Mechanics
Transport
Communications
Academic journal http://www.mtc-aj.com

ISSN 1312-3823 (print)
ISSN 2367-6620 (online)
volume 19, issue 3, 2021
article № 2123

STUDY THE TRAJECTORIES OF A TENNIS BALL

Anastas Ivanov Ivanov
aii2010@abv.bg

Todor Kableshkov University of Transport
1574 Sofia, 158 Geo Milev Str
THE REPUBLIC OF BULGARIA

Key words: *trajectories, tennis ball, strokes with rotations, Magnus effect, numerical solution, MatLab*

Abstract: *Modern tennis sport is characterized by constant change in several directions. The most significant of these changes are connected to the design of tennis rackets. The composite material of the rocket's frame is continuously improved. It is also working hard on improving the tennis strings. Tennis balls are also being modified. All these innovations lead to a change in the shot technique of the athletes. Modern tennis players use a variety of strokes. From the physical point of view, these blows can be classified into eight main groups depending on the way the racket interacts with the tennis ball. These are the strokes with top, bottom, side, top-side, and bottom-side rotation, as well as strokes without rotation. These eight interactions lead to main types of trajectories at the same initial position and the same initial linear velocity. The main purpose of the work is to study the two most important strokes in professional tennis: forehand with upper rotation and backhand with lower rotation. The calculations and graphical images are performed with the MatLab mathematical package. The conclusions are important not only for specialists in the field of Theoretical and Applied Mechanics, but they would also be useful for sports specialists, coaches, and tennis players.*

INTRODUCTION

In recent years, there has been a significant increase in studies related to the motion of a variety of sports balls in a real fluid environment. For example, modern researches on tennis ball aerodynamics can be seen in the works [1-7].

It is known that all strokes in tennis sports are determined not only by the initial linear velocity, but also by the initial angular velocity that tennis players transmit to the tennis ball through their racquets, [8-10]. The effect of spin using experimental and computational methods is studied in the work [11], where the computer simulation results are compared with experimental findings.

The correct determination of the drag and lift coefficient significantly affects accurately calculating the trajectory of the tennis balls. The studies of spinning and non-spinning tennis ball aerodynamics, as well as the determining of the drag and lift coefficients, are made in the publications [12-15].

The measurements of drag and lift for new tennis balls in flight are presented in the study [16]. Two video cameras are used to measure the velocity and flight height of the balls. They are fired from a ball launcher at a velocity between 15 and 30 m/s and with topspin or backspin at rates up to 2000 rpm. The authors measured an average drag coefficient 0.507 ± 0.024, independent of ball speed or spin, and this value is lower than the value usually observed in wind tunnel experiments.

An in-depth study of the influence of drag and lift coefficients on the trajectories of tennis balls is performed in the work [17].

A three-dimensional study of tennis ball flight is done in the article [18], where a classical analytical formula is used for the lift coefficient.

From the point of view of Kinetics, all tennis strokes can be classified into eight main groups depending on how the tennis racquet interacts with the tennis ball. These are the strokes with top, bottom, side, topside, and bottom-side rotation, as well as strokes without rotation. These eight interactions lead to eight characteristic types of trajectories at the same initial position and the same initial velocities.

The main purpose of this work is to study the two most important strokes in professional tennis, namely, forehand with upper rotation (topspin forehand) and backhand with lower rotation (backhand slice).

DYNAMICAL MODEL

The tennis ball is a round, hollow, several-layered sphere with a certain average thickness. The ball is an elastic body with strong damping characteristics. The air inside the competition balls is under pressure higher than atmospheric pressure. A fluffy woolen knit is glued on the rubber surface of the ball (Fig.1 and Fig.2).

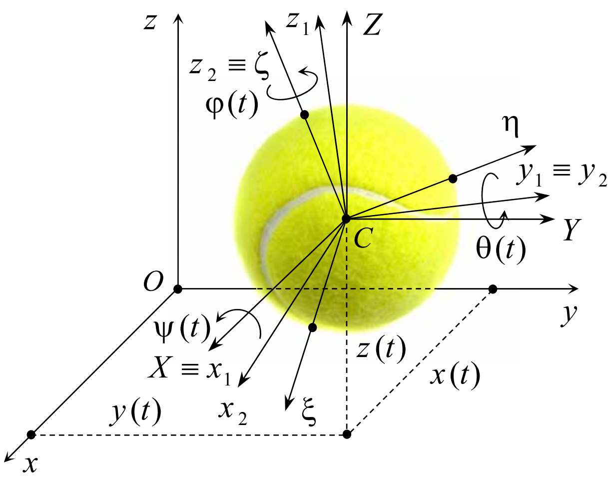

Fig.1 Generalized coordinates.

