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## APPLICATION OF A COVERAGE MODEL FOR ASSIGNMENT THE LOCATION OF LOGISTICS FACILITIES

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**Key words:** *set covering problem; location of the center; police*

**Abstract:** *The problem, which is considered in the paper is how to determine the number of police offices and their area and population coverage according to the time frame of the specified standards. The goal is to find the minimum number of police centers to provide maximum service (coverage), both to the population and to the territory of the city for a given maximum time and/or distance. In order to determine the coverage by districts in the city of Sofia, it is necessary to apply known mathematical models for choosing the location of the police station, and in this paper is used the set covering problem, which is a mixed integer and linear programming problem. The objective function and the constraints and the binary variables are defined in the Microsoft Solver Add-in. These police teams must provide maximum "coverage" for a limited time to the population of all areas. Considering the population of each area and the service times of existing and potential centers, it is necessary to find additional centers with their respective locations that can provide service within these time frames. As a result of solving the task with a varying number of police centers (1, 2 and 3), their location and degree of coverage (served population) with the previously set time standards for police triage (to 15 min., which includes the time for transmission of the signal by the dispatcher (about 2 min.) and the reaction time of the designated team as necessary (between 3-5 min.)).*

### INTRODUCTION

Police offices are working at their full capacity, but the main problem is still untimely service to the population. The Ministry of the Interior (MoI) is a system ensuring the security of the population. Therefore, it is necessary to deploy one or more police teams to meet the needs of the people. The location of these police teams is critical to being able to reach the crime scene more quickly and help. Therefore, these police teams must provide maximum "coverage" for a limited time to the population of all areas.

One of the ways of leaving people in the queue for service is the developed regulation, where the standards for signal service at MoI are confirmed [1]. According to the Road Traffic Act [2], the cars of MoI drive with a special traffic mode and use the right of way. This practically means that the cars can to go through crossroads with sound and light signaling on, when they have to reach the place where the crime is located, considering also [6].

## INTRODUCTION TO THE PROBLEM AND ITS SOLUTION IN EXCEL [4]

The problem under consideration is how to determine the number of the Police offices and their coverage by area and population according to the time frame of the standards set. The goal is to find the minimum number of police centers to provide maximum service (coverage), both for the population and the city territory, for a given maximum time and/or distance. An improved version of the model presented in [5], where the task of choosing centers and locations is defined with the aim of maximum coverage under constraints: the number of population included within a certain distance. Using binary variables and logical constraints are the main ways to solve any location model. The objective function, constraints, and binary variables are defined in Solver Add-in.

The territory of the city of Sofia is served by taking into account the Euclidean metric of the distances between two points, corrected by a factor taking into account the street network, and the time travel was obtained at an assumed average speed of movement in the city of the police team. As a result of solving the task at a varying number of additional police centers (1, 2 and 3), their location and degree of coverage (served population) with the previously set time standards for police triage (to 15 min., including the time for transmission of the signal from the dispatcher (about 2 min.) and the reaction time of the designated team as necessary (between 3-5 min.)). For the application of the task, the city of Sofia is preliminarily divided by administrative division, with each district consisting of one or more neighborhoods. Distances along the transport network are determined [4] using a matrix of Euclidean distances between all regions, multiplied by the coefficient of non-linearity, which for the city of Sofia is calculated at 1.4. In table 1, the rightmost column contains the population by regional administrations.

**Table 1. Matrix of distances (times) between regions and population by region**

Distance on the transport network (in km)

8 min.

coef.	1,4	PY1	PY2	PY3	PY4	PY5	PY6	PY7	PY8	PY9	Population
RG1		0	5,8	5,2	4,6	2,9	8,5	7,7	4,3	14	166000
RG2		5,8	0	3,3	4,7	3,5	6,5	11,6	10,8	9,5	180000
RG3		5,2	3,3	0	3,3	4,4	4,8	11,8	11	5,8	188000
RG4		4,6	4,7	3,3	0	3,3	5,7	8,2	9	9,5	120000
RG5		2,9	3,5	4,4	3,3	0	6,9	10	10,2	8	147500
RG6		8,5	6,5	4,8	5,7	6,9	0	11,6	13,5	8,1	226867
RG7		7,7	11,6	11,8	8,2	10	11,6	0	5,4	16,6	350000
RG8		4,3	10,8	11	9	10,2	13,5	5,4	0	15,7	96000
RG9		14	9,5	5,8	9,5	8	8,1	16,6	15,7	0	190700
USED Center		1	1	1	0	0	0	0	0	0	1665067

### Goals and tasks:

- To develop a mixed linear and integer programming model, the problem is formulated mathematically with an objective function and constraints.
- To implement a mixed linear and integer programming model with binary variables to determine the minimum number of centers that can cover the population of all districts.
- The objective function of the model maximizes the total number of populations by area covered by fixing the integer variables determining the location and number of centers, subject to constraints on the total number of centers and the service time standard.

