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A PENDULUM SUSPENDED ON AN ELASTIC BEAM

Anastas Ivanov Ivanov
aii2010@abv.bg

Todor Kableshkov University of Transport
1574 Sofia, 158 Geo Milev Str
THE REPUBLIC OF BULGARIA

Key words: *pendulum, elastic beam, geometric nonlinearity, nonlinear oscillations, simulation, MatLab*

Abstract: *The oscillations of a pendulum suspended on an elastic beam are examined. The mathematical pendulum is a material point suspended on an ideal rigid and massless rod. The upper end of the rod is connected by a joint with an elastic beam on two supports. The beam is considered to be perfectly elastic and massless. The system has two degrees of freedom. The nonlinearity is due only to a geometric nature. A nonlinear system of two differential equations is derived. A numerical solution was made with the mathematical package MatLab. The laws of motion, generalized velocities, generalized accelerations, and phase trajectories are obtained. The internal force in the rod as a time function is also determined. The dynamical coefficient for the rod is calculated. In order to continue the task by preparing an actual model and conducting experimental research, the projections of the velocity and acceleration of the material point along the horizontal and vertical axes, as well as their magnitudes, are determined. The obtained results are presented graphically and analyzed in detail. The research has a theoretical and applied character.*

INTRODUCTION

The pendulum has been the subject of researches from ancient times. During the Renaissance, the phenomenal periodic movements of the pendulum were studied by Leonardo da Vinci and Galileo Galilei, [1].

In 1656, the Dutch scientist Christiaan Huygens constructed a pendulum clock. Such clocks remained the most accurate instruments for measuring time until the 1930 year, [2].

In 1851, Jean-Bernard Leon Foucault constructed a mathematical pendulum to prove the rotation of the Earth around the North-South axis, [3].

Analytical solutions related to the study of small and large oscillations of the pendulum provide a field for the development of a number of branches of Mathematics. To this day, many scientists, when studying the large nonlinear oscillations of the pendulum, apply analytical solutions, [4, 5, 6].

The pendulum usually has a fixed center. In publication [7] the pendulum is wrapped around a stationary cylinder. In this case, the length of the cord changes, and the mathematical pendulum has a momentary center of rotation.

The dynamical behavior of rigid weightless rod and concentrated mass, sliding periodically along the axis of the rod is studied in the article [8].

The dynamical behavior of rigid weightless rod and concentrated mass, moving simultaneously along and across the axis of the rod, according to given periodical laws is studied in the article [9].

In the present work, the rotational center of the mathematical pendulum is not fixed. It is connected by a joint for an ideal elastic beam.

The main goals of this study are the following:

1. To obtain the differential equations that describe the big oscillations of the pendulum and the beam taking into account the geometric nonlinearity of the model.

2. To compile a program for numerical integration of the derived nonlinear system of differential equations in the area of the mathematical package MatLab, and after then, to perform a computer simulation.

3. The study should be a basis for continuing the task by preparing a real pendulum and conducting experimental researches.

DYNAMICAL MODEL

The dynamic model of the mathematical pendulum is shown in Figs. 1. It consists of a material point M with a mass m and a perfectly rigid and weightless rod MN, which has a length L. The rod is connected to the simple beam AB by a joint N. The beam has the length l and stiffness of bending $E.I$, where E is the modulus of elasticity (modulus of Young), and I is the moment of inertia of the cross-section of the beam. The joint N is located in the middle of the beam.

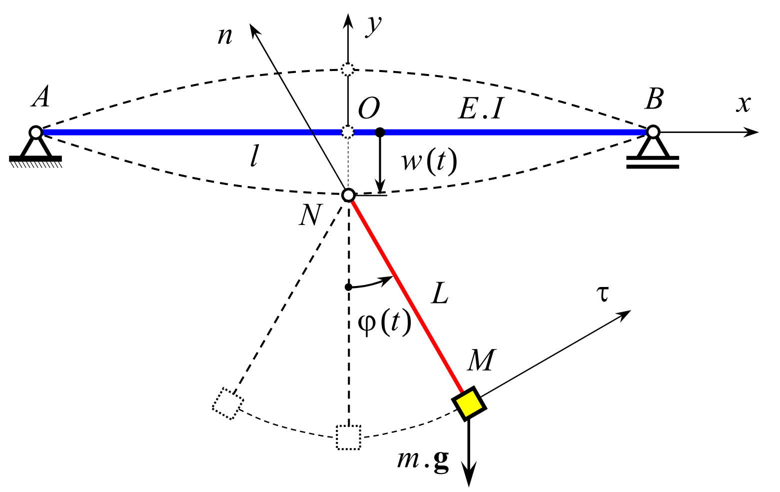

Fig. 1 Dynamical model of the system.

