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# CONTRIBUTIONS TO THE STUDY OF THE ELASTIC CONSTANTS FOR SUSPENSIONS USED ON HIGH SPEED STEERING BOGIES

**Ioan Sebeşan, Mădălina Dumitriu, Cristina Tudorache**

*ioan_sebesan@yahoo* *madalinadumitriu@yahoo.com* *cristi_tudorache2003@yahoo.com*

*POLITEHNICA University of Bucharest*
*Faculty of Transports, Rolling Stock Engineering Department*
*313 Splaiul Independentei, sector 6, 77206, Bucharest. http://www.pub.ro*
**ROMANIA**

***Abstract:*** *The elastic characteristics of the suspension influence the own vibrations of the vehicle's suspended masses. These vibrations must be situated in some specific areas. Starting from the boundary condition of achieving the mandatory values for the own frequencies the necessary stiffness and the static loads for the suspension springs may be determined.*

***Key words*****:** *elastic characteristics for the suspension, own vibrating frequencies, steerable-axle bogies*

## INTRODUCTION

In order to ensure a smooth rolling quality and to avoid the resonance phenomena, the suspensions of the railway vehicles are designed to operate in a very strict range of frequencies. The necessary stiffness of elastic elements may be determined starting from the boundary conditions for the own frequencies.

## BOUNDARY CONDITIONS FOR ADOPTING THE ELASTIC CONSTANTS OF THE SUSPENSION

In the particular case of the double suspension bogies, the primary suspension's vertical vibrations are coupling the ones generated by the secondary, thus resulting two own frequencies – the "low" and the "high". Strictly by comfort means, it is a common custom among rolling stock manufacturers to lower the "low" frequency at around 1 Hz.

The "high" own frequency is currently situated between 5 and 8 Hz. These frequencies depend by the flexibilities of the springs mounted in both suspension decks and by the suspended masses (the bogie frame and the car body). When calculating the suspension parameters it is mandatory to note that the suspended masses ratio might suffer very slight modifications and a greater flexibility of the primary suspension will consequently generate high amplitudes for the gallop oscillations. These might be transmitted to the car body thus generating unwanted resonance effects, together with excessive stresses on the wheel set and large variations of the axle loads, which combined might seriously affect the safety running [1]. It is also necessary to establish the bogies own frequencies outside some critical values (e.g. the perturbation frequencies generated by the rail joints) and the avoidance of the resonance between the gallop oscillations and the bending vibrations induced by the car body.

The car body's bending vibrations might couple to the vertical high-frequency ones and generate important effects when the own frequencies are close. Therefore, the elastic constants have to be adopted in a manner that the "high" own frequency of the vertical suspension and the frequency of the gallop vibrations of the bogie must be situated outside the range of the own bending frequencies of the car body (8-13 Hz).

The need to "un-couple" the vibrations for the high-speed railway vehicles assume the following values for the own frequencies [4]:

- for vertical movements: the "low" at 1 Hz and the "high" among 5 to 7 Hz;
- for the hunting movement of the car body: 0,8 ... 0,9 Hz;
- for the own shaking-rolling movements – the "low" at 0,5...0,6 Hz and the "high" at 1,2...1,3 Hz.

When distributing the flexibilities on the suspension decks it is mandatory to note that there are situations in which the flexibility of the primary suspension is limited to some specific values due to the vehicle design, transmission, braking system etc. If the vertical suspension is highly flexible the own rolling frequency of the car body is lowered, thus influencing the transversal behaviour when running in straight line. Achieving the 1 Hz vertical frequency requires an elastic suspension, but the curves determine an unacceptable quasi-statically rolling angle. In this case, it is necessary to equip the vehicle with anti-roll bars or find other constructive solutions, such as the tilting car body [2].

Transversally, the dynamic behaviour of the vehicle is determined by the bogie characteristics and, on the other hand, by the traction transfer device.