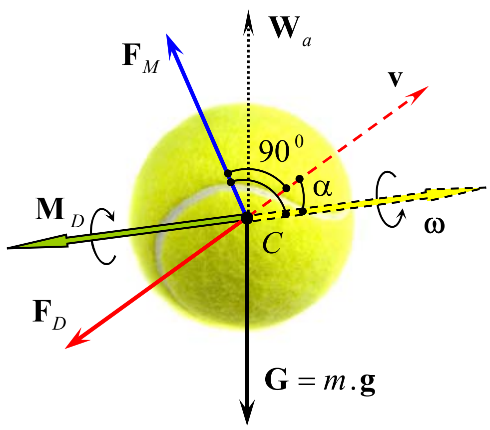

Fig.2 Aerodynamic forces.

The ball is assumed to be a perfectly rigid hollow closed two-layer spherical shell. The rubber wall is defined by an inner radius $r = 0.026\ m$ and an outer radius $R_1 = 0.030\ m$. Wool knit gives the total size of the ball with a radius $R = 0.0325\ m$.

The main physical characteristics of the tennis ball are: the mass of the tire $m_1 = 0.045\ kg$, the mass of the knit $m_2 = 0.012\ kg$, the total mass $m = m_1 + m_2$, and the mass moment of inertia $J = 3.155 \times 10^{-5}\ kg.m^2$.

The flight of a tennis ball is represented as a general motion of a hollow spherical rigid body in an air environment, the influence of which is taken into account by means of aerodynamic forces.

DIFFERENTIAL EQUATIONS

To determine the law of motion of the tennis ball, namely, $x(t)$, $y(t)$, $z(t)$, $\psi(t)$, $\theta(t)$ and $\varphi(t)$, it is necessary to integrate the following system of differential equations:

(1) $m.\ddot{x} = F_{Dx} + F_{Mx}$,

(2) $m.\ddot{y} = F_{Dy} + F_{My}$,

(3) $m.\ddot{z} = F_{Dz} + F_{Mz} - G + W_a$,

(4) $J_X.\dot{\omega}_X - \left(J_Y - J_Z\right).\omega_Y.\omega_Z = M_{Dx}$,

(5) $J_Y.\dot{\omega}_Y - \left(J_Z - J_X\right).\omega_Z.\omega_X = M_{Dy}$,

(6) $J_Z.\dot{\omega}_Z - \left(J_X - J_Y\right).\omega_X.\omega_Y = M_{Dz}$.

In equations (4), (5) and (6), $J_X = J_Y = J_Z = J$ $[kg.m^2]$ is the mass moment of inertia of the tennis ball.

The components of the angular velocity satisfy the following kinematic equations:

(7) $\omega_X = \dot{\psi} + \dot{\varphi}.\sin\theta$,

(8) $\omega_Y = \dot{\theta}.\cos\psi - \dot{\varphi}.\cos\theta.\sin\psi$,

(9) $\omega_Z = \dot{\varphi}.\cos\theta.\cos\psi + \dot{\theta}.\sin\psi$.

They are structured using the Cardan angles.

AERODYNAMIC FORCES

- Force of weight.

(10) $G = m.g = 0.057 \times 9.81 \approx 0.56\ N$.

- Lift force, according to Archimedes Law.

It is determined by the formula:

(11) $W_a = 4/3.\pi.R^3.\rho_a.g = 4/3.\pi \times 0.0325^3 \times 1.205 \times 9.81 \approx 0.0017\ N$.

It is obvious this force can be ignored.

- Drag force.

It is in the opposite direction of the velocity $\mathbf{v}$ and is determined by the formula:

(12) $\mathbf{F}_D = -1/2.C_D.\rho_a.A.v.\mathbf{v}$,

(13) $\mathbf{v} = \left\langle \dot{x} \quad \dot{y} \quad \dot{z} \right\rangle^T$,

(14) $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$.

For this study, the air density is taken as an average value of $\rho_a = 1.205\ kg/m^3$ at a temperature of $T = 20^o C$.