The system has two degrees of freedom. The angle $\varphi(t)$ and the vertical displacement $w(t)$ of the joint N are taken as independent generalized coordinates, (Fig. 1).

In the position of stable equilibrium and rest, the joint N coincides with the initial point O that is considered the center of the fixed coordinate system Oxy. In the same position, the rod MN is vertical.

DIFFERENTIAL EQUATIONS

The mechanical system is conservative. Its research is carried out with the Lagrange equations of the second kind:

(1) $$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{w}}\right) - \frac{\partial E_k}{\partial w} = -\frac{\partial E_p}{\partial w} \ ,$$

(2) $$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} = -\frac{\partial E_p}{\partial \varphi} \ .$$

The kinetic energy of the pendulum is determined by the formula:

(3) $$E_k = \frac{1}{2}.m.\left(\dot{\varphi}.L - \dot{w}.\sin\varphi\right)^2 + \frac{1}{2}.m.\dot{w}^2.\cos^2\varphi \ .$$

The full potential energy has the form:

(4) $$E_p = C_0 - m.g.w + m.g.L.\left(1-\cos\varphi\right) + \frac{1}{2}.c.w^2 \ ,$$

where $C_0 = const$ is the potential energy of the system in a stable equilibrium position, and c is the quasi-elastic coefficient for the beam determined for point N.

This coefficient is calculated by the formula:

(5) $$c = 48.\frac{E.I}{l^3} \ .$$

Formulas (3) and (4) are put in equations (1) and (2), and then the following system of differential equations is reached as follows:

(6) $$\ddot{w} - L.\sin\varphi.\ddot{\varphi} - L.\cos\varphi.\dot{\varphi}^2 + \frac{c}{m}.w - g = 0 \ ,$$

(7) $$\sin\varphi.\ddot{w} - L.\ddot{\varphi} - g.\sin\varphi = 0 \ .$$

This nonlinear system of two ordinary second-order differential equations, under appropriate initial conditions, is performed numerically with a specially prepared program in the area of the mathematical package MatLab.

For future experimental measurements of the velocity and the acceleration of point M, some formulas for the algebraic projections of absolute velocity and absolute acceleration relative to the axes of the fixed coordinate system Oxy are needed, [10-12].

These expressions are the following:

(8) $$v_x = \dot{\varphi}.L.\cos\varphi,$$

(9) $$v_y = \dot{\varphi}.L.\sin\varphi - \dot{w} \ ,$$

(10) $$a_x = \ddot{\varphi}.L.\cos\varphi - \dot{\varphi}^2.L.\sin\varphi \ ,$$

(11) $$a_y = \ddot{\varphi}.L.\sin\varphi + \dot{\varphi}^2.L.\cos\varphi - \ddot{w} \ .$$

The magnitudes of the velocity and acceleration of point M are as follows:

(12) $$v = \sqrt{v_x^2 + v_x^2} \ ,$$

(13) $$a = \sqrt{a_x^2 + a_x^2} \ .$$

The inner force of the rod is determined by the formula:

(14) $$S = m.\left((g - \ddot{w}).\cos\varphi + \dot{\varphi}^2.L\right) .$$

The advantages of the numerical solution consist in the possibility to vary with the values of the parameters in order to obtain the desired final results.

The numerical solution also allows optimization of some parameters, [13].

NUMERICAL SOLUTION

The numerical integration is carried out using the following numerical parameters: $E = 2\times10^{11}\ Pa$; $I = 1.125\times10^{-8}\ m^4$; $L = 2\ m$; $m = 20\ kg$; $l = 3.20\ m$.

The initial conditions are: $w_0 = 0.05\ m$, $\varphi_0 = \pi/4\ rad$, $\dot{w}_0 = 0\ m/s$, $\dot{\varphi}_0 = 0\ rad/s$.

The calculations are performed using an integration time, $t = 10\ s$.

