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## APPROXIMATION OF IDEAL CHARACTERISTIC OF RAILWAY VEHICLE SUSPENSION

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**Key words:** Freight wagons, stiffness, coil springs.

**Abstract:** Proper functioning of the suspension of the railway vehicles is of great importance for the normal functioning of railway traffic. The coil springs are a very common element of the railway vehicles suspension. They are obtained by winding cold drawn wire along the line of the coil with equal distances between the threads. Thanks to these distances, the height of the spring changes easily under the load. Cold drawing is the process of kneading the surface layer of the wire by passing it through a calibrated hole in the cold state. This significantly improves the structure of both the surface layer and the core of wire. All coil springs are made of steel wires, usually round in diameter, and other cross-sectional shapes are possible (square, etc.). The stiffness characteristic of these elements is linear. This paper shows that the ideal characteristic of suspension stiffness of railway vehicles should be exponential. Due to the expensive production of suspension elements that would have an exponential stiffness characteristic, the standard suspension elements which in a certain combination approximates the ideal stiffness characteristic, are used. The whole procedure must satisfy the theoretical postulates presented in this paper. The quality of suspension has a key influence on the quiet running and running safety of railway vehicles.

### 1. INTRODUCTION

The most commonly used elements of suspension of the railway vehicles are leaf springs (Fig. 1) and coil springs (Fig. 2). Their basic task is to provide adequate elasticity of the system. In addition to elasticity, a very important characteristic of suspension system is damping. In order to reduce the amplitudes of the vehicle's oscillations in running and to avoid the danger of resonance, special devices that dampen oscillations are very often used. Damping of oscillations is usually solved through appropriate friction surfaces (Fig. 3), by installing hydro or pneumatic shock absorbers, rubber shock absorbers, etc [1-3]. The suspension of the vehicle mitigates the shocks transmitted from the track over the wheels to the carbody, and also reduces the amplitudes of oscillations that occur when running. Quality suspension enables more conformal and safer train movement [4]. Higher speeds of the vehicle running causing the greater shocks in the vehicle-track interaction (Fig. 4). These impacts have a very detrimental effect on the passengers and cargo, reducing the service life of both vehicles and tracks.



Fig. 1. Leaf spring

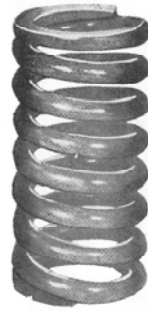


Fig. 2. Coil spring

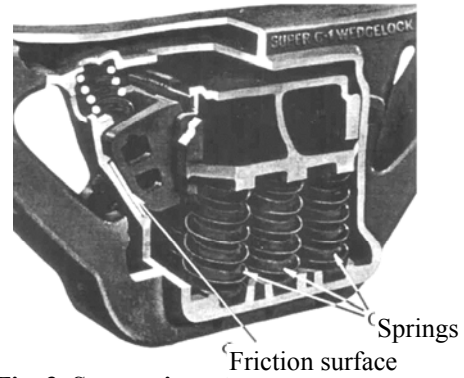


Fig. 3. Suspension with coil springs and friction surfaces

Apart from the track quality and the running speed, the intensity of the vertical oscillations of the vehicle also depends on the elastic and damping properties of the suspension system. The suspension reduces the dynamic load, and thus reduces the stresses in the wheelsets, bearings, bogies, underframe and carrying structure as well as track. The impacts on the railway vehicle are reduced because the suspension elements absorb some of the kinetic energy, thus achieving a safer and quieter vehicle running [5]. The vertical load of railway vehicles in static conditions comes from the weight of the structure and the weight of the load. The vertical static load of empty and loaded vehicles differs significantly. In running, due to the dynamic impact, the already large difference between the vertical load of loaded and empty vehicle increases and this fact significantly complicates the optimal design of suspension system [6-7]. In order to approach the solution of this problem in the right way, it is necessary to consider the vertical oscillations of the vehicle carbody.

