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MODIFICATIONS ERLANG' S DISTRIBUTION OF RANDOM VARIABLE.

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Abstract: *It is valid for the normally divided random variable that in boundaries ± 3σ (σ = standard deviation) from the average value can be found 99,73 % of all the values of distribution function* ***F(x).*** *Having used a computer experiment with certain limitation the possibility of application of "3σ rule" for the random variable with Erlang' s distribution was proved.*

1. Gauss-Laplace' distribution of the random variable and the 3σ rule.

The continuous random variable φ is defined by a certain distribution characterized by so called frequency function ***f (x)*** (known as well as a density of probability) or by the distribution function ***F(x)***.

The random variable **x** occurs in certain boundaries according to *Fig. 1.*

The function ***f(x)*** is in the graphical representation shown as a certain curve (mostly bell-shaped). ***F(x)*** represents an area limited by the curve above the horizontal centre line to the adequate elected point ***x.***

The function ***f(x)*** is usually adapted so that the whole area below the curve is equal to one the lower and upper boundary thus being ± ∞**.**

The probability of occurance of the value ***x*** in the vicinity of the boundary lines is negligible. Therefore it is advantageous to set so called minimal and maximal values (boundaries) of the variable ***x*** which relatively precisely describe the original distribution. These boundaries are called x_{min} and x_{max}.

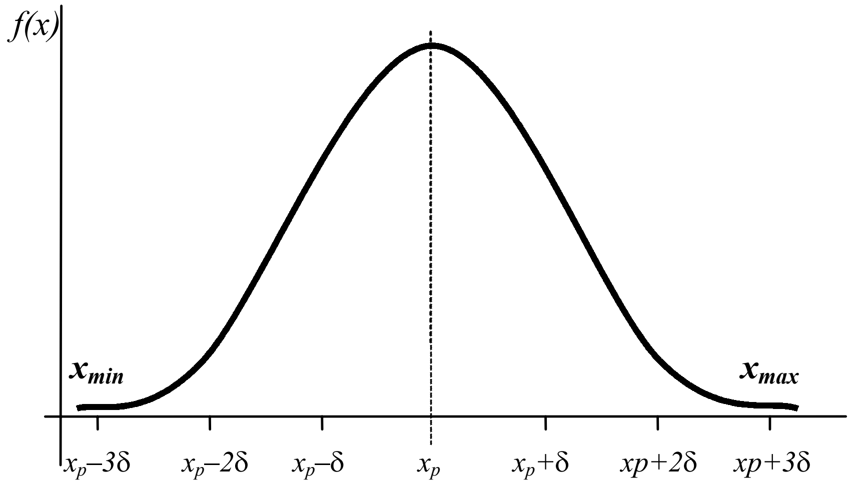

Fig.1. 3σ rule of the continuous random variable

The normal distribution depends on two parameters, standard deviation (σ) and average value (**x_p**). To the selected value of random variable ***x*** responds also the corresponding part of the area ***F(x)*** expressed in (%). This datum represents also the probabilities of occurance of random variable **x.** This relation is given for the normal distribution with the application of normal deviation.

To demonstrate these conclusions most often is used **Gauss-Laplace'** so called **normal distribution** of random variable which is set by the frequency function f(x)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_p)^2}{2\sigma}}$$

where is σ - standard deviation
x - value of random variable in boundaries ± ∞
x_p - average value of random valuable

The size of area ***F(x)*** in the distance ±σ from the average value x_p in normal, Gauss-Laplace distribution .

Variable *x* in boundaries	**Part of area *F(x)* (%)**
$x_p \pm 1\sigma$	**68,26**
$x_p \pm 1{,}96\sigma$	**95**
$x_p \pm 2\sigma$	**95,44**
$x_p \pm 2{,}58\sigma$	**99**
$x_p \pm 3\sigma$	**99,73**
$x_p \pm 3{,}29$	**99,9**

The measured value of random value φ does not almost definitely differ from its average value φ more than ± *3*σ . It means that the random variable φ gains in the interval ***xp*** ± *3*σ nearly 100 % of its values and so it is sufficient to calculate the probability of random valuable φ mainly in this interval. This rule is called "rule (theory) *3*σ" . Using the *3*σ rule the parameters of normal distribution express average

value $x_p = \frac{x_{\max} - x_{\min}}{2}$, standard deviation $\sigma = \frac{x_{\max} - x_{\min}}{6}$.

2. The application of "3σ theory" to Erlang' s distribution of random variable.

Erlang' s distribution of random variable is sufficiently general and can cover great mount of phenomena. Mathematically it is less complicated. So, because of these qualities it is often applied to the mathematical simulation . It is defined by these parameters ***a, b, y*** where are:

y - shift of file
a, b - parameters which are related to the average value xp and the standard deviation as

$$x_p = \frac{a}{b}, \qquad \sigma = \frac{\sqrt{a}}{b}$$

If we know only parameters ***a, b, y*** of Erlang' s distribution, it is difficult to imagine its course. If we know its shape, it is complicated to obtain the corresponding parameters a,b,y . More advantageous is to come out from the average and boundary values. These date are easy to imagine and can be relatively simply obtained by estimation, observation or measurements. This simplification enables the application of the *3*σ rule in Erlang' s distribution.

If average value x_p , minimal value x_{min} and maximum value x_{max} are known, then parameters ***a, b, y*** can be calculated :

$$a = x_p b, \qquad y = x_{\min}, \qquad b = \frac{x_p}{\sigma^2}, \qquad a = x_p b$$

In generating the pseudo-random numbers with Erlang' s distribution using the *3*σ rule there are certain limitations. The aim of the computer xperiments was to set the range of these limitations.