The hunting movement is particularly important for the vehicle stability – thus for the safety – and in order to ensure the transversal comfort.

The axles hunting movements are transmitted to the bogie frame and to the car body through the traction transfer device. The anti-yawing dampers (situated on both sides of the car body and bogie frame) are playing a key role, especially when the hunting frequencies of the bogie and the car body are significantly close. The achievement of a friction momentum between the bogie and the car body reduces the hunting but, in return, increases the transversal stresses transmitted to the rails when running in curves. It is essential for the suspension to ensure the maximum reduction possible for both, the hunting movement and the combined shaking-rolling movements of the car body. In order to achieve this, it is essential to ensure a" controlled independence" between the bogie and the car body.

By adopting sufficiently lower frequencies for the car body – secondary suspension system, compared to the frequency of the hunting movement at the bogie level, a reduction in hunting effects occurs and, in addition, the risk of resonance at high speeds is avoided [3].

Another important role in extending the stability domain associated with the hunting movement is given by the longitudinal and transversal flexibilities of the primary suspension.

The axle guiding system must carry the horizontal forces with no damage to the suspension system, ensuring the correct axle position as well. The rigid guiding systems (that amplify the wavelength of the hunting movement) proved to be satisfactory for speeds up to 140 km/h. At higher speeds, the elastic guiding systems are much reliable because they enable the axle to gain a radial orientation, which is preferable when running in curves. The new generation of steering-axle bogies fulfil the essential requirements for hunting when running in straight line and the extension of vehicle capabilities for negotiating the curvature radiuses as well [3].

## DETERMINIG THE ELASTIC CONSTANTS OF THE SUSPENSION BY CONSIDERING SOME MANDATORY OWN FREQUENCIES

The first step in designing the suspension consists in setting the vertical suspension stiffness and the spring static load. Studying the suspension of the railway vehicles is difficult due to the great number of degrees of freedom (dof) for the vibrant system. For a vehicle equipped with bogies, there are 18 dof (taking into account only the suspended masses of the two bogies and the car body). If the longitudinal and recoil movements are neglected there are still 15 dof's left.

A major criterion followed in order to design a railway vehicle is to minimise the reciprocal effects of the vibrating elements. This is doable by "un-coupling" the movements or, to say so, by ensuring the "controlled independence" of these elements. Thus, the complex suspension system of a railway vehicle may be practically decomposed into individual simple systems with one or two dof's.

A simple mechanical model used for railway vehicles takes into account the suspended masses of the bogies and the car body mass linked through elastic and damping elements. With the elastic and geometric symmetrical conditions fulfilled, the equivalent model for a vehicle equipped with double suspension bogies is shown below (fig.1).

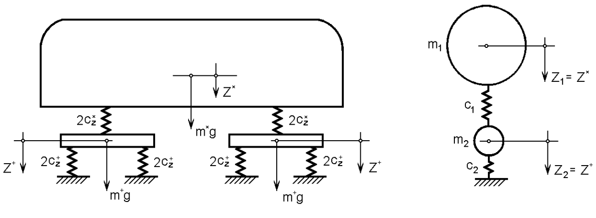


**Fig. 1. Mechanical models for studying the shaking vibrations**

a – vehicle equipped with double suspension bogies; b – equivalent model with two dof's

If $m^+$ - the suspended mass of a bogie; $m^*$- the car body mass; $2c_z^+$- the equivalent spring stiffness for the primary suspension; $2c_z^*$- the equivalent spring stiffness for the secondary suspension, the equivalent model will have:

$$m_1 = m^*/2; \qquad m_2 = m^+;$$
$$(1) \quad c_1 = 2c_z^*; \qquad c_2 = 4c_z^+;$$
$$\omega_1^2 = c_1/m_1; \qquad \omega_2^2 = (c_1+c_2)/m_2.$$