The area of the middle cross-section of the tennis ball has the following value:

(15) $A = \pi.R^2 = \pi \times 0.0325^2 \approx 0.00332\ m^2$.

The drag coefficient C_D is considered to be a turbulent flow regime. When the tennis balls are flying at medium velocities, the coefficient C_D retains a relatively constant value for a wide range of Reynolds numbers, namely, $5 \times 10^4 \leq R_e \leq 7.5 \times 10^4\ m^2/s$, [12-16].

•Magnus force. It is determined by the formula:

(16) $$\mathbf{F}_M = 1/2.C_L.\rho_a.A.v^2.\frac{\mathbf{\Omega.v}}{|\mathbf{\Omega.v}|},$$

(17) $$\mathbf{\Omega} = \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix}.$$

The lift coefficient C_L of the Magnus force, for the same turbulent flow regime, depends not only on the Reynolds number but also on the spin parameter, namely:

(18) $$S = \frac{\omega.R}{v}.$$

When the spin parameter S increases, then the value of the lift coefficient C_L also increases. This phenomenon is described in the works [13-17].

•Drag aerodynamic moment. It is determined by the formula:

(19) $$\mathbf{M}_D = -1/2.C_M.\rho_a.A.v^2.\frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|},$$

(20) $$\boldsymbol{\omega} = \langle \omega_X \quad \omega_Y \quad \omega_Z \rangle^T.$$

The coefficient C_M depends on the Reynolds number, the spin rate S, and the condition of the uniform felt fabric of the tennis ball. It has been adopted $C_M = 0.01$.

NUMERICAL SOLUTION

For the numerical solution of the system of differential equations (1)-(6) in the time area, the program using MatLab is prepared.

The laws of motion, velocities, and accelerations of all generalized coordinates are obtained, as well as the laws of velocity and acceleration of the tennis ball mass center: $x(t)$, $y(t)$, $z(t)$, $\psi(t)$, $\psi(t)$, $\theta(t)$, $\varphi(t)$, $\dot{x}(t) \equiv v_x$, $\dot{y}(t) \equiv v_y$, $\dot{z}(t) \equiv v_z$, $\dot{\psi}(t)$, $\dot{\theta}(t)$, $\dot{\varphi}(t)$, $\ddot{x}(t) \equiv a_x$, $\ddot{y}(t) \equiv a_y$, $\ddot{z}(t) \equiv a_z$, $\ddot{\psi}(t)$, $\ddot{\theta}(t)$ and $\ddot{\varphi}(t)$.

The laws of angular velocity and angular acceleration of the tennis ball are obtained, namely, $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$, $\varepsilon_x(t) \equiv \dot{\omega}_x(t)$, $\varepsilon_y(t) \equiv \dot{\omega}_y(t)$ and $\varepsilon_z(t) \equiv \dot{\omega}_z(t)$.

Finally, the projections of the trajectory of the tennis ball mass center on the three coordinate planes, Oxz, Oyz and Oxy, are obtained.

A computer simulation of two main strokes, forehand and backhand, is realized. The initial conditions and some parameters of the study are listed in the following two tables.

Forehand	v_o [m/s]	α	ω_o [rad/s]	z_o [m]	S	C_D	C_L
First stroke	33.333	6°	314.159	1.00	0.306	0.55	0.184
Second stroke	30.556	6°	261.799	1.00	0.278	0.53	0.167
Third stroke	27.778	6°	209.440	1.00	0.245	0.51	0.147

Backhand	V_o [m/s]	α	ω_o [rad/s]	z_o [m]	S	C_D	C_L
Fourth stroke	33.333	3°	- 209.440	1.00	0.204	0.55	0.122
Fifth stroke	30.556	3°	- 261.799	1.00	0.278	0.53	0.167
Sixth stroke	27.778	3°	- 314.159	1.00	0.368	0.51	0.221

