



---

## ANALYTICAL CALCULATION OF STRENGTH OF FREIGHT WAGON AXLE IN ACCORDANCE WITH EUROPEAN STANDARDS

**Milan Bižić, Dragan Petrović,**

[bizic.m@mfkv.kg.ac.rs](mailto:bizic.m@mfkv.kg.ac.rs), [petrovic.d@mfkv.kg.ac.rs](mailto:petrovic.d@mfkv.kg.ac.rs)

*Faculty of Mechanical and Civil Engineering in Kraljevo,  
University of Kragujevac, Dositejeva 19, 36000 Kraljevo, SERBIA*

**Key words:** *Strength calculation, Freight wagon axle, Railway vehicles*

**Abstract:** *This paper presents a methodology for analytical calculation of strength of freight wagon axle in accordance with European standards. The axle of standard freight wheelset for normal track gauge and axle-load of 22.5 t was considered. The authoritative loads of the axle, which originate from vehicle mass and braking, are discussed and analyzed. The principal scheme of loads of axle is given as well as the way of determination of quantitative values of all loads in accordance with European standards. The procedure of determination of operating and allowable stresses and proving the axle strength was performed and exposed. The obtained results are verified by the results of numerical calculation performed in Ansys software package.*

### 1. INTRODUCTION

Axles of railway vehicles belong to the most important and responsible components in the whole railway system. The failure of the railway axle may lead to the derailment not only one vehicle but also the whole train. The consequences are well known and they are reflected in a huge direct and indirect material damage, and very often in human victims. That is why railway axles must be designed and produced in order to satisfy a lot of very rigorous conditions during their long-term operating life. It is about rotational and distinctly dynamically loaded part which was a very common cause of serious derailments and accidents during the past of the railway. During the operating life, railway axle is subjected to the constant bending under the weight of the vehicle. It rotates in the same time, which causes the alternating change of the stress-strain state in the whole axle. There are very distinctly fatigue under this characteristic dynamic load and thus to serious risk of axle fracture. Precisely because of the frequent failures of railway axles in the past, Wöhler had been studying this problem and established his famous theory of material fatigue [1].

Bearing in mind all the above, the great attention has been paid to the problem of design, calculation and production of railway axles, which is still today a very actual topic in the area of railway engineering [2-6]. Although numerical and experimental methods are irreplaceable today and provide wide range of possibilities in design and development phases, the importance of analytical methods should not be neglected. They are especially important in the cases where it is necessary to perform optimization on the basis of precisely described target functions and constraints in mathematical sense. Thus, the aim of this paper is to

presents the methodology for analytical calculation of strength of axle of freight wagon in accordance with the European standards, specifically EN 13103 [7].

## 2. AUTHORITATIVE LOADS FOR AXLES CALCULATION

The reliable identification and determination of loads for axles calculation is very complex task. In order to prevent eventual omissions and mistakes of individuals during these activities, international standards are very precisely defining all loads which should be taken into account in axle design, calculation and testing. In this sense, the authoritative loads for considered axle of standard freight wheelset for normal track gauge and axle-load of 22.5 t are defined in standard [7]. In accordance with given standard, the authoritative loads for axle calculation originate from mass of the vehicle leaning on the axle journals and braking. Forces from mass of the vehicle are caused by influence of weight of vehicle and cargo, as well as vertical oscillations. Also, there are lateral forces and lateral oscillations. These forces cause vertical forces on axle journals  $P_1$  and  $P_2$ , as well as lateral force  $H$  (Fig. 1). The forces  $Q_1$ ,  $Q_2$ ,  $Y_1$  and  $Y_2$  are reactions in the wheel-rail contact. Schematic representation of authoritative loads of considered axle is shown in Fig. 1.

Forces  $P_1$ ,  $P_2$ ,  $Y_1$  and  $Y_2$  are determined from the following expressions [7]:

$$\begin{aligned} P_1 &= \left( 0.625 + 0.075 \frac{h_o}{b} \right) m_1 \cdot g & P_2 &= \left( 0.625 - 0.075 \frac{h_o}{b} \right) m_1 \cdot g & Y_1 &= 0.3 \cdot m_1 \cdot g \\ Y_2 &= 0.15 \cdot m_1 \cdot g & H &\approx Y_1 - Y_2 \end{aligned} \quad (1)$$

All forces are determined in function of mass  $m_1$  per one axle, which is defined as the difference between axle-load mass  $m_{al}$  and wheelset mass ( $m_1 = m_{al} - m_{ws}$ ). Forces  $Q_1$  and  $Q_2$  are defined with the following expressions [7]:

$$\begin{aligned} Q_1 &= \frac{1}{2s} [P_1(b+s) - P_2(b-s) + (Y_1 - Y_2) \cdot R] \\ Q_2 &= \frac{1}{2s} [P_2(b+s) - P_1(b-s) - (Y_1 - Y_2) \cdot R] \end{aligned} \quad (2)$$

The standard [7] also defines the forces due to the braking, in dependence of way of braking realization (one-side shoe brake, two-sided shoe brake, disc brake, etc.).

## 3. STRENGTH CALCULATION

The calculation is performed for the axle of standard wheelset of freight wagon for normal track gauge and axle-load of 22.5 t. The following input parameters are taken into account for a given wheelset:  $m_{ws}=1350$  kg – wheelset mass; axle material EA1T;  $h_o=1900$  mm – height of center of gravity of suspended mass for loaded wagon;  $2b=2000$  mm – distance between axle-box cases;  $2s=1500$  mm – distance between nominal rolling circles;  $D=920$  mm – nominal wheel diameter.

