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TORSIONAL STIFFNESS OF MULTIPLE-UNITS RAILWAY VEHICLE WITH LINEAR AND SYMMETRIC ACTION OF SUSPENSION SYSTEM

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Abstract: *This paper presents analytical expression for torsional stiffness of railway vehicles which consist of multiple carbodies connected via links providing symmetric action of suspension system to connected units.*
Keywords: *railway, wagon, multiple units, torsional stiffness*

INTRODUCTION

During recent years, analyses of torsional stiffness of railway vehicles in non-standard cases occupied attention of the authors [1-5]. Initially inspired by demands of manufacturers [1], the analysis of derailment conditions of railway wagons with two units and three axles led to development of more general approach [3], and eventually, to more general conclusions.

The approach applied to the case of vehicle with two units [3] is suitable for extension to the case of vehicle with multiple units. However, analytical expressions for torsional stiffness of the vehicle that are obtained are too intricate to be of practical importance. Therefore, it is important to analyze cases of vehicles where structural properties and reasonable approximations enable derivation of forms of the analytical expression for torsional stiffness, which are suitable for analyses.

Important simplifications can be achieved in case of vehicles that have units linked so that action of suspension system, arising as reaction to movement of units, is symmetric respective to those units. Being that it is the case with vast majority of types of railway vehicles, this simplification, applied in this paper, leads to results of wide importance.

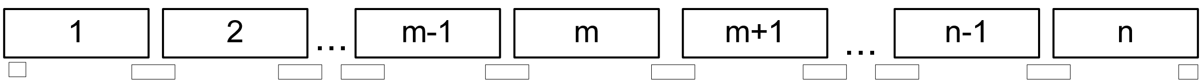

Figure 1. Composition and numeration of railway vehicle with multiple units

MODEL

The model used for analysis of torsional stiffness is extension of the model applied to analyses of torsional stiffness of railway vehicle with two units and three axles [5]. The investigated structure consists of *n* identical units; each unit is linked with two other units except for the first and last unit (Fig 1.) so as that connected units have common suspension; suspension system is attached to the lower plane of carbody of unit *m* in points denoted as $1^{(m)}$ to $4^{(m)}$, forming a rectangle; vertical displacements of those points are denoted as $z_i^{(m)}$ (i =1,2,3,4); on the other side, suspension system is attached to wheelset; vertical displacements of wheelset on which suspension system acting in point $i^{(m)}$ is supported is denoted as $h_i^{(m)}$ (Fig. 2).

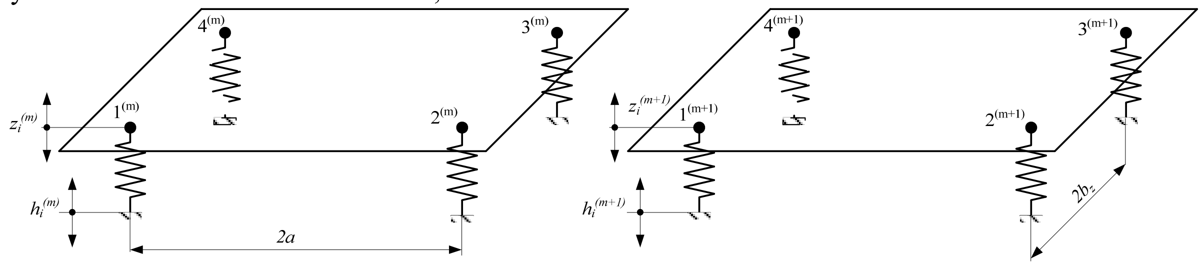

Figure 2. Model of carbodies and suspension system of considered vehicle

Due to track irregularities, wheelsets undergo vertical displacements, and, through the action of suspension system, carbodies of units undergo inclination and torsional deformation; the amounts of vertical displacements of the considered points due to torsional deformation are [6]

$$\delta_1^{(m)} = +\delta^{(m)}, \delta_2^{(m)} = -\delta^{(m)}, \quad \delta_3^{(m)} = +\delta^{(m)}, \delta_4^{(m)} = -\delta^{(m)} \qquad (1)$$

where [5]

$$\delta^{(m)} = \frac{1}{4}\left\{z_1^{(m)} - z_2^{(m)} + z_3^{(m)} - z_4^{(m)}\right\}. \qquad (2)$$