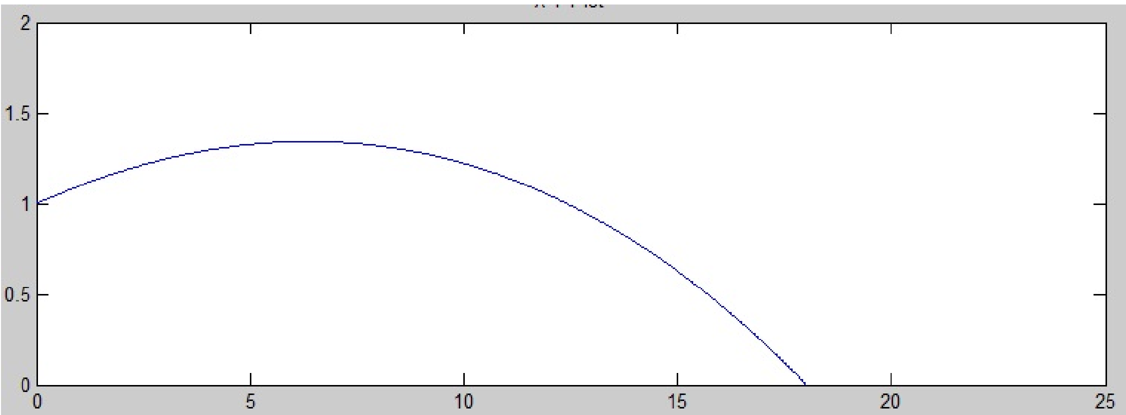

Fig. 3 **Forehand**, topspin stroke, $v_0 = 120\ km/h$, $3000\ rpm$, $\alpha = 6^0$.

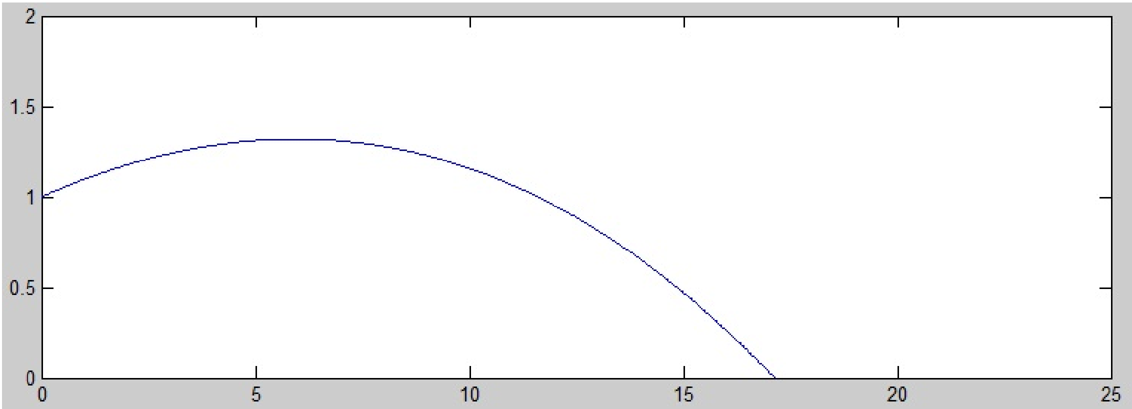

Fig. 4 **Forehand**, topspin stroke, $v_0 = 110\ km/h$, $2500\ rpm$, $\alpha = 6^0$.

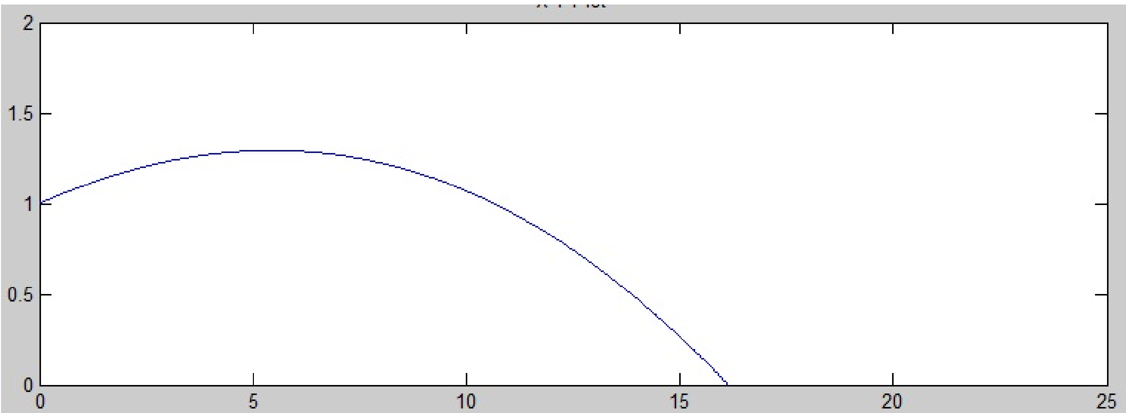

Fig. 5 **Forehand**, topspin stroke, $v_0 = 100\ km/h$, $2000\ rpm$, $\alpha = 6^0$.