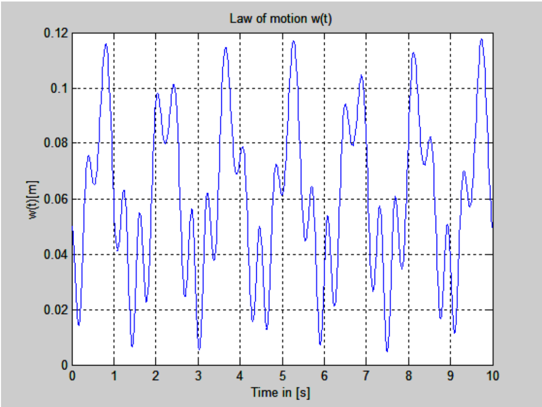

Fig.2 Generalized coordinate $w = w(t)\ [m]$.

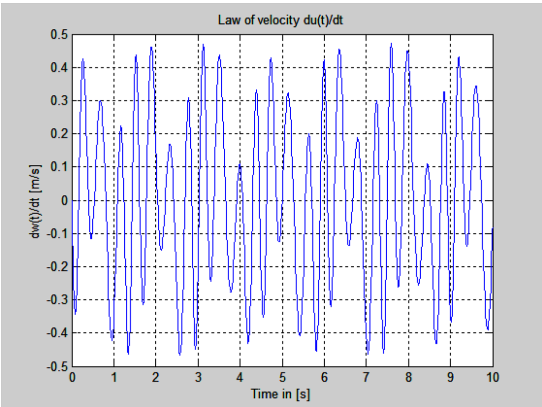

Fig.3 Generalized velocity $\dot{w} = \dot{w}(t)\ [m/s]$.

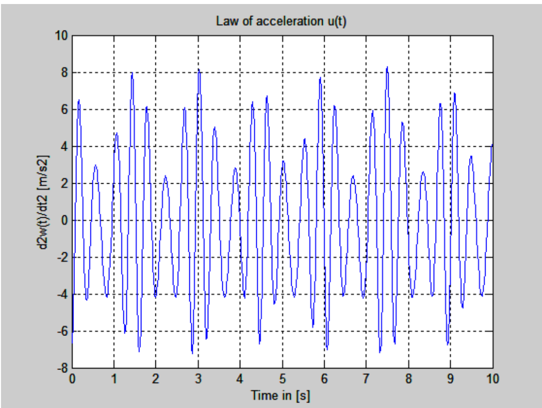

Fig.4 Generalized acceleration $\ddot{w} = \ddot{w}(t)\ [m/s^2]$.

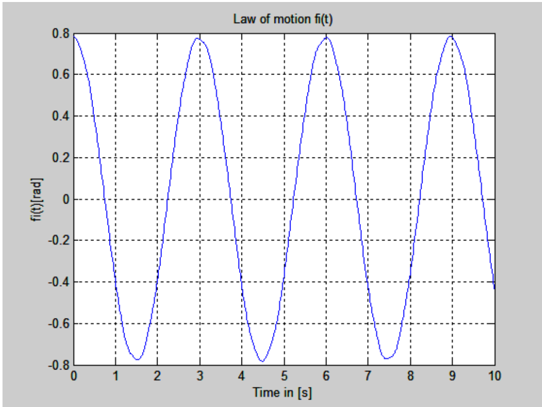

Fig.5 Generalized coordinate $\varphi = \varphi(t)\ [rad]$.

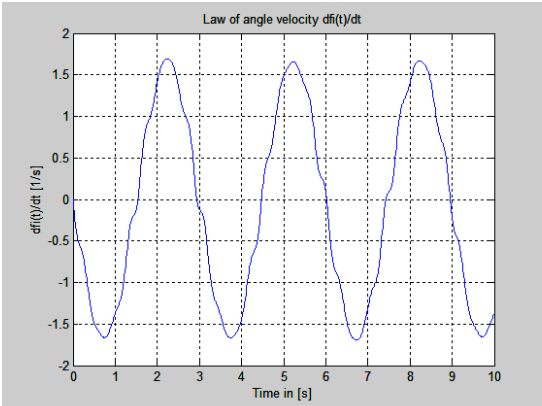

Fig.6 Generalized angle velocity $\dot{\varphi} = \dot{\varphi}(t)\ [rad/s]$.