### Mathematical statement of the problem [5]

### Designations:

- I* -many of the areas generating emergency calls;  
*J*- many of the regions with additional potential hubs, generating emergency calls;

$T$  - travel time (distance) to which the areas can be considered covered;

$x_j = (0,1), j \in J$ . When  $x_j = 1$ , it then  $j$  a potential additional center has been found, and at  $x_j = 0$  not found;

$Nd$  - multiple of the preselected centers;

$x_j = 1 \quad \forall j \in Nd$

$Pe = |Nd|$  - number of pre-selected centers;

$Nused = J \cup Nd$  - multiplicity of all centers-permanent and additional;

$Ni = \{j \in Nused \mid t_{ij} \leq T\}$  - the multitudes of centers (permanent and additional),  $i \in I$ , which "cover" the respective region  $i$ , i.e. with travel time less than set  $T$ .

$Pop_i$  - population in an area  $i \in I$

The modified task for the maximum coverage of the regions with a given number of additional centers  $Pd$ , with existing ones  $Pe = |Nd|$ , i.e. with a given total number of centers  $P = Pd + Pe$ , as  $Nd$  the set of pre-selected centers, is defined as follows:

The objective function:

$$(1) \quad \text{Max } Z = \sum_{i \in I} Pop_i y_i$$

with constraints:

$$(2) \quad \sum_{j \in Ni} x_j \geq y_i \quad \forall i \in I$$

$$(3) \quad \sum_{j \in Nused} x_j = P$$

$$(4) \quad x_j = \{0,1\} \quad \forall j \in J$$

$$(5) \quad y_i = \{0,1\} \quad \forall i \in I$$

The variables that change in the model are accordingly  $x_j$  and  $y_i$ .

If the center is operationally at full capacity (ie no police car is available at any given time) to ensure the 8 minute standard, the call needs to be serviced by another center. If so, how many centers are needed and where should they be located in order to guarantee that the service to the population of all regions is realized within 8 minutes from two or more centers. Considering the population of each area and the service times of existing and potential centers, it is necessary to find additional centers with their respective locations that can provide service within these time frames.

#### CONSIDERED:

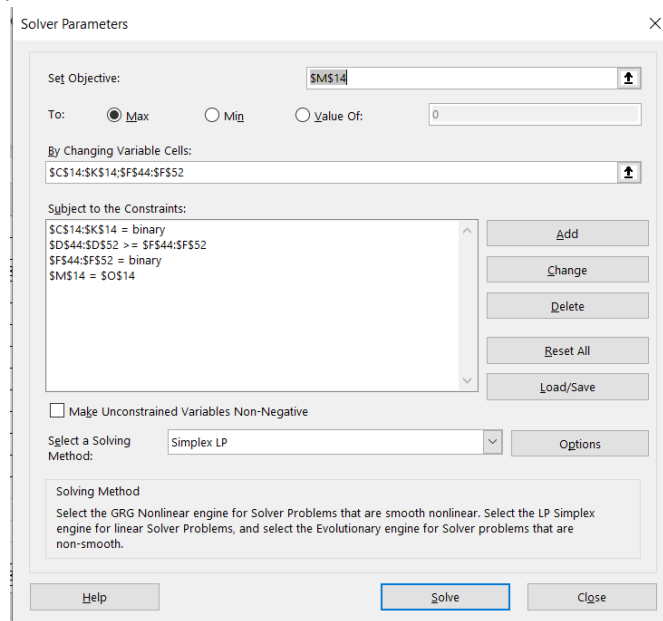
1) Variant 1, additionally detecting 1st to 3rd centers. The coverage with 1 to 3 additional centers at  $T=8$  min was analyzed.

2) Variant 2 - with 1 to 3 additional centers opened at standard  $T=10$  min.

**Table 2 Coverage of centers**

	A	B	C	D	E	F	G	H	I	J	K	L
43			Coverage of centers			Coverage		Served		Unserved		
44		RU1		1 >=		0		166000		0		1
45		RU2		1 >=		0		180000		0		1
46		RU3		1 >=		0		188000		0		1
47		RU4		1 >=		0		120000		0		1
48		RU5		1 >=		0		147500		0		1
49		RU6		0 >=		0		0		226867		1
50		RU7		1 >=		0		350000		0		1
51		RU8		1 >=		0		96000		0		1
52		RU9		0 >=		0		0		190700		0
53												
54					Population	0		1247500		417567		74,92%

The task thus defined is solved with the Solver Add-in, after checking the Simplex LP option and the Solve button.



