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ANALYSIS OF THE EFFECT OF LOCAL INERTIA FORCES ON THE EHD LUBRICATION OF DYNAMICALLY LOADED JOURNAL BEARINGS

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Abstract: *The aim of the presented investigation is to demonstrate the influence of lubricants local inertia forces on the bearing performance. The 2-D model of HD journal bearing in isothermal and isoviscous conditions is considered. The bearing shaft is covered with thin resilient layer and its elastic distortions are taken into consideration. The elasticity part of the problem is investigated in accordance with Winkler hypothesis. The generalized Reynolds equation is obtained by Fourier transform methods. The results demonstrate the dynamic film effect.*

1. INTRODUCTION

Most fluid film bearing analyses are based on the assumption that the contribution of the inertia forces to the bearing performance is negligible. It is important to note a fact that for analysis of dynamically loaded journal bearings the effect of local inertia forces could be importance [1, 2]. This additional inertia effects are due to the unsteady velocity changes. However, this not mean, that the so-mentioned contribution is of the same order as the contribution of the shearing forces. In the studies, which deal the acceleration effects, the models are referred only to one-dimensional bearing with rigid surfaces.

The current paper presents elastohydrodynamic solution for a dynamically loaded HD finite journal bearing (2-D problem). The bearing shaft is covered with thin resilient layer and its elastic distortions are of the same order of magnitude as the film thickness.

The objective is to demonstrate the influence of local inertia terms of the lubricant on the HD pressure distribution and on the load capacity parameter values.

It is assumed that the material of the layer on the shaft deforms linearly according to Winkler hypothesis.

The generalized Reynolds equation for the pressure distribution is obtained by Fourier transform methods. The differential equation is solved numerically by FDM and iterative technique. The solution is extended to prescribed loci of the shaft centre as the results demonstrate the dynamic film effect.

Notations:

c	Radial clearance;
d	Shaft liner thickness;
E	Young modulus;
e	Eccentricity
h	Film thickness
L	Axial length
p	HD pressure
r	Outer radius of shaft liner;
t	Time;
u,v,w	Velocity components;
γ	Angle deviation;
δ	Radial distortion;
υ	Kinematical viscosity;
η	Dynamic viscosity;
μ	Poisson ratio;
ρ	Density of the lubricant
ω	Angular velocity of the shaft

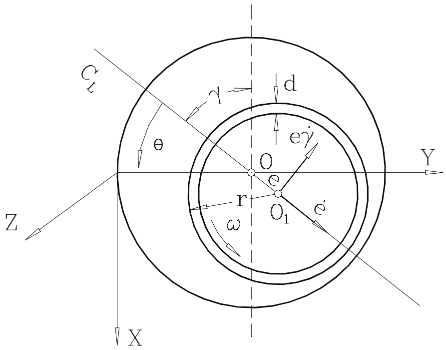

Fig. 1
Journal bearing geometry

Non-dimensional parameters

$$H=\frac{h}{c};\quad \Pi=\frac{c^2}{6\eta\omega r^2}.p;\quad \tau=t\frac{\omega}{2};$$

$$z_1=\frac{z}{L/2};\qquad \bar{\delta}=\frac{\delta}{c};$$

$\alpha=\left(\frac{2r}{L}\right)^2$ - diameter to length ratio; $\varepsilon=\frac{e}{c}$ - eccentricity ratio;

$\theta=\frac{x}{r}$ - angular coordinate measured from the centers line;

$K_h=\frac{6\eta\omega r^2}{c^3}2\frac{1-\mu^2}{\pi E}d$ - elasticity factor; $S=\frac{Wc^2}{\eta\omega r^3 L}$ - Sommerfeld number

$\mathrm{Re}=\frac{c\omega r}{\upsilon}$ - Reynolds number; $\lambda=\frac{c}{r}\mathrm{Re}$ - generalized Reynolds number.

2. FLOW EQUATIONS

In the case of two-dimensional journal bearing the governing equations for the incompressible lubricant flow with local inertia terms are:

(1) $$\frac{\partial u}{\partial t}-\upsilon\frac{\partial^2 u}{\partial y^2}=-\frac{1}{\rho}\frac{\partial p}{\partial x};\qquad \text{(a)}$$

$$0=\frac{\partial p}{\partial y};\qquad \text{(b)}$$

$$\frac{\partial w}{\partial t}-\upsilon\frac{\partial^2 w}{\partial y^2}=-\frac{1}{\rho}\frac{\partial p}{\partial z};\text{(c)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 . \qquad \text{(d)}$$

Obviously, from (1.b.) follows the independence from y of the left-hand side terms in (1.a) and (1.c). Thus both equations become homogenous after using the following expressions

(2) $u(x,y,z,t)=u^*(y,t)-f_1(x,y,z,t)$;

(3) $w(x,y,z,t)=w^*(y,t)-f_2(x,y,z,t)$,

where

(4) $f_1(x,y,z,t)=\frac{1}{\rho}\frac{\partial}{\partial x}\int p dt=\frac{\partial q}{\partial x}$;

(5) $f_2(x,y,z,t)=\frac{1}{\rho}\frac{\partial}{\partial z}\int p dt=\frac{\partial q}{\partial z}$.