If $f_1$ and $f_2$ stand for the static displacement of the springs in the secondary and primary suspension, the stiffnesses of the springs used in the equivalent model will be:

$$(2) \quad c_1 = \frac{m_1 g}{f_1}\ ;\ c_2 = \frac{(m_1+m_2)g}{f_2} \quad .$$

Between the own angular speeds and static displacements the following relations occur:

$$\omega_{z1}^2 + \omega_{z2}^2 = g\left(1+m_1/m_2\right)\left(f_1+f_2\right)/\left(f_1 f_2\right);$$
$$(3)$$
$$\omega_{z1}^2\,\omega_{z2}^2 = g^2\left(1+m_1/m_2\right)/\left(f_1 f_2\right),$$

From which, depending on the own frequencies

$\nu_{z1} = \omega_{z1}/(2\pi)$ and $\nu_{z2} = \omega_{z2}/(2\pi)$ will obtain

$$f_1 + f_2 = \frac{g}{4\pi^2}\left(\frac{1}{\nu_{z1}^2} + \frac{1}{\nu_{z2}^2}\right);$$
$$(4)$$
$$f_1 f_2 = \frac{g^2}{16\pi^4}\left(1+\frac{m_1}{m_2}\right)\frac{1}{\nu_{z1}^2\,\nu_{z2}^2}.$$

The static displacements of the suspension will be (5)

$$f_{1,2} = \frac{g}{8\pi^2}\cdot$$
$$\cdot\frac{\nu_{z1}^2 + \nu_{z2}^2 \pm \sqrt{\left(\nu_{z1}^2+\nu_{z2}^2\right)^2 - 4\left(1+\dfrac{m_1}{m_2}\right)\nu_{z1}^2\,\nu_{z2}^2}}{\nu_{z1}^2\,\nu_{z2}^2}$$

If the following condition is fulfilled

$$(6) \quad \left(\nu_{z1}^2+\nu_{z2}^2\right)^2 - 4\left(1+m_1/m_2\right)\nu_{z1}^2\,\nu_{z2}^2 \ge 0.$$

The total system stiffness will be

$$(7)\ c_\Sigma = \frac{c_1 c_2}{c_1+c_2} = \frac{\left(m_1+m_2\right)g}{f+\left(m_2/m_1\right)f_1} \approx \frac{\left(m_1+m_2\right)g}{f}$$

And the apportionment coefficients are

$$\eta_1 = \frac{c_\Sigma}{c_1} = \frac{\left(1+m_2/m_1\right)f_1}{f+\left(m_2/m_1\right)f_1} \approx \frac{f_1}{f};$$
$$(8)$$
$$\eta_2 = \frac{c_\Sigma}{c_2} = \frac{f_2}{f+\left(m_2/m_1\right)f_1} \approx \frac{f_2}{f}.$$

If neglecting $1/\nu_{z2}^2$ compared to $1/\nu_{z1}^2$ in (4), the following approximate relation for the total static displacement results:

$$(9) \quad f = f_1 + f_2 = \frac{g}{4\pi^2\nu_{z1}^2} \approx \frac{1}{4\nu_{z1}^2}.$$

For $\nu_{z1} = 1\,\mathrm{Hz}$, f = 0,25 and m = 250 mm, the total static displacement required for the passenger vehicles. For vehicles with variable degree of loading, the mandatory condition is that the height of the buffers (the variation of the total static displacement, Δf) should remain below the maximal admitted value of $\Delta f_{max}$. If $m_i$ stands for the payload mass, then

$$\Delta f = \frac{m_i g}{c_\Sigma} = \frac{m_i}{m_1+m_2}\left(f + \frac{m_2}{m_1}f_1\right)$$
$$(10) \quad \Delta f \approx \frac{m_i}{m_1+m_2} f \le \Delta f_{max},$$

in which, if taken into account the "f" given by (9), results

$$(11) \quad \nu_{z1} \ge \frac{1}{2}\sqrt{\frac{m_i}{\left(m_1+m_2\right)\Delta f_{max}}} = \nu_{z1\min},$$

equation which will be used in order to determine the lower limit of the vehicle's own "low" frequency. Having the $\nu_{z1}$ value now settled, from (6) results:

$$\nu_{z2} \geq \left( \sqrt{1+\frac{m_1}{m_2}} + \sqrt{\frac{m_1}{m_2}} \right) \nu_{z1} = \nu_{z2\,min} , \tag{12}$$

which is the lower limit of the "own" high frequency.