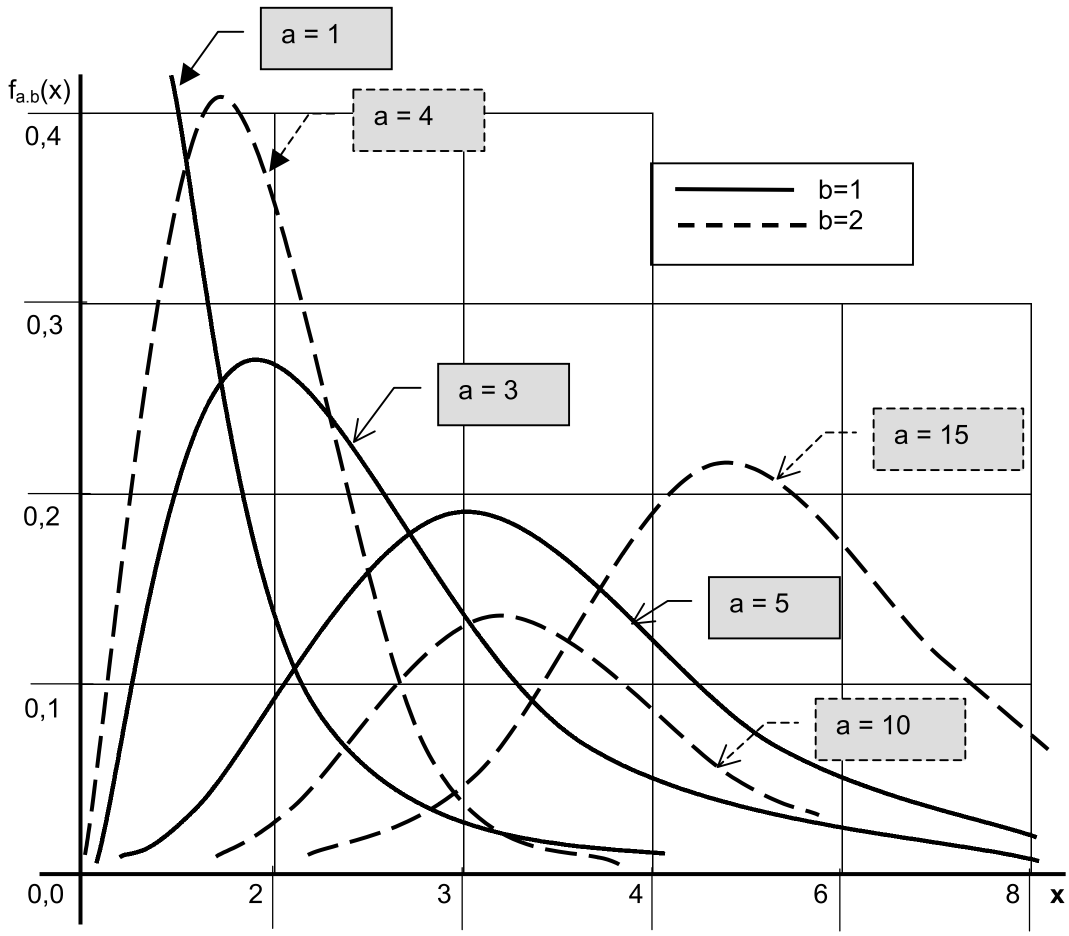

Fig.2. **Erlang' s distribution of random variable**

The interval of validity for the ***3***σ rule will be restricted by the left boundary inclination of Erlang' s distribution as far as the exponentional, and by the right inclination. Next restriction will be the ability of computer or the programme algorithm to handle big numbers (factorial).

The frequency function of random variable with Erlang' s distribution is shown in the relation

$$f_{(x)} = \frac{b^a x^{a-1}}{(a-b)!} e^{-bt}$$

The computer experiment was based on the following method :

(1) definition of $\boldsymbol{x_{max}}$, $\boldsymbol{x_p}$, $\boldsymbol{x_{min}}$,

(2) calculation of parameters of Erlang' s distribution ***a, b, y*** from given values $\boldsymbol{x_{max}}$, $\boldsymbol{x_p}$, $\boldsymbol{x_{min}}$,

(3) generation of the set of random variables with Erlang' s distribution with the obtained parameters ***a, b, y,***
(4) calculation of new $\boldsymbol{x_p}$ out of the set of generated values of random variable.

Next step was to compare the equality of the ordered values with those obtained from the generated set ($\boldsymbol{x_{min}}$, $\boldsymbol{x_p}$, $\boldsymbol{x_{max}}$). As suitable were characterized all the cases where the compared values were equal to the coefficient 0,98.

The results of the experiment set the following limitations for the possibility of application of 3σ rule to the generating of random valuable with Erlang' s distribution:
- value xp - x min > l.5 - this limitation means to eliminate the extreme inclination of the left distribution.
- values $\mathbf{x_{max}}$ - $\mathbf{x_p}$ > $\mathbf{x_{min}}$ - this limitation represents the right inclination elimination.

Parameter ***a*** - is to be a natural number processable by the programme algorithm. All the around average values were admissible.

3. Conclusion

Except the above mentioned limitations the ***3***σ rule is applicable for the simple input mission by generating of pseudo- random numbers with Erlang' s distribution.

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