Figure 3 shows the projection of the trajectory in Oxz plane of topspin forehand stroke. The horizontal length of the flight is $L = 18.017\ m$.

Figure 4 shows the projection of the trajectory in Oxz plane of topspin forehand stroke. The horizontal length of the flight is $L = 17.149\ m$.

Figure 4 shows the projection of the trajectory in Oxz plane of topspin forehand stroke. The horizontal length of the flight is $L = 16.148\ m$.

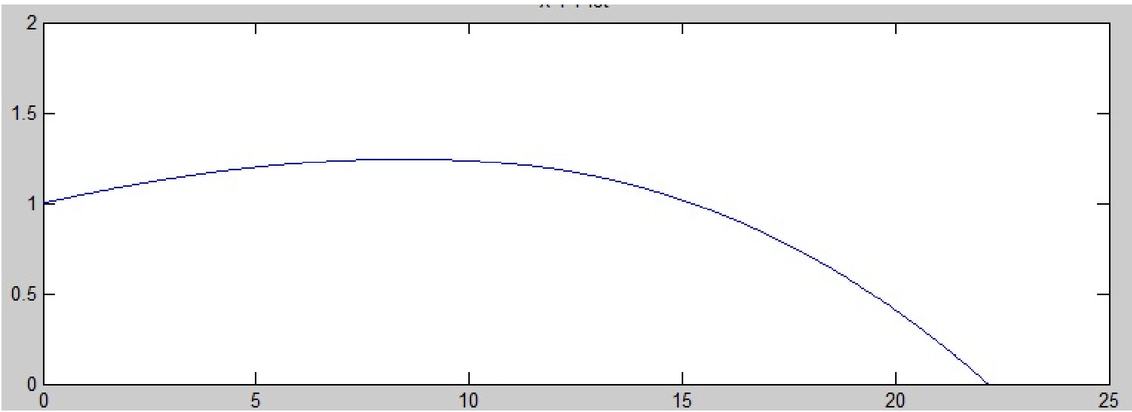

Fig. 6 **Backhand**, backspin stroke, $v_0 = 120\ km/h$, $-2000\ rpm$, $\alpha = 3^0$.

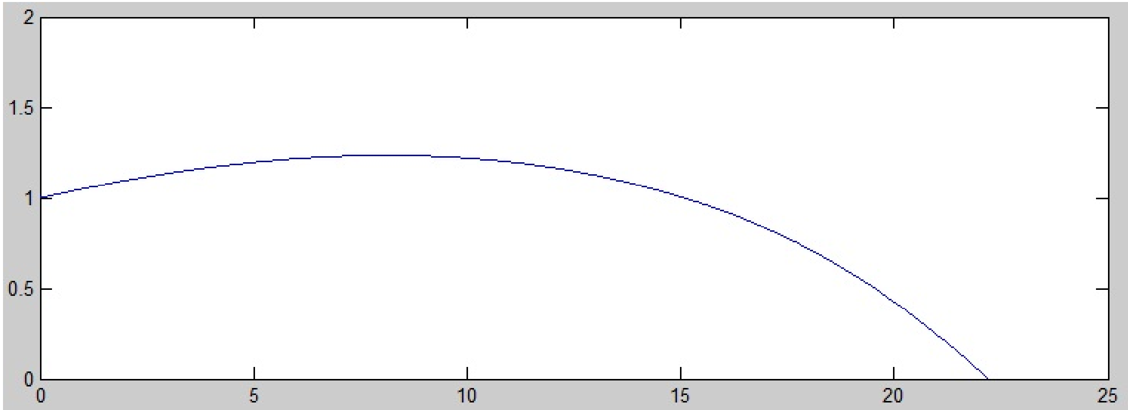

Fig. 7 **Backhand**, backspin stroke, $v_0 = 110\ km/h$, $-2500\ rpm$, $\alpha = 3^0$.