## ADOPTING THE ELASTIC CONSTANTS FOR THE VERTICAL SUSPENSION OF A PASSENGER CAR EQUIPPED WITH STEERING BOGIES TYPE Y32R

In order to illustrate the methodology that governs the settlement of the stiffness for the vertical suspension of a double suspension vehicle, the AVA 200 passenger car will be considered. This vehicle is equipped with bogies type Y32R. The following data is available: the fully inspected empty car tare – 47 metric tonnes, bogie mass – 6,5 tonnes, wheel set mass – 2 tonnes, max. number of passengers – 84. Considering the appropriate mass for a passenger (including luggage) at about 80 kg, the payload mass is 6,72 tonnes.

The elastic characteristics of the suspension will be determined considering a medium load of the car body and setting the values for the own vibrating frequencies in very specific ranges. As mentioned above, the "low" frequency is set at around 1 Hz and the "high" own frequency is situated between 5 and 8 Hz. The results acquired through calculus using the relations presented in section 2 are presented in table 1.

The main issue is to adopt the correct values for the stiffness in order to achieve the optimal solution. The specialized literature offers highly specialized advice for choosing the right values of the static displacements (depending on the vehicle type) and their apportionment on every suspension deck. For passenger car bogies, the recommended total static displacement is 250 mm and should be apportioned according to the optimal ratio [1].

$$\eta_2 / \eta_1 = (0,15 \ldots 0,25) / (0,85 \ldots 0,75) .$$

By analyzing the results (table 1) it occurs that for the considered own "high" frequency of 7 Hz, the total static displacement is 253 mm and the ratio of the apportionment coefficients is situated between the boundary conditions.

For the fulfilled conditions the following will be adopted: the secondary suspension stiffness $c_1$ = 0,915 kN/mm; the primary suspension stiffness: $c_2$ = 3,897 kN/mm. The corresponding values for the static displacements are:

- $f_1$ = 200 mm;
- $f_2$ = 53 mm;
- the total static displacement, $f$ = 254 mm;
- the total system stiffness: $c_\Sigma$ = 0,741 kN/mm;
- the apportionment coefficients: $\eta_1$ = 0,81; $\eta_2$ = 0,19.

As for fulfilling the condition that the height of the buffers (the variation of the total static displacement, $\Delta f$) should remain below the maximal admitted value of $\Delta f_{max} = 0{,}08$ m, the study revealed that for high frequencies $\nu_{z2}$ = 5,7 ... 8 Hz, $\Delta f = 0{,}022$ m.

**Table 1**

| $\nu_{z1}$ | $\nu_{z2}$ | $f_1$ | $f_2$ | $f$ | $c_1$ | $c_2$ | $c_\Sigma$ | $\eta_1$ | $\eta_2$ | $\nu_{z1min}$ | $\nu_{z2min}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Hz | Hz | mm | mm | mm | kN/mm | kN/mm | kN/mm | - | - | Hz | Hz |
| 1 | 5,7 | 145 | 111 | 256 | 1,260 | 1,876 | 0,753 | 0,59 | 0,40 | 0,49 | 5,64 |
| | 6,0 | 170 | 86 | 255 | 1,079 | 2,428 | 0,747 | 0,69 | 0,30 | | |
| | 6,5 | 189 | 66 | 254 | 0,970 | 3,168 | 0,743 | 0,76 | 0,23 | | |
| | 7,0 | 200 | 53 | 253 | 0,915 | 3,897 | 0,741 | 0,81 | 0,19 | | |
| | 7,5 | 208 | 45 | 253 | 0,880 | 4,653 | 0,740 | 0,84 | 0,15 | | |
| | 8,0 | 214 | 38 | 252 | 0,854 | 5,445 | 0,739 | 0,86 | 0,13 | | |