The torsional deformation causes elastic forces acting upon the carbody structure in the considered points; the intensity of the forces, denoted as F_m^*, can be calculated as

$$F_m^* = c_t^* \cdot \delta^{(m)}, \qquad (3)$$

where c_t^* represents appropriate equivalent stiffness given by expression

$$c_t^* = \frac{C_t^*}{2ab_z^2}, \qquad (4)$$

where C_t^* is torsional stiffness of carbody according ORE B55 Rp.8 [7] and ORE B12 DT135 [8], *2a* represents longitudinal measuring base (distance between axles or bogies) and $2b_z$ represents distance between spring sets on opposite sides of the vehicle (Fig. 2).

Elastic forces caused by torsional deformation, which act on carbody in considered points, are balanced by forces exerted by suspension system. Those forces depend on design of suspension system and displacements of wheels and carbody. In this paper, we consider suspension systems designed so as to provide linear dependence of forces on displacements, so

$$\frac{\partial F_i^{(m)}}{\partial z_j^{(l)}} = -\frac{\partial F_i^{(m)}}{\partial h_j^{(l)}} = \mathrm{K}_{ij}^{(m)(l)} = const. \qquad (5)$$

where $F_i^{(m)}$ represents the force acting upon *i*-th point of *m*-th unit and K represents matrix describing influence of displacements of structure on forces of suspension system. Suspension system joins adjacent units, so that K matrix has nonzero elements only in the following cases:

- $l = m$ and $i = j$ (influence of displacement of carbody of the considered unit)
- $l = m-1, i=1, j=2$ and $i=4, j=2$ (influence of displacement of rear part of carbody of previous unit on front part of the considered unit),
- $l = m+1, i=2, j=1$ and $i=2, l=4$ (influence of displacement of front part of carbody of next unit on rear part of the considered unit)

The model analyzed in the paper also assumes symmetry of action of suspension system, expressed through the following equation

$$\mathrm{K}_{ij}^{(m)(l)} = \mathrm{K}_{ji}^{(l)(m)} = \mathrm{K}_{ii}^{(m)(m)} = K, \qquad (6)$$

which essentially means that displacement of any unit causes the same force on both units connected via the suspension system. This assumption is valid in the case of symmetric design of suspension system, which is the most frequent case of interest in practice.

The only exception to equation (6) are elements of K matrix which describe the forces acting upon the front part of the first unit and rear part of the last unit of the vehicle, being that these parts of vehicle must have suspension system different in comparison to rest of the vehicle (there is no more units in front of the first unit or behind the last unit), so it will be put

$$\mathrm{K}_{11}^{(1)(1)} = \mathrm{K}_{44}^{(1)(1)} = \mathrm{K}_{22}^{(n)(n)} = \mathrm{K}_{33}^{(n)(n)} = C_z \qquad (7)$$

in accordance to usual notification used in ORE B55 Rp.8 and ORE B12 DT135.

It is of interest to notice that torsional stiffness of railway vehicle with multiple units can be considered and defined in different manners, which is to be topic of separate paper. Torsional stiffness of the vehicle, applied in this paper, denoted as C_{tA} and developed with aim to demonstrate the approach to determination of torsional stiffness, is based on the concept applied in ORE B55 Rp.8 and ORE B12 DT135, expressed through the following equation:

$$C_{tA} = \frac{\Delta F}{\alpha} = \frac{\Delta F}{\left(\dfrac{h}{2a}\right)} = 2a\frac{\Delta F}{h}. \qquad (8)$$

with h being vertical track distortion at wheel of leading axle of the vehicle, and ΔF being resulting variation of vertical component of force in the wheel-rail contact. It is, of course, worthy of noticing that deformation of railway vehicle with multiple units is not strictly along lines of concept of deformation of structure considered in ORE documents, but the concept is still adopted being that the essence of its application is not calculation of torsional stiffness, but calculation of variation of vertical force in wheel-rail contact, and its influence on derailment of railway vehicle.