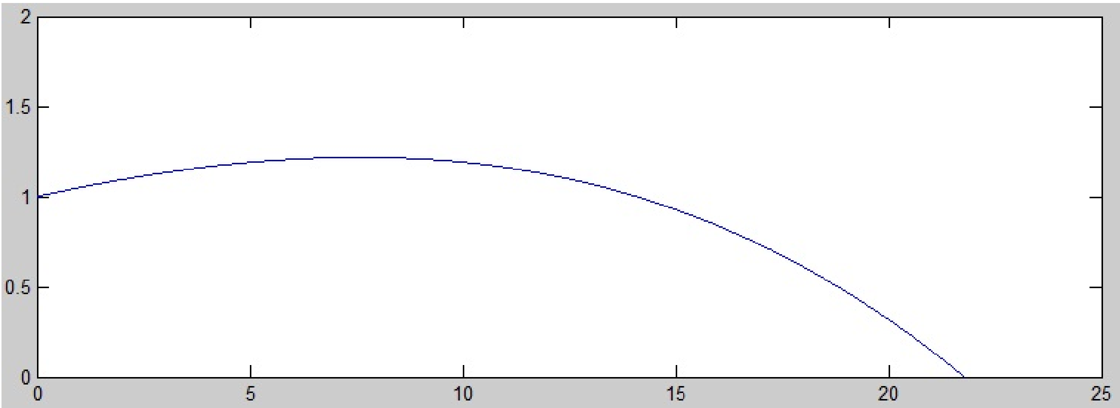

Fig. 8 **Backhand**, backspin stroke, $v_0 = 100\ km/h$, $-3000\ rpm$, $\alpha = 3^0$.

Figure 6 shows the projection of the trajectory in Oxz plane of backspin backhand stroke (backhand slice). The horizontal length of the flight is $L = 22.170\ m$.

Figure 7 shows the projection of the trajectory in Oxz plane of backspin backhand stroke (backhand slice). The horizontal length of the flight is $L = 22.206\ m$.

Figure 8 shows the projection of the trajectory in Oxz plane of backspin backhand stroke (backhand slice). The horizontal length of the flight is $L = 21.785\ m$.

All trajectories, shown in Figures 3-8, are determined by the Magnus force.

The first three trajectories (Figures 3, 4, and 5) refer to the topspin forehands (strokes with upper rotation). At these shots, the Magnus force is directed downward. These trajectories are more convex, reach a greater height, their initial angle is greater (in this case 6^0), and horizontal lengths of their trajectories are about 16 up to 18 meters. If these strokes are performed from the bottom of the court, these strokes cannot go beyond the outline, which is located at a length of $23.77\ m$. Tennis balls pass at a great height above the tennis net, which has a height of $0.914\ m$ up to $1.045\ m$. These strokes are very safe. That is why these strokes are preferred for an attack by professional tennis players.

The second three trajectories (Figures 6, 7, and 8) refer to backspin backhand (strokes with lower rotation). At these shots, the Magnus force is directed upwards. These trajectories are more sloping, reach a lower height, their initial angle is smaller (in this case 3^0), and the horizontal lengths of their trajectories are about 21 up to 22 meters. If these strokes are performed from the bottom of the court, these shots can be much easier to get out the field. Tennis balls pass at a very close distance over the tennis net. These strokes are more risky and insecure. They are defensive, but they are also used by professional tennis players.

CONCLUSION

The trajectories of the two most important tennis strokes are studied: three topspin forehand strokes and three backspin backhand strokes. The first and fourth strokes, the second and fifth strokes, and finally, the third and sixth strokes have the same initial linear velocity, but different initial angular velocities and different initial angles towards the horizon. The main differences, advantages and disadvantages of the two types of strokes are described. The study is useful for a wide range of specialists in the area of Mechanics and the Theory of tennis sport.

REFERENCES

[1] Cross R. Dynamics properties of tennis balls. Physics Department, Sport Engineering, 2, 1999, pp. 23-33.

[2] Cooke, A.J., An overview of tennis ball aerodynamics. Sport Engineering, 3 (2), 2000, pp. 123-129.

[3] Mehta R.D., Pallis J.M., Sports ball aerodynamics: effects of velocity spin and surface roughness. Materials and Science in Sports, TMS, 2001, pp. 185-197.

[4] Chadwick S.G., The aerodynamics properties of tennis balls. The University of Sheffield, Department of Mechanical Engineering, PhD dissertation, 2003, p. 331.

[5] Gonzales O., Graf A.B., Maddocks J.H., Dynamics of rigid body in a Stockes fluid, Jour. Fl. Mech., 519, Cambridge, DOI: 10.1017/S0022112004001284, 2004, pp 133-160

[6] Cohen C., Clanet C., Physics of Ball Sports. EPN 47/3, 2016, pp. 13-16,
doi: http://dx.doi.org/10.1051/epn/2016301.