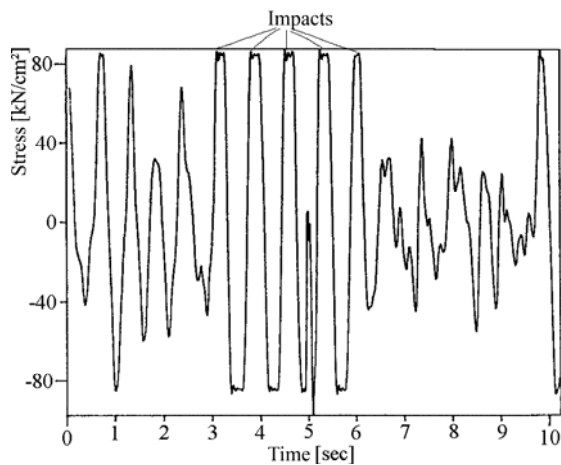


Fig. 4. Change of stress on the elements of suspension system during the vehicle running

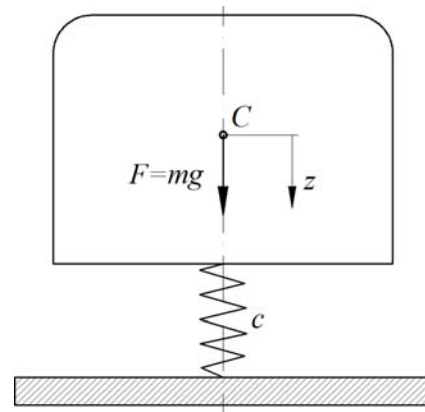


Fig. 5. The basic dynamic model of vertical oscillations of carbody of railway vehicles

## 2. VERTICAL OSCILLATIONS OF CARBODY AND IDEAL CHARACTERISTIC OF SUSPENSION OF RAILWAY VEHICLES

The basic dynamic model of vertical oscillations of the carbody of railway vehicles is shown in Fig. 5. The frequency of vertical oscillations of the basic dynamic model of oscillation of a railway vehicle carbody shown in Fig. 5 is obtained by solving the following expression:

$$m\ddot{z} = F - c(f_{st} + z) \quad (1)$$

where:

$m$  – mass of carbody

$f_{st}$  – static deflection of spring

$c$  – stiffness of suspension

$z$  – vertical movement of carbody

$F = m \cdot g = f_{st} \cdot c$  – static equilibrium condition

Taking into account previous considerations, the initial equation (1) can be written in the following way:

$$(2) \quad m \cdot \ddot{z} + c \cdot z = 0, \text{ or } \ddot{z} + \omega^2 \cdot z = 0$$

where:

$$(3) \quad \omega = \sqrt{\frac{c}{m}} \text{ [Hz]} - \text{angular frequency}$$

$$(4) \quad \omega = 2 \cdot \pi \cdot \nu - \text{ratio of angular frequency and frequency of vertical oscillations}$$

$$(5) \quad \nu = \frac{1}{2\pi} \sqrt{\frac{c}{m}} = \text{const. [Hz]} - \text{frequency of vertical oscillations}$$

The stiffness is defined by the following expressions:

$$(6) \quad c = k \cdot m = k_1 \cdot g \cdot m = k_1 \cdot F; \quad c = \frac{dF}{df}$$

By equalizing the expressions for stiffness is obtained:

$$(7) \quad \frac{dF}{df} = k_1 \cdot F \Rightarrow \frac{dF}{F} = k_1 \cdot df$$

After integrating, the following expression is obtained:

$$(8) \quad \ln F = k_1 \cdot f + k_2$$

Finally, the dependence of the force and deflection of the suspension is:

$$(9) \quad F = e^{k_1 \cdot f + k_2} = k_3 \cdot e^{k_1 \cdot f}$$

From the previous expression it is obviously that, as far as the dynamic behavior of a railway vehicles is concerned, the ideal dependence of the suspension deflection  $f$  on the force  $F$  is exponential.