RESULTS

In the case considered by definition of torsional stiffness in ORE documents ($h_1^{(1)}=h$, $h_i^{(m)}=0$, for $i\neq 1$ or $m\neq 1$), equilibrium of forces acting in points $i^{(m)}$ (i=1,2,3,4, m=1,2,…,n) is described by the following equations:

$$\begin{aligned} C_z(z_1^{(1)} - h) + c_t^*\delta^{(1)} &= 0 \\ K(z_2^{(1)} + z_1^{(2)}) - c_t^*\delta^{(1)} &= 0 \\ K(z_3^{(1)} + z_4^{(2)}) + c_t^*\delta^{(1)} &= 0 \\ C_z z_4^{(1)} - c_t^*\delta^{(1)} &= 0 \end{aligned} \qquad (9)$$

for the points of the first unit,

$$\begin{aligned} K(z_1^{(2)} + z_2^{(1)}) + c_t^*\delta^{(2)} &= 0 \\ K(z_2^{(2)} + z_2^{(3)}) - c_t^*\delta^{(2)} &= 0 \\ K(z_3^{(2)} + z_4^{(3)}) + c_t^*\delta^{(2)} &= 0 \\ K(z_4^{(2)} + z_3^{(1)}) - c_t^*\delta^{(2)} &= 0 \end{aligned} \qquad (10)$$

for the points of the second unit,

$$\begin{aligned} K(z_1^{(m)} + z_2^{(m-1)}) + c_t^*\delta^{(m)} &= 0 \\ K(z_2^{(m)} + z_2^{(m+1)}) - c_t^*\delta^{(m)} &= 0 \\ K(z_3^{(m)} + z_4^{(m+1)}) + c_t^*\delta^{(m)} &= 0 \\ K(z_4^{(m)} + z_3^{(m-1)}) - c_t^*\delta^{(m)} &= 0. \end{aligned} \qquad (11)$$

for the points of middle units (m = 2,3,…,n-1) and

$$\begin{aligned} K(z_1^{(n)} + z_2^{(n-1)}) + c_t^*\delta^{(n)} &= 0 \\ C_z z_2^{(n)} - c_t^*\delta^{(n)} &= 0 \\ C_z z_3^{(n)} + c_t^*\delta^{(n)} &= 0 \\ K(z_4^{(m)} + z_3^{(m-1)}) - c_t^*\delta^{(m)} &= 0. \end{aligned} \qquad (12)$$

for the points at the last unit.

Comparing the second and the third of equations (9) with the first and the fourth of equations (10), one comes to conclusion that $\delta^{(2)} = -\delta^{(1)}$. Performing similar comparison for other units, it can be easily concluded that the consequence of symmetric design of car is that

$$\delta^{(1)} = -\delta^{(2)} = \ldots = (-1)^{m+1}\delta^{(m)} = \ldots = \delta. \qquad (13)$$

Introducing now the result (13) and the following designations

$$u = \frac{c_t^*}{C_z}\delta \text{ and } w = \frac{c_t^*}{K}\delta \qquad (14)$$

in equations (9)-(12), the following chain of equations can be obtained

$$\begin{aligned} z_1^{(1)} &= h - u \\ z_2^{(1)} + z_1^{(2)} &= w \\ z_3^{(1)} + z_4^{(2)} &= -w \\ z_4^{(1)} &= u \end{aligned} \qquad (9a)$$

$$\begin{aligned} z_1^{(2)} + z_2^{(1)} &= w \\ z_2^{(2)} + z_1^{(3)} &= -w \\ z_3^{(2)} + z_4^{(3)} &= w \\ z_4^{(2)} + z_3^{(1)} &= -w \end{aligned} \qquad (10a)$$

$$\begin{aligned} z_1^{(m)} + z_2^{(m-1)} &= (-1)^m w \\ z_2^{(m)} + z_1^{(m+1)} &= (-1)^{m+1} w \\ z_3^{(m)} + z_4^{(m+1)} &= (-1)^m w \\ z_4^{(m)} + z_3^{(m-1)} &= (-1)^{m+1} w \end{aligned} \qquad (11a)$$

$$\begin{aligned} z_1^{(n)} + z_2^{(n-1)} &= (-1)^n w \\ z_2^{(n)} &= (-1)^{n+1} u \\ z_3^{(n)} &= (-1)^n u \\ z_4^{(n)} + z_3^{(n-1)} &= (-1)^{n+1} w \end{aligned} \qquad (12a)$$