This lead to homogenous forms for (1.a) and (1.c), namely:

(6) $\frac{\partial u^*}{\partial t}-\upsilon\frac{\partial^2 u^*}{\partial y^2}=0$

(7) $\frac{\partial w^*}{\partial t}-\upsilon\frac{\partial^2 w^*}{\partial y^2}=0$.

In accordance with [3] one of the bounded particular solutions of equations like (6) и (7) has a form

(8) $\psi(y,t)=Ae^{\pm(1+i)\sqrt{\frac{\omega}{2\upsilon}}y+i\omega t}$,

where A is an arbitrary constant. Since the functions $f_1(x,y,z,t)$ and $f_2(x,y,z,t)$ are unknown, it is necessary to use solutions of these functions in a general form, which could be achieved by applying the Fourier integrals of (8), namely

(9) $u^*(y,t)=\int_{-\infty}^{\infty}A_1(\omega)e^{\varphi y+i\omega t}d\omega+\int_{-\infty}^{\infty}A_2(\omega)e^{-\varphi y+i\omega t}d\omega$;

(10) $w^*(y,t)=\int_{-\infty}^{\infty}A_3(\omega)e^{\varphi y+i\omega t}d\omega+\int_{-\infty}^{\infty}A_4(\omega)e^{-\varphi y+i\omega t}d\omega$,

where

(11) $\varphi=(1+i)\sqrt{\frac{\omega}{2\upsilon}}$

and $A_i(\omega)$ are the spectral densities of $u^*(y,t)$ and $w^*(y,t)$.

The determination of $A_i\ (i=1,2,3,4)$ could be performed by the application of Fourier integral transforms, (where ω is the transform parameter) at taking into account the actual boundary conditions:

(12.a) $u^*(0,t)=u_0^*=\frac{\partial q}{\partial x}=\int_{-\infty}^{\infty}\left[A_1(\omega)+A_2(\omega)\right]e^{i\omega t}d\omega$;

(12.b) $u^*(h,t)=u_h^*=u_h+\frac{\partial q}{\partial x}=\int\limits_{-\infty}^{\infty}A_1(\omega)e^{\varphi h+i\omega t}d\omega+\int\limits_{-\infty}^{\infty}A_2(\omega)e^{-\varphi h+i\omega t}d\omega$;

(12.c) $w^*(0,t)=w^*(h,t)=\frac{\partial q}{\partial z}$;

(12.d) $u(x,0,z,t)=u_0=v(x,0,z,t)=v_0=w(x,0,z,t)=w_0=w(x,h,z,t)=w_h=0$;

(12.e) $u(x,h,z,t)=u_h=\omega r+\dot{e}\sin\theta-e\dot{\gamma}\cos\theta$;

(12.f) $v(x,h,z,t)=v_h=\omega r\frac{\partial h}{\partial x}+\dot{e}\cos\theta+e\dot{\gamma}\sin\theta$.

In applying the following Fourier transform

(13) $$f(x,z,t)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}C(x,z,t)e^{i\omega t}d\omega$$

and inverse transform

$$C(x,z,\omega)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}f(x,z,t)e^{-i\omega t}dt,$$

as after satisfying the boundary conditions (12.a, b, c) the expressions for the velocity components are obtained in the form:

(14) $$u(x,y,z,t)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\frac{(C_1+C_2)sh\varphi y+C_2 sh\varphi(h-y)}{sh\varphi h}e^{i\omega t}d\omega-\frac{\partial q}{\partial x};$$

(15) $$w(x,y,z,t)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}C_3\frac{ch\varphi\left(\frac{h}{2}-y\right)}{ch\varphi\frac{h}{2}}e^{i\omega t}d\omega-\frac{\partial q}{\partial z},$$

where

(16) $$C_1(x,z,\omega)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}u_h e^{-i\omega t}dt;$$

$$C_2(x,z,\omega)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\frac{\partial q}{\partial x}e^{-i\omega t}dt;$$

$$C_3(x,z,\omega)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\frac{\partial q}{\partial z}e^{-i\omega t}dt.$$