### 3. APROXIMATION OF IDEAL CHARACTERISTIC OF SUSPENSION OF RAILWAY VEHICLES

The suspension system should be designed to approximately follow the ideal (exponential) dependence of deflection and force. Therefore, a two-spring set is most often used in the designing the vehicle suspension (Fig. 6). Each spring has a linear dependence between deflection and force  $c'=c^s$ , where these linear dependences have different inclinations. When, due to the action of load  $F_2$ , the parallel operation of the outer and inner springs begins, the equivalent stiffness is  $c''=c^s+c^u$ . In this way, the ideal deflection characteristic is approximated by means of constant stiffnesses of springs  $c'$  and  $c''$  (Fig. 7).

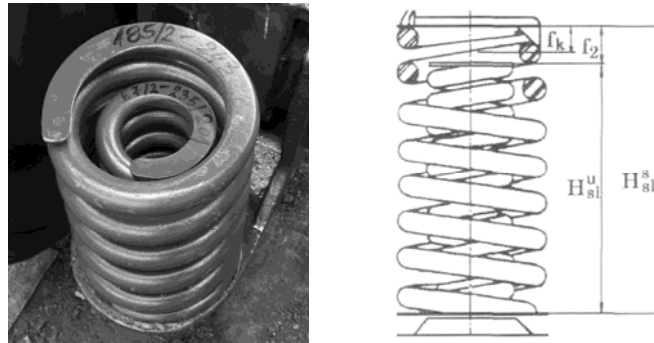


Fig. 6. Set of outer and inner spring

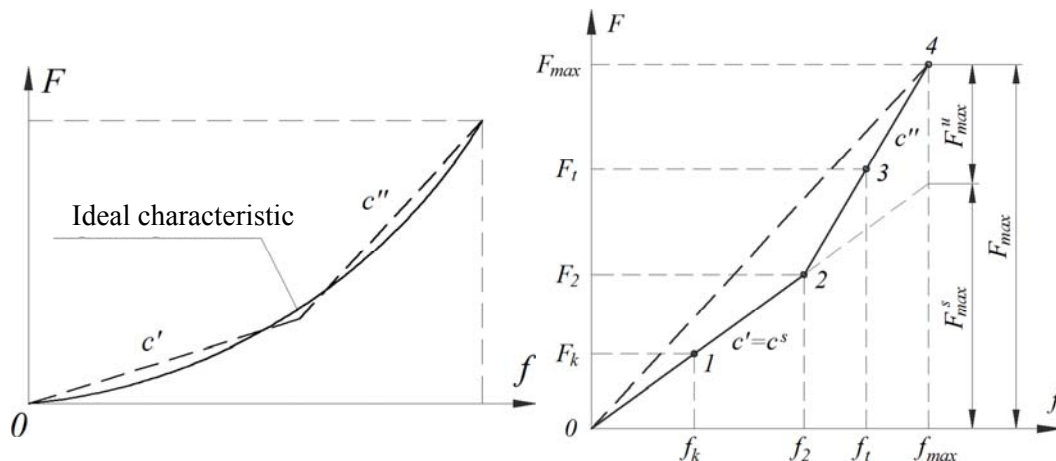


Fig. 7. Approximation of ideal stiffness by using two springs

Fig. 8. Suspension stiffness diagram with bilinear characteristic

The meaning of significant parameters shown in the previous diagrams is:

- $c'=c^s$  – stiffness of the outer spring;
- $c''=c^s+c^u$  – stiffness of one set of springs; parallel connection of external and internal spring;
- $c^u$  – stiffness of the inner spring;
- $H_{sl}^u$  – free height of unloaded inner spring;
- $H_{sl}^s$  – free height of unloaded outer spring;
- $f_k$  – deflection due the weight of the wagon structure;
- $f_2$  – deflection when the inner spring starts working;
- $f_t$  – deflection of the spring set due the weight of the load;
- $f_{max}$  – maximum deflection of the spring set with the addition of dynamic shocks;
- $F_k$  – force of the weight of the wagon structure;
- $F_2$  – force when the inner spring starts working;
- $F_t$  – force of the weight of the load;
- $F_{max}$  – maximum force with the addition of dynamic shocks;

$F_{max}^u$  – maximum force on the inner spring with dynamic shocks;

$F_{max}^s$  – maximum force on the outer spring with dynamic shocks.

The criterion for choosing the ratio of the stiffness of the outer and the inner spring, as well as for determining the position of point 2 (the point where the inner spring starts working), is set from consideration of the basis of vehicle oscillation. The key condition is that the frequency of vertical oscillations in the interval 1-2 changes in the same way as the frequency of vertical oscillations in the interval 2-3 (Fig. 8). It can be assumed that in the point 2 infinity close to the left side is stiffness  $c'$ , so the frequency of vertical oscillations is  $\nu_2'$ , while infinity close to the right side is stiffness  $c''$ , so the frequency of vertical oscillations is  $\nu_2''$ . Therefore, there is the following equations:

$$\nu_1 = \nu_2''; \quad \nu_2' = \nu_3 \quad (10)$$

In this way, the condition for approximately equal quiet running of an empty and laden railway vehicle is created. The frequency of vertical oscillations of empty vehicle with the mass  $m_k$  is:

$$\nu_1 = \frac{1}{2 \cdot \pi} \sqrt{\frac{c'}{m_k}} \quad (11)$$

The frequency of vertical oscillations of vehicle loaded with a cargo of mass  $m_2$  on a spring  $c'$  is:

$$\nu_2' = \frac{1}{2 \cdot \pi} \sqrt{\frac{c'}{m_2}} \quad (12)$$

The frequency of vertical oscillations of vehicle loaded with a cargo of mass  $m_2$  on a spring  $c''$  is:

$$\nu_2'' = \frac{1}{2 \cdot \pi} \sqrt{\frac{c''}{m_2}} \quad (13)$$

The frequency of vertical oscillations of vehicle fully loaded with a cargo of mass  $m_t$  on a spring  $c''$  is:

$$\nu_3 = \frac{1}{2 \cdot \pi} \sqrt{\frac{c''}{m_t}} \quad (14)$$

Substitution the previous equations in condition (10) gives the following equations:

$$c' = \frac{m_k}{m_2} c'' = \frac{F_k}{F_2} c''; \quad c' = \frac{m_2}{m_t} c'' = \frac{F_2}{F_t} c'' \quad (15)$$

Equating the previous expressions, the following expression is obtained:

$$F_2 = \sqrt{F_k \cdot F_t}$$

(16)

Thus, the condition for determination of load  $F_2$ , i.e., the moment when both springs start working at the same time, have obtained. In this way, the determined value of the force  $F_2$  best approximates the ideal characteristic of suspension, which consists of two springs with constant stiffness. The most commonly used suspension system of two-axled freight bogies type Y25 (Fig. 9) is designed precisely based on the previous considerations. It is based on double coil springs and bilinear stiffness characteristic.



**Fig. 9. Suspension of freight bogie Y25**

#### **4. CONCLUSION**

The paper presents the way of approximation of ideal characteristic of stiffness of suspension of railway vehicles which has exponential nature. The technical solution for this problem is based on the usage of the double coil springs which have bilinear characteristic of stiffness. Thanks to the difference in heights, the outer springs receive the load from the empty vehicle, while the load of partially or fully loaded vehicle is received together by both the outer and inner springs. The results of the paper are very significant because they are the ground for designing the suspensions of most today's commercial bogies of railway vehicles.

#### **ACKNOWLEDGEMENTS**

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