**Fig. 3. Solver Add-in**

## RESULTS

### *Variant 1 (to 8 min.)*

The minimum number and specific location of the Police officers in the city of Sofia, which provide maximum service to the population. As a result, 3 centers were obtained. These are RU1, RU2 and RU3.

In the table 3 and table 4, the results obtained in the detection of 1, 2 and 3 Pd centers from 1 to 3 are systematized.

**Table 3**

Number of additional centers		Served population to T=8 min.		Number and % of areas covered by 1, 2 and 3 centers			Kcover <sup>(1)</sup>
Pd	Центрове	Брой	%	1	2	3	
1	RU1	1 247 500	35,7%	7	0	0	7
				77,8%	0,0%	0,0%	0,78
2	RU1, RU2	2 466 567	70,6%	4	5	0	14
				44,4%	55,6%	0,0%	1,56
3	RU1, RU2, RU3	3 494 934	100,0%	3	1	5	20
				33,3%	11,1%	55,6%	2,22

(1) remark. Kcover- average number of centers that cover the service areas

### Variant 2 (to 10 min.)

The minimum number and specific location of the Police offices in the city of Sofia, which provide maximum service to the population. As a result, 3 centers were obtained. These are the Police offices RU1, RU4 and RU5 SDVR.

In the table 5 the results obtained in the detection of 1, 2 and 3 Pd centers from 1 to 3 are systematized.

Table 4

Number of additional centers		Served population to T=10 min.		Number and % of areas covered by 1, 2 and 3 centers			Kcover <sup>(1)</sup>
Pd	Центрове	Брой	%	1	2	3	
1	RU1	1 665 067	47,6%	9	0	0	9
				100,0%	0,0%	0,0%	1,00
2	RU1, RU4	3 139 434	89,8%	1	8	0	17
				11,1%	88,9%	0,0%	1,89
3	RU1, RU4, RU5	4 708 501	134,7%	0	2	7	25
				0,0%	22,2%	77,8%	2,78

(1) remark. Kcover- average number of centers that cover the service areas

As a result of solving the problem with a varying number of centers (1, 2, 3), their location was obtained, plotted according to the degree of coverage with set time standards (Variant 1 - to 8 min. and Variant 2 - to 10 min.).

### CONCLUSION

The developed and implemented model of mixed linear and integer programming with binary variables determines the minimum number of centers that can cover all areas in the time frames for Variant 1 - 8 min. and for Variant 2 - 10 min. In the considered Variant 1 location of the centers, which serve the population of Sofia-city are:

RU 1; RU 2 and RU 3.

For Variant 2, the location of the centers serving the population of Sofia-city are:

RU 1; RU 4 and RU 5.

The considered Variant 2 shows that it would be more appropriate to choose RU4 and RU5 with a coverage rate of 89.8%, one uncovered area and an average number of centers that cover the served areas 1.89. In the existing Variant 1 with RU1 and RU2, the coverage rate is 70.6%, 4 uncovered areas and an average number of centers that cover the served areas 1.56.

### REFERENCES:

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## ПРИЛОЖЕНИЕ НА МОДЕЛ ЗА ПОКРИТИЕ ПРИ ИЗБОР НА МЕСТОПОЛОЖЕНИЕ НА ЛОГИСТИЧНИ ОБЕКТИ

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***Ключови думи:** проблем при покритие; местоположение на центъра; полиция*

***Резюме:** Проблемът, който се разглежда в доклада, е как да се определи броят на полицейските управления, покритието на тяхната площ и обхват на населението според времевата рамка на предварително определени стандарти. Целта е да се намери минимален брой полицейски центрове, които да осигурят максимално обслужване (покритие), както на населението, така и на територията на града за дадено максимално време и/или разстояние. За определяне на покритието по райони в гр. София е необходимо да се приложат известни математически модели за избор на местоположението на РПУ, като в настоящия доклад се използва задачата за покритие на множеството, която е смесена целочислена и задача за линейно програмиране. Целевата функция, ограниченията и променливите са дефинирани в допълнителната функция на Microsoft – Solver Add-in. Следователно тези полицейски екипи трябва да осигурят максимално „покритие“ на населението от всички области за ограничено време. Като се има предвид населението на всяка област и времето за обслужване на съществуващи и потенциални центрове, е необходимо да се намерят допълнителни центрове със съответните им местоположения, които могат да предоставят обслужване в рамките на тези времеви рамки. В резултат на решаване на задачата с различен брой полицейски центрове (1, 2 и 3), тяхното местоположение и степен на покритие (обслужвано население) с предварително зададените времеви стандарти за полицейски сортиране (до 15 мин., което включва време за предаване на сигнала от диспечера (около 2 мин.) и време за реакция на определения екип при необходимост (между 3-5 мин.)).*