Because of the film thickness is sufficiently small, the radial velocity component can be presented in a linear function:

$$v(x,y,z,t)=D_1(x,z,t)y+D_2(x,z,t),$$

and through (12.d и 12.f) this velocity is written as

(17) $$v(x,y,z,t)=\frac{1}{h}v_h y=\frac{1}{h}\left(\omega r\frac{\partial h}{\partial x}+\dot{e}\cos\theta+e\dot{\gamma}\sin\theta\right)y.$$

The substitution of the expressions for the *u, v, w* velocity components in the continuity equation (1.d) and integration of it across the film

$$\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dy = 0$$

yields

(18) $$\frac{\partial}{\partial x}\int_0^h u dy - u_h \frac{\partial h}{\partial x} + v_h + \frac{\partial}{\partial z}\int_0^h w dy = 0 ,$$

where

(19) $$\int_0^h u dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(C_1 + 2C_2 \right) \frac{1}{\varphi} th\varphi \frac{h}{2} e^{i\omega t} d\omega - \frac{\partial q}{\partial x} h ;$$

(20) $$\int_0^h w dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 2C_3 \frac{1}{\varphi} th\varphi \frac{h}{2} e^{i\omega t} d\omega - \frac{\partial q}{\partial z} h .$$

By representing the hyperbolic function $th\left(\varphi h/2\right)$ with power series in $\left(\varphi h/2\right)$ and after neglecting the higher order terms, the expressions (19) and (20) could be rewritten as

(21) $$\int_0^h u dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(C_1 + 2C_2 \right) \left(\frac{h}{2} - i \frac{\omega h^3}{24\upsilon} + ... \right) e^{i\omega t} d\omega - \frac{\partial q}{\partial x} h ;$$

(22) $$\int_0^h w dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} C_3 \left(h - i \frac{\omega h^3}{12\upsilon} + ... \right) e^{i\omega t} d\omega - \frac{\partial q}{\partial z} h .$$

Noting that

$$\left(i\omega \right)^n e^{i\omega t} = \frac{d^n}{dt^n} e^{i\omega t}$$

the above expresses are transformed to

(24) $$\int_0^h u dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(C_1 + 2C_2 \right) \left(\frac{h}{2} - i \frac{h^3}{24\upsilon} \frac{d}{dt} + ... \right) e^{i\omega t} d\omega - \frac{\partial q}{\partial x} h ;$$

(25) $$\int_0^h w dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} C_3 \left(h - i \frac{h^3}{12\upsilon} \frac{d}{dt} + ... \right) e^{i\omega t} d\omega - \frac{\partial q}{\partial z} h .$$

The substitution of inverse transforms (16) in (24) and (25) yields

(26) $$\int_0^h u dy = \frac{h}{2} u_h - \frac{h^3}{24\upsilon} \frac{\partial u_h}{\partial t} - \frac{h^3}{12\upsilon} \frac{d}{dx} \left(\frac{\partial q}{\partial t} \right);$$

(27) $$\int_0^h w dy = -\frac{h^3}{12\upsilon} \frac{d}{dz} \left(\frac{\partial q}{\partial t} \right).$$

After substitution of the last two expresses into (18) is obtained the partial differential equation for the pressure distribution, which represents a generalized form of Reynolds equation

(28) $$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z} \right) = 6\eta \frac{\partial}{\partial x}\left(h u_h \right) - 12\eta u_h \frac{\partial h}{\partial x} - 12\eta v_h - \frac{1}{2}\rho \frac{\partial}{\partial x}\left(h^3 \frac{\partial u_h}{\partial t} \right).$$

Here

(29) $$h = c + e\cos\frac{x}{r} + kp = h_0 + \delta \, .$$

The last term on the right-hand side of (28) is connected with the presence of inertia forces due to the unsteady linear velocity of the journal surface.

By introducing the dimensionless parameters the above equation could be rewritten as:

(30) $$\frac{\partial^2 \Pi}{\partial \theta^2} + \alpha \frac{\partial^2 \Pi}{\partial {z_1}^2} + \frac{3}{H}\frac{\partial H}{\partial \theta}\frac{\partial \Pi}{\partial \theta} + \alpha \frac{3}{H}\frac{\partial H}{\partial z_1}\frac{\partial \Pi}{\partial z_1} = \frac{1}{H^3}\frac{\partial H}{\partial \theta} + \frac{1}{H^3}\left(\dot{\varepsilon}\cos\theta + e\dot{\gamma}\sin\theta\right) -$$
$$-\frac{\lambda}{48}\left[\ddot{\varepsilon}\cos\theta + \left(\dot{\varepsilon}\dot{\gamma} + \varepsilon\ddot{\gamma}\right)\sin\theta\right] - \frac{\lambda}{16}\frac{1}{H}\frac{\partial H}{\partial \theta}\left[\ddot{\varepsilon}\sin\theta - \left(\dot{\varepsilon}\dot{\gamma} + \varepsilon\ddot{\gamma}\right)\cos\theta\right],$$