[7] Li F., Liu L., Wang Q., Qin K., Hu Q., Yang Q., Liu Y., Zhang B., Tennis balls judgment model based on numerical simulation. National Convention on Sports Sc. Of Cina, 01018, 2017, pp. 1-3, doi: 10.1051/ncssc/201701018.

[8] Kwon S., Pfister R., Hager R.L., Hunter I., Steely M.K., Influence of tennis racquet kinetics of ball topspin angular velocity and accuracy during the forehand grandstroke. Journal of Sports Science and Medicine, 16, 2017, pp. 505-513.

[9] Landlinder J., Lindiger S.J., Stöggl T., Wagner H., Müller E., Kinematic differences of elite and high-performance tennis players in the cross court and down the line forehand. Sport Biomechanics, 9 (4), 2010, pp. 280-285.

[10] Ivančević T., Jovanović B., Dikić M., Marković S., Dukić N., Biomechanical analysis of shots and ball motion in tennis and analogy with handball throws. Facta universitatis, series: Physical Education and Sport, vol. 6, № 1, 2008, pp. 51-66.

[11] Alam F., Subic A., Naser J., Rasul M.G., Khan M.M.K., A sydy of sin efects on tennis ball aerodynamics. WSEAS Transactions on Fluid Mechanics, vol. 3. Issue 3, ISSN: 1790-5087, 2008, pp. 271-278.

[12] Chadwick S.G., Haake S.J., The drag coefficient of tennis balls. Engineering of Sport Conference, Sydney, June 2000, pp. 169-176.

[13] Goodwill S.R., Chin S.B., Haake S.I., Aerodynamics of spinning and non-spinning tennis balls. Journ. Wind Eng. and Indust. Aerodyn. vol. 92, Issue 11, 2004, pp. 935-958.

[14] Mehta R., Alam F., Subic A., Review of tennis ball aerodynamics. Sports Technology, No 1, John Wiley and Sons Asia Pte Ltd, 2008, pp. 1-16, doi: 10.1002/jst.11.

[15] Nakajima T., Hiratsuka M., Ito S., Konno A., Aerodynamics characteristics and PIV analyses concerning tennis balls. IOP Conf. Series: Materials Sciene and Engineering 249, 012021, 2017, pp. 1-4.

[16] Cross R., Londsey C., Masurements of drag and lift on tennis balls in flight. Sports Engineering 17 (2), doi: 10.1007/s12283-013-0144-9, 2013, pp. 89-96.

[17] Cross R., Calculations of groundstroke trajectories in tennis. School of Physics, University of Sydney, 2020, pp.1-20, doi: 10.1007/s12283-020-00322-1.

[18] Ivanov A.I., Three-dimensional study of tennis ball flight. Journal Mechanics of Machines, year XXII, book 2, ISSN 0861-9727, 2014, pp. 34-37.

ИЗСЛЕДВАНЕ ТРАЕКТОРИИТЕ НА ТЕНИС ТОПКА

Анастас Иванов Иванов
aii2010@abv.bg

Висше Транспортно Училище „Тодор Каблешков”
1574 София, ул.”Гео Милев” 158,
РЕПУБЛИКА БЪЛГАРИЯ

Ключови думи: *траектории, тенис топка, удари с ротация, ефект на Магнус, числено решение, MatLab*

Резюме*: Съвременният тенис на корт се характеризира с постоянно изменение в няколко направления. Най-съществено от тях се отнася до изменения в конструкцията на ракетите за тенис. Усъвършенства се композитният материал, от който се изработва рамката на ракетата и кордажа. Топките за тенис също се модифицират. Всичко това води до промяна на техниката на ударите на състезателите. Съвременните тенисисти използват най-различни удари. Те се характеризират с различни начални линейни и ъглови скорости, както и с различни начални ъгли спрямо хоризонта. Основната цел на работата е да се изследват двата най-важни удара в тениса: форхенд с горна ротация и бекхенд с долна ротация. Изчисленията и графичните изображения се провеждат с математическия пакет MatLab. Направените изводи и заключения са важни не само за специалистите в областта на Теоретичната и Приложна механика, но и биха били полезни за спортните специалисти, треньори и тенисисти.*