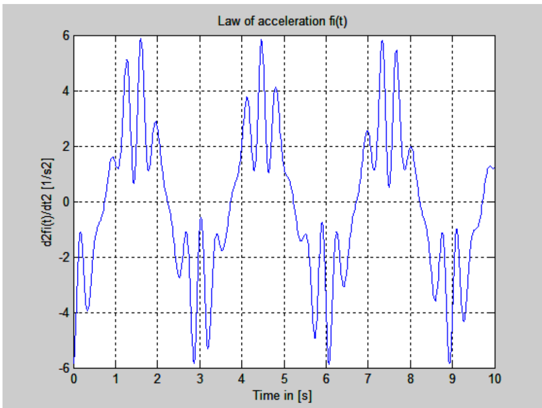

Fig.7 Generalized angle acceleration $\ddot{\varphi} = \ddot{\varphi}(t)\ [rad/s^2]$.

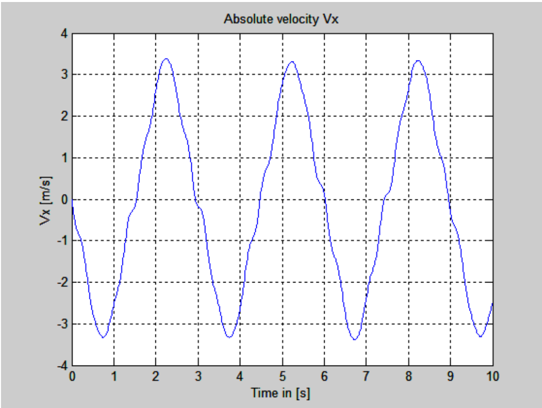

Fig.8 Velocity along the axis Ox
$v_x = v_x(t)\ [m/s]$.

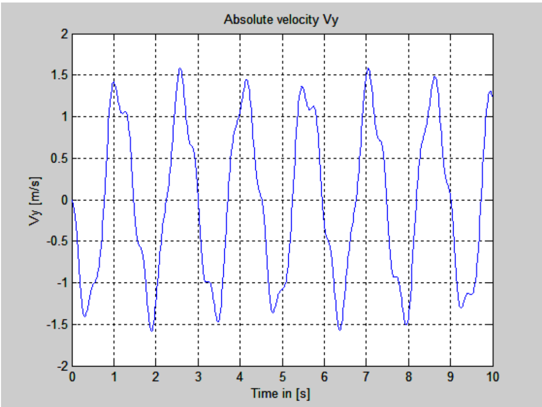

Fig.9 Velocity along the axis Oy
$v_y = v_y(t)\ [m/s]$.

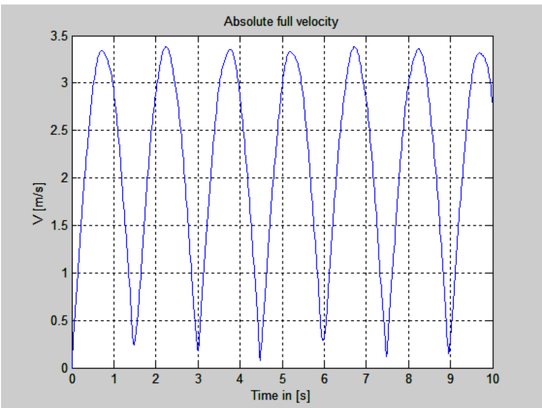

Fig.10 Full velocity
$v = \left(v_x^2 + v_y^2\right)^{0.5}\ [m/s]$.

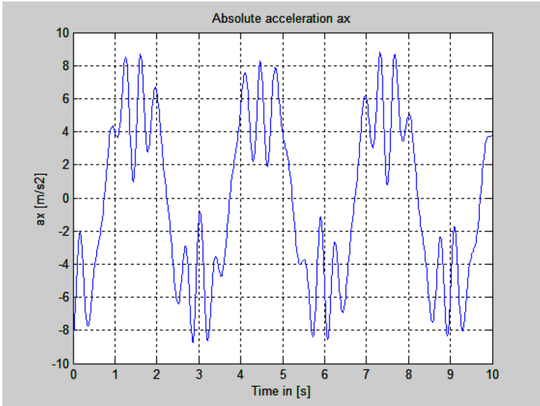

Fig.11 Acceleration along the axis Ox
$a_x = a_x(t)\ [m/s^2]$.

