be transformed in the system of k linearly independent equations. The symbol of "k" means number of points of the structure in which the output on dynamic loads are recorded. The left hand side of matrix equation (1) expresses the dependence of vector time series values on former values of the series and the right hand side shows the relationship of stochastically random deviations.

The Possibilities and Advantages of VARMA Models

The application of VARMA models as an alternative to the systems of differentials equations for stochastically loaded structures identification is suitable from different point of view too.

If we can express the system of differential equations in a simplified form (1), we can judge the matrix of damping **K** and the matrix of stiffness **C** as a mathematical expression of the relationship among the individual parts of structure. Its matrix formulation is in form (10). The matrix coefficients as $\mathbf{a_1, a_2, \dots, a_m}$ of a vector model VARMA are the mathematical expressions of individual parts interaction.

$$(10)\quad \begin{bmatrix} m_1 & 0 & \dots & 0 \\ 0 & m_2 & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & m_n \end{bmatrix} . \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \dots \\ \ddot{x}_n \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{n2} \\ \dots & & & \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix} . \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_n \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{n2} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \dots \\ f_n(t) \end{bmatrix}$$

The advantages of VARMA models:

- ♦ they can show the physically base of problem studied (this means that they to obtain the natural frequencies and natural modes of vibrations) [9],
- ♦ they can describe a wanted accuracy of real system [9,7,3],
- ♦ the mathematical apparatus of methods is relative simple (use for "real time" control [9,6]).

A NEW SOFTWARE SUPPORT OF A PROPOSED METHOD OF IDENTIFICATION

The scalar models of a simple description of dynamic system can not express statistically adequate description of complex systems. For this reason there was developed an effective software system which enables to create the statistically models of dynamic stochastic system by using VARMA models. The accuracy and the reliability of the developed methods and algorithms were verified by use of commercial software packages.

The creation of a software support which is able to identify some stochastic loaded parts of structures is just the firs step for applying of the adaptive control of mechanical system activity. The result of a proposed application of this methodology is software product ArmaGet (Fig.3, Fig.4). This developed software is able to create an adequate mathematical model for describing a matrix model of a tested stochastic loaded mechanical system.

There was developed a FEM (Finite Element Method) model of a crane jib (Fig.5) and in MATLAB environment was realized simulation of its loading. The acting loads were described as a stochastic excitation in form of a stochastic time series. There were used as an application of a numeric Crank-Nicholson method [3] of direct integration the deformation of all nodes of FEM model (20 nodes). The time intervals were selected as $\Delta t_{vz} = 0.01$ s.

Resulting outputs for deformation were organized in corresponding vector time series. There was selected in a testing example a vector time series of deflection in “z” axe direction.

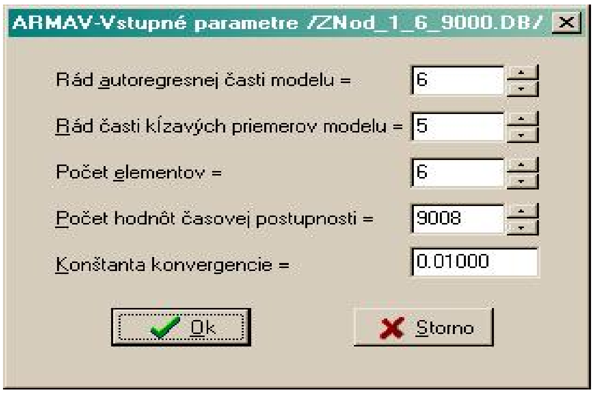

Fig. 3. Settings of input parameters.

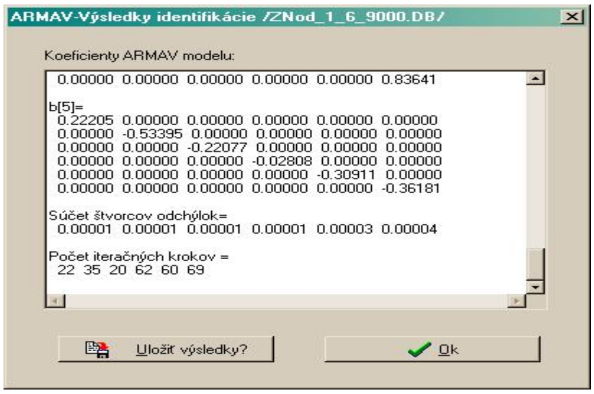

Fig. 4. Results of a jib upper boom identification – an optimal model VARMA (6,5).

The determination of vector time series in direction of “z” axe is introduced on Fig.3 and results of identification are introduced on Fig.4 (an optimal order of model is VARMA (6,5)). The verification of developed software support (ArmaGet) was realized by two different ways. At first it was the simulation of time series with determined parameters and their “back way” identification (this option is available by menu item *Simulation→ Model VARMA*). The second way was a comparison with results of software ARMASA Package [2]. There are showed in Tab.1 the results of an application of ArmaGet software and their comparison with results obtained using other product - ARMASA Package.