Inserting obtained expressions for vertical displacements $z_i^{(m)}$ in equations (2) for the cases m=1,2,…,n-1, describing geometrical conditions that first n-1 units of deformed vehicle satisfy, it is obtained

$$\begin{aligned} z_1^{(1)} - z_4^{(1)} &= h - 2u \\ z_1^{(2)} - z_4^{(2)} &= 2w + 4\delta - (h - 2u) \\ z_1^{(n)} - z_4^{(n)} &= \\ &= (-1)^n \left[(n-1)(2w + 4\delta) - (h - 2u)\right] \end{aligned} \tag{15}$$

Finally, combining the last of equations (15) with (12a) and (13), inserting them into geometrical condition (2), that is to satisfy the last unit (m=n), it is obtained

$$\begin{aligned} &(-1)^n \left[(n-1)(2w + 4\delta) - (h - 4u)\right] \\ &= (-1)^{n+1} 4\delta \end{aligned}, \tag{16}$$

wherefrom, using (14) it can finally be derived expression for displacement of considered points due to torsional deformation:

$$\delta = \frac{h}{4n + 4\dfrac{c_t^*}{C_z} + 2(n-1)\dfrac{c_t^*}{K}}, \tag{17}$$

Once δ is known, it is straightforward to determine expression for variation of vertical component of force in wheel-rail contact

$$\Delta F = \frac{h}{\dfrac{4n}{c_t^*} + \dfrac{4}{C_z} + \dfrac{2(n-1)}{K}}, \tag{18}$$

and expression for torsional stiffness of considered railway vehicle

$$\begin{aligned} \frac{1}{C_{tA}} = 10^3 \Bigg[& n\frac{(2b_A)^2}{C_t^*} + \\ & + \frac{1}{2a}\left(\frac{2b_A}{2b_z}\right)^2 \left(\frac{4}{C_z} + \frac{2(n-1)}{K}\right) \Bigg] \end{aligned} \tag{19}$$

ANALYSIS

The derived expression for torsional stiffness (19) may be analyzed for its special cases, which were already investigated. For n=1 the expression reduces to

$$\frac{1}{C_{tA}} = 10^3 \left[\frac{(2b_A)^2}{C_t^*} + \frac{1}{2a}\left(\frac{2b_A}{2b_z}\right)^2 \frac{4}{C_z} \right] \tag{20}$$

which is well known expression for torsional stiffness of railway vehicle with two axles derived in [ORE B55]. For n=2 and K=C_z/4 one obtains expression

$$\frac{1}{C_{tA}} = 10^3 \left[2\left(\frac{(2b_A)^2}{C_t^*} + \frac{1}{2a}\left(\frac{2b_A}{2b_z}\right)^2 \frac{6}{C_z} \right) \right] \tag{21}$$

identical to the one derived during investigations of torsional stiffness of DDam wagon.. Agreement of derived formula with already obtained results serves as one mean for checking its validity.

The results (17), (18) and (19) show that torsional deformation of carbody, variation of vertical force in wheel-rail contact and torsional stiffness of the vehicle decrease when number of units is increased. The explanation is that, according to proposed model, increased number of units is dividing deformation through action of linkages and suspension system. The validity of the result depends on validity of critical assumption that action of suspension system is symmetric, being that this assumption leads to conclusion of equal amounts of torsional deformation of all units, expressed through equation (13). Experimental verification of the assumption is not available at the moment.

CONCLUSION

The paper considers torsional stiffness of railway vehicle with multiple units. Suspension system of the vehicle was considered linear and symmetric in previously described meaning. Developed model led to analytical expression for torsional stiffness of vehicle, which enables calculation of the quantity and predicts its reduction with increasing of number of units.

The obtained result still needs experimental verification, but in the case it is validated, the obtained expression has wide applicability, being that it is derived under assumptions which are valid for wide class of vehicles.

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КОРАВИНА НА УСУКВАНЕ НА МНОГОКОМПОНЕНТНО ЖЕЛЕЗОПЪТНО ВОЗИЛО С ЛИНЕЙНО И СИМЕТРИЧНО ДЕЙСТВИЕ НА СИСТЕМАТА ЗА ОКАЧВАНЕ

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ИТАЛИЯ

***Ключови думи**: железница, вагон, многобройни компоненти, коравина на усукване*

***Анотация**: Тази статия представя аналитичен израз на коравината на усукване на железопътните возила, състоящи се от многобройни каросерии, съединени посредством връзки, осигуряващи симетричното действие на системата за окачване към свързаните компоненти.*