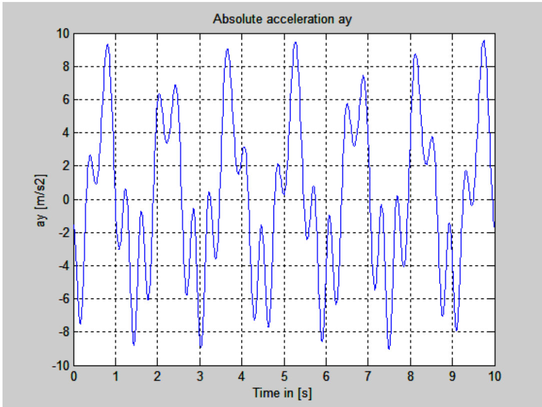

Fig.12 Acceleration along the axis Oy
$a_y = a_y(t)\ [m/s^2]$.

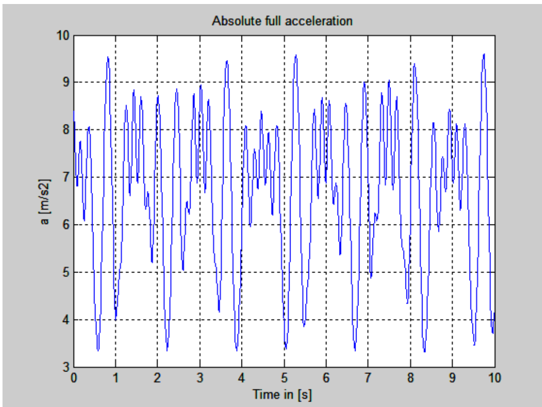

Fig.13 Full acceleration
$a = \left(a_x^2 + a_y^2\right)^{0.5}\ [m/s^2]$.

The fourth-order Runge-Kutta method with a fixed step at relative accuracy 10^{-6} and absolute accuracy 10^{-8} was used.

Figures 2, 3, and 4 show the graphs of the generalized coordinate $w(t)$, the generalized velocity $\dot{w}(t)$, and the generalized acceleration $\ddot{w}(t)$ as functions of time t.

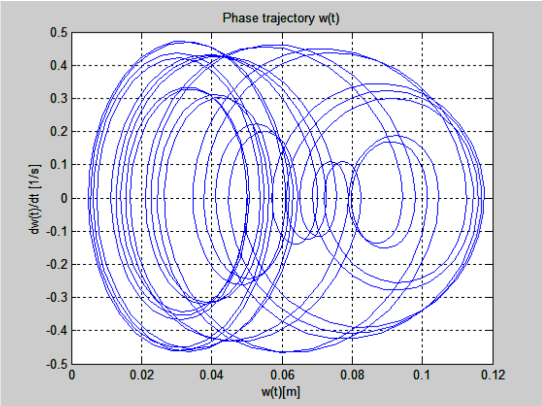

Fig.14 Phase trajectory $\dot{w} = \dot{w}(w)$.

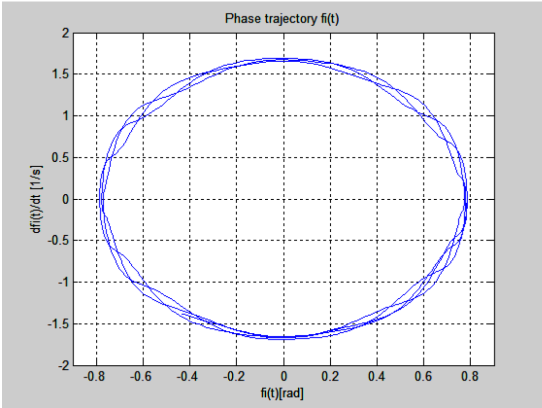

Fig.15 Phase trajectory $\dot{\varphi} = \dot{\varphi}(\varphi)$.

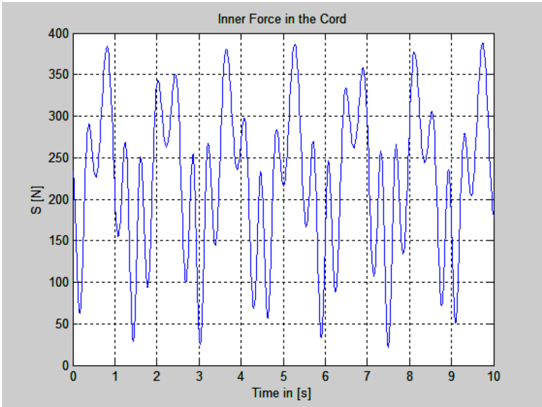

Fig.16 Inner force of the rod
$S = S(t)\ [N]$.