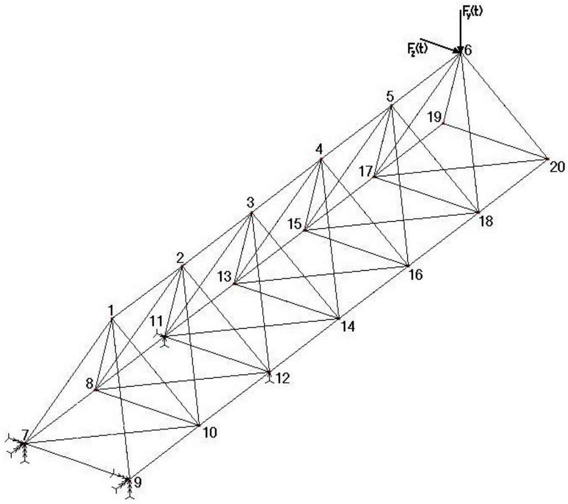

Fig. 5. FEM model of a crane jib

Tab. 1. The verification of the identification results (ArmaGet and Excel).

	VARMA(6,5)		VARMA(8,7)		VARMA(10,9)	
	ArmaGet	*ARMASA*	*ArmaGet*	*ARMASA*	*ArmaGet*	*ARMASA*
Node 1	$7{,}9591.10^{-6}$	$7{,}9606.10^{-6}$	$7{,}9251.10^{-6}$	$7{,}8991.10^{-6}$	$7{,}8985.10^{-6}$	$7{,}8398.10^{-6}$
Node 2	$1{,}0659.10^{-5}$	$1{,}1381.10^{-5}$	$1{,}0493.10^{-5}$	$1{,}1243.10^{-5}$	$1{,}0365.10^{-5}$	$1{,}1953.10^{-5}$
Node 3	$1{,}1204.10^{-5}$	$1{,}2789.10^{-5}$	$1{,}0689.10^{-5}$	$1{,}1868.10^{-5}$	$1{,}0424.10^{-5}$	$1{,}1307.10^{-5}$
Node 4	$1{,}4823.10^{-5}$	$2{,}4128.10^{-5}$	$1{,}4327.10^{-5}$	$2{,}1467.10^{-5}$	$1{,}4028.10^{-5}$	$2{,}0833.10^{-5}$
Node 5	$2{,}5955.10^{-5}$	$4{,}3874.10^{-5}$	$2{,}3396.10^{-5}$	$4{,}2690.10^{-5}$	$2{,}2255.10^{-5}$	$3{,}5906.10^{-5}$
Node 6	$4{,}1240.10^{-5}$	$8{,}4167.10^{-5}$	$3{,}8608.10^{-5}$	$6{,}4402.10^{-5}$	$3{,}7822.10^{-5}$	$6{,}2980.10^{-5}$

CONCLUSION

It introduces problems were proposed and verified in a frame of grant research VEGA # 9/0430/09 named „Stochastic Methods of Identification of Mechanical Structures Dynamic Systems” where some possible applications of the proposed identification procedure were investigated. Advantage of

ARMA models utilization is in fact that their parameters can one obtains directly from adequate models without necessity of transfer function determination.

Paper is dealing with a newly developed method of identification of mechanical dynamic structures. This method is based on building of statistically significant Vector Auto-Regressive Moving Average (VARMA) models of random excited structures that define statistically significant modes of structure vibration. The relationship between continuous and discrete models developed in former work is mentioned also. Once having VARMA models structure of a structure it is very easy to simulate its behavior by both stochastic and deterministic excitation in form of time series. The application is demonstrated on an example of Finite Element Model of simple structure using adaptive method of identification that can deal with non-stationary behaviour of structure also.

Further, any subjective judgment is eliminated because the tests of statistical adequacy are strictly defined. Results of further problems using proposed procedure by dynamic analysis and identification of modal characteristics of mechanical systems showed a relatively good agreement between theoretical and identified values.

From presented facts one can develop that above shown assumptions and theoretical starting points are correct and developed procedure can reduce number of calculation in an expressive way and improve efficiency of mechanical structures dynamic calculation.

It was shown that by using of a suitable mathematical apparatus can forecast the future behaviour of a mechanical structure. The vector time series and its models were chosen as a suitable mathematical apparatus and the suitability of this choice were proven by use of computer simulation of stochastically excited mechanical systems parameters.

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ТЕОРЕТИЧНА ОСНОВА И СОФТУЕРНА ПОДДРЪЖКА НА МЕТОДА ЗА ПАРАМЕТРИЧНА ИДЕНТИФИКАЦИЯ НА МЕХАНИЧНИ КОНСТРУКЦИИ НА ДИНАМИЧНИ СИСТЕМИ

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Ключови думи: *механично структура, стохастични товари, идентификация, векторни авторегресивни модели VARMA, виртуален модел на кран, поддръжка на софтуера - ArmaGet, пакет ARMASA*

Резюме: *В статията се разглежда един от възможните начини за идентификация на стохастично натоварени механично структури. Целта на този подход е да се намери алгоритъм за прогнозиращ контрол на работата им в реални работни условия. Докладът представя предложение за прилагане на движещи се средна модели със серия вектори за време (VARMA). Докладът съдържа теоретичен принцип на решените проблеми и описание на истинска методика за тестване.*