where the non-dimensional film thickness is in a form

(31) $$H = 1 + \varepsilon\cos\theta + K_h \Pi \, .$$

3. NUMERICAL SOLUTION AND RESULTS

The solution of the differential equation for pressure distribution is done by using finite difference method. Unknowns are the dimensionless pressure and film thickness at every grid point (*j, k*). The derivates in abovementioned equation are expressed via finite differences in two variables.

The calculating procedure presupposes at the beginning un-deformability of the elastic layer. At this assumption the HD pressure is calculated. As result of this pressure the radial displacements of the liner surface points and the relevant film thickness are determined. The obtained values of this changed film thickness are base for pre-calculation of the pressure. This process is repeated until a satisfactory convergence rate for pressure values is reached.

The Reynolds boundary conditions are taken and because of satisfy them the negative values of pressure are immediately put to zero.

The numerical results are obtained for Newtonian fluid at different values of the diameter-to-length ratio and angular velocity of the precession. The other conditions are: $\omega = 314$ [s^{-1}]; $r = 15.10^{-2}$ [m]; $c = 3.10^{-4}$ [m]; $\eta = 4.10^{-2}$ [Pa.s]; $d = 2.10^{-3}$ [m]; $E = 7{,}33.10^{7}$ [Pa]; $\mu = 0{,}4$. The presented solution and results correspond only to the dynamic film effect ($\dot{\varepsilon} = \ddot{\varepsilon} = 0$).

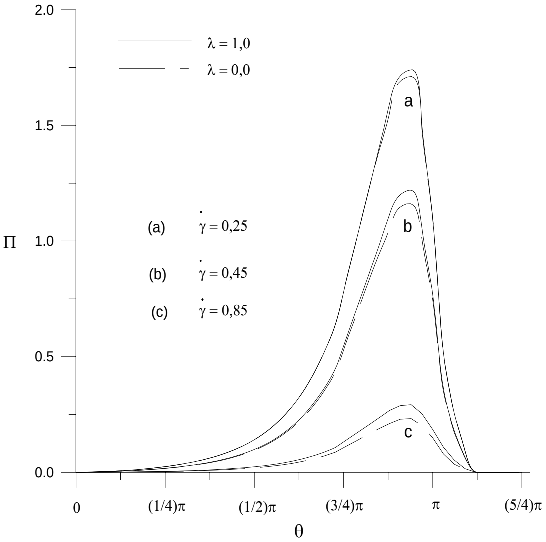

Fig. 2 HD pressure distribution

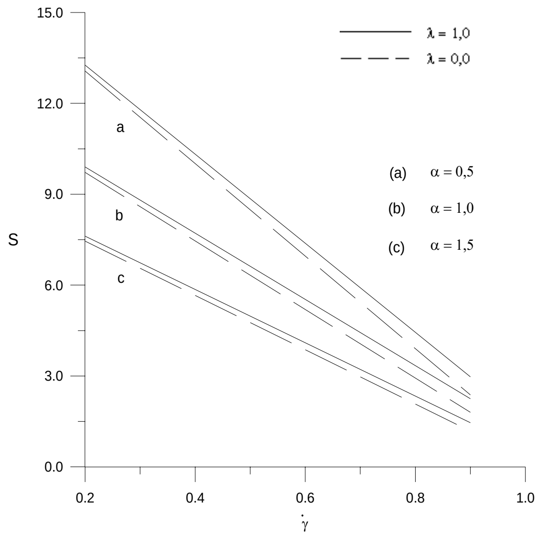

Fig. 3 Sommerfeld number versus precession velocity

The numerical results are shown in Figs. 2, 3. On the first of them is presented the HD pressure distribution at three different values of angular velocity of the precession. On the next figure is given variation of Sommerfeld number with precession velocity. The contribution of the local inertia forces is presented by plotted curves and lines, which correspond to $\lambda = 1,0$ (the "inertia-free" solution is marked as $\lambda = 0,0$).

4. CONCLUSIONS

On the base of the obtained numerical results can be generalized, that the consideration of local inertia terms increases the values of HD pressure and Sommerfeld number. The maximal inertia effects on the pressure and load capacity are of order of 10-12 %. It is evident that the contribution of local inertia forces to the HD forces is small as whole, but for the extreme values of pressure it is of significance. Obviously in a more precise bearing analysis the inertia forces should be taken into consideration.

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