**Table 2**

| $\nu_{z1}$ | $\nu_{z2}$ | $f_1$ | $f_2$ | $f$ | $c_1$ | $c_2$ | $c_\Sigma$ | $\eta_1$ | $\eta_2$ | $\nu_{z1min}$ | $\nu_{z2min}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Hz | Hz | mm | mm | mm | kN/mm | kN/mm | kN/mm | - | - | Hz | Hz |
| 1,04 | 6,92 | 182 | 49 | 231 | 0,915 | 3,897 | 0,5824 | 0,83 | 0,16 | 0,49 | 5,64 |

**Table 3**

| $\nu_{z1}$ | $\nu_{z2}$ | $f_1$ | $f_2$ | $f$ | $c_1$ | $c_2$ | $c_\Sigma$ | $\eta_1$ | $\eta_2$ | $\nu_{z1min}$ | $\nu_{z2min}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Hz | Hz | mm | mm | mm | kN/mm | kN/mm | kN/mm | - | - | Hz | Hz |
| 0,95 | 7,13 | 276 | 58 | 276 | 0,915 | 3,897 | 0,5824 | 0,83 | 0,16 | 0,49 | 5,64 |

Using the same conditions of medium loaded car body for the lower limits of the vibration frequencies, one may acquire $\nu_{z1\min} = 0.498$, the inferior limit of the ”low” own frequency and $\nu_{z2\min} = 5.644$ Hz, the inferior limit of the ”high” own frequency. Although the study was performed considering the medium load, it is mandatory to check the vertical vibrations for booth situations, either when the car is empty, or fully loaded. This additional step aims that the values calculated for these extreme situations are superior to the ones previously determined.

The results are presented in tables 2 and 3.

## CONCLUSIONS

In order to operate safely and in comfort at high speeds both, the vehicle and the rail must fulfil some special constructive requirements. Knowing that the costs for major track improvements are very high, the most effective solution is to enforce a series of special constructive requirements for the vehicle suspension. Nowadays, this issue has become more acute as the demand for higher transport capacities and speeds arose significantly.

The elastic characteristics of the suspension are influencing the own frequencies of the vehicle's suspended masses. Starting from the boundary condition of achieving the mandatory values for the own frequencies the necessary stiffness and the static loads for the suspension springs may be determined.

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# ПРИНОСИ КЪМ ИЗУЧАВАНЕТО НА ЕЛАСТИЧНИТЕ КОНСТАНТИ НА ОКАЧВАНИЯТА, ИЗПОЛЗВАНИ ПРИ ВИСОКОСКОРОСТНИТЕ ТАЛИГИ С КОРМИЛЕН МЕХАНИЗЪМ


**Иоан Себешан, Мадалина Димитриу, Кристина Тудораче**

ioan_sebesan@yahoo madalinadumitriu@yahoo.com cristi_tudorache2003@yahoo.com

*Университет в Букурещ "ПОЛИТЕХНИКА" ; Транспортен факултет*

*Площад "Независимост" 313, сектор 6, 77206, Букурещ http://www.pub.ro*

**РУМЪНИЯ**



***Ключови думи***: *еластични характеристики на окачването, собствени вибриращи честоти, управляеми осови талиги*



***Анотация***: *Еластичните характеристики на окачването влияят на собствените вибрации на окачените маси на превозното средство. Тези вибрации трябва да бъдат разположени в някои специфични области. Необходимите коравина и статично натоварване на ресорите за окачване могат да бъдат определени чрез граничното условие за постигане на задължителните стойности на собствените честоти.*