Figures 2, 3 and 4 show the graphs of the generalized coordinate $w(t)$, the generalized velocity $\dot{w}(t)$, and the generalized acceleration $\ddot{w}(t)$ as functions of time t.

Figures 5, 6, and 7 show the graphs of the generalized angle coordinate $\varphi(t)$, the generalized angle velocity $\dot{\varphi}(t)$, and the generalized angle acceleration $\ddot{\varphi}(t)$ as functions of time t.

Figures 8, 9, and 10 show the graphs of the velocity $v_x(t)$ along the axis Ox, the velocity $v_y(t)$ along the axis Oy, and finally, the full velocity $v(t)$ as functions of time t.

Figures 11, 12, and 13 show the graphs of the acceleration $a_x(t)$ along the axis Ox, the acceleration $a_y(t)$ along the axis Oy, and finally, the full acceleration $a(t)$ as functions of time t.

The laws $w(t)$, $\dot{w}(t)$ and $\ddot{w}(t)$ are periodic indeterminate functions of time. The most important obtained values are: $0.0054\,m \le w(t) \le 0.1176\ m$, $\max|\dot{w}| = 0.4688\ m/s$, and $\max|\ddot{w}| = 8.2904\ m/s^2$.

The maximum angular velocity is $\max|\dot{\varphi}| = 1.6909\ s^{-1}$. The range of variation of the angular acceleration is $-5.8622\ s^{-2} \le \ddot{\varphi}(t) \le 5.8872\ s^{-2}$.

The laws of velocity, $v_x(t)$, $v_y(t)$ and $v(t)$, are periodic functions with the following maximum values: $\max\left|v_x\right| = 3.3818\ m/s$, $\max\left|v_y\right| = 1.5845\ m/s$ and $0.0750\ m/s \le v(t) \le 3.3822\ m/s$.

The laws of acceleration, $a_x(t)$, $a_y(t)$ and $a(t)$, are periodic functions that vary in the following ranges: $-8.7331\ m/s^2 \le a_x \le 8.7721\ m/s^2$, $-9.0102\ m/s^2 \le a_y \le 9.4692\ m/s^2$ and $3.3104\ m/s^2 \le a \le 9.5828\ m/s^2$.

The inner force $S(t)$ of the rod has always a positive value, or in other words, the rod is always loaded with tensile force.

This force is in the range $24.7609 N \le S(t) \le 386.589\ N$.

The dynamical coefficient for the rod is $k_{dyn} = 1.97$.

CONCLUSION

It could definitely be said that the conducted research enriches the knowledge in the field of nonlinear Mechanics. It also shows the rich capabilities of the MatLab package for numerical integration of highly nonlinear systems of ordinary differential equations. The obtained numerical results are within real and acceptable limits. This proves that the dynamic model is adequate and the entered input parameters, as well as the initial conditions, are correctly chosen. The study opens the possibility for continuation by building a real model on which some experimental measurements can be made to compare the numerical results.

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МАХАЛО ОКАЧЕНО НА ЕЛАСТИЧНА ГРЕДА

Анастас Иванов Иванов
aii2010@abv.bg

Висше Транспортно Училище „Тодор Каблешков”
1574 София, ул.”Гео Милев” 158,
РЕПУБЛИКА БЪЛГАРИЯ

Ключови думи: *махало, еластична греда, геометрична нелинейност, нелинейни трептения, симулация, MatLab*

Резюме*: Изследват се трептенията на махало, окачено на еластична греда. Математичното махало представлява материална точка окачена на идеално корав безмасов прът. Горният край на пръта е свързана чрез става за проста греда на две опори. Гредата се приема за идеално еластична и безмасова. Системата има две степени на свобода. Нелинейността е само от геометричен характер. Изведена е нелинейна система от две диференциални уравнения. Извършено е числено решение с математическия пакет MatLab. Получени са законите на движение, обобщените скорости, обобщените ускорения и фазовите траектории. Определена е вътрешната сила в пръта като функция на времето. Изчислен е коефициента на динамичност за пръта. С оглед на продължаване на задачата чрез изготвяне на действителен модел и провеждане на експериментални изследвания, са определени проекциите на скоростта и ускорението на материалната точка по хоризонталата и вертикалата, както и техните големини. Получените резултати са изобразени графично и подробно анализирани. Изследването има теоретичен и приложен характер.*