For given input parameters, the following values of authoritative loads for calculation of axle are obtained on the basis of expressions (1) and (2):  $m_1=21150$  kg;  $P_1=159.24$  kN;  $P_2=100.11$  kN;  $Y_1=62.24$  kN;  $Y_2=31.12$  kN;  $H=31.12$  kN;  $Q_1=178.64$  kN;  $Q_2=80.71$  kN. On the basis of these values, the diagram of bending moments in yz plane is formed, as shown in Fig. 2.

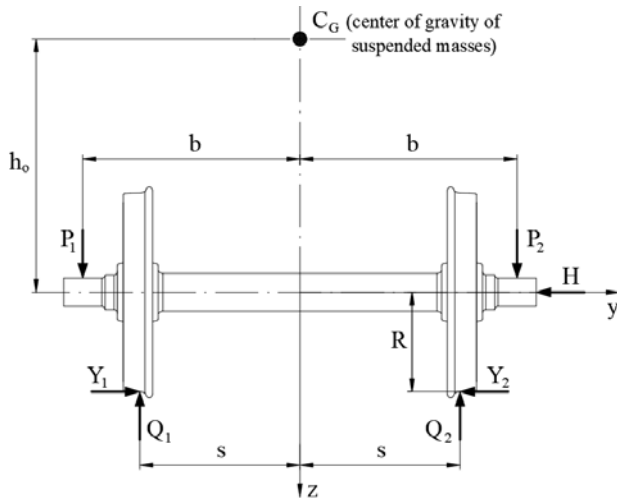


Fig. 1. Schematic representation of loads of axle

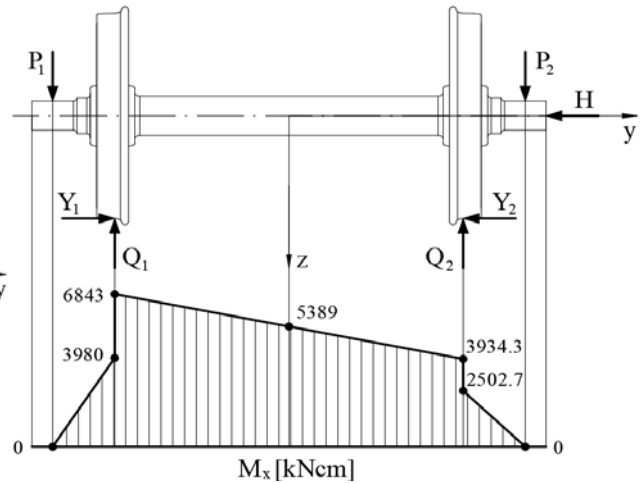


Fig. 2. Diagram of bending moments of axle in yz plane due to the authoritative loads from Fig. 1

In developments [8, 9] wheel axles with disc brake were studied.

In this study, it is supposed that given wheelset is braked with two-sided shoe brake made of gray cast iron. It is impossible to achieve the equal braking forces on both shoe brake on the one wheel, as well as between two wheels of the same wheelset. This causes bending moments of axle in yz plane and xy plane, as well as torsion moment around y axis. The difference between braking forces on shoe brakes of one wheel has been identified by measurements and it is approximately equal  $F_{s1}-F_{s2}=0.3 \cdot F_s$  (see Fig. 3).

The maximal bending moment in yz plane due to braking can be determined from the following expression:

$$M_{xb\max} = \mu \cdot 0.3 \cdot F_s (b - s) \quad (3)$$

In the previous expression  $\mu$  is friction coefficient (for braking with shoe brake made of gray cast iron  $\mu=0.1$ ) and  $F_s$  is braking force which can be determined by multiplication of braking coefficient  $B_k$  (maximal allowable ratio between braking force and static load per wheel in order to prevent wheel blocking) and static load per wheel  $P_{wheel}$ , as follows:

$$F_s = B_k \cdot P_{wheel} \quad (4)$$

For known values of  $\mu=0.1$ ,  $B_k=0.75$  and  $P_{wheel}=110.4$  kN, the value of braking force  $F_s=82.8$  kN is obtained. Thereafter, the value of moment  $M_{xb\max}=62.1$  kNcm is obtained from expression (3). The diagram of bending moments in yz plane due to braking is shown in Fig. 3.

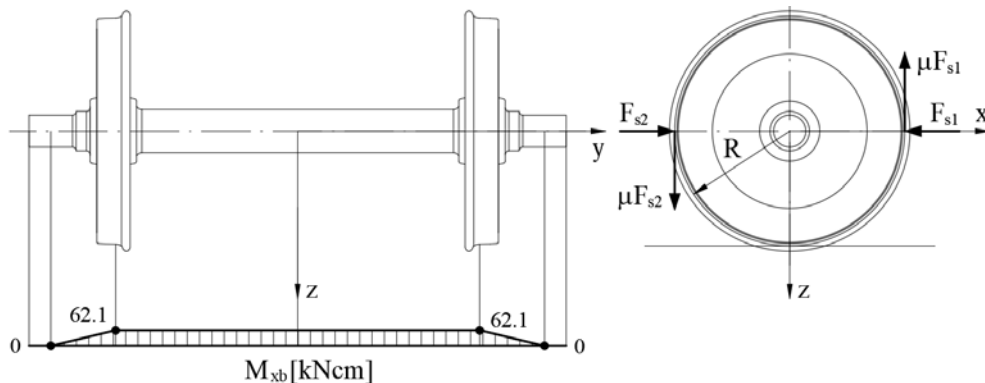


Fig. 3. Diagram of bending moments of axle in yz plane due to braking

The maximal bending moment in xy plane due to braking can be determined from the following expression:

$$M_{zb\max} = 0.3 \cdot F_s (b - s) \quad (5)$$

By substituting known values, the value of moment  $M_{zb\max}=621$  kNcm is obtained. The diagram of bending moments in xy plane due to braking is shown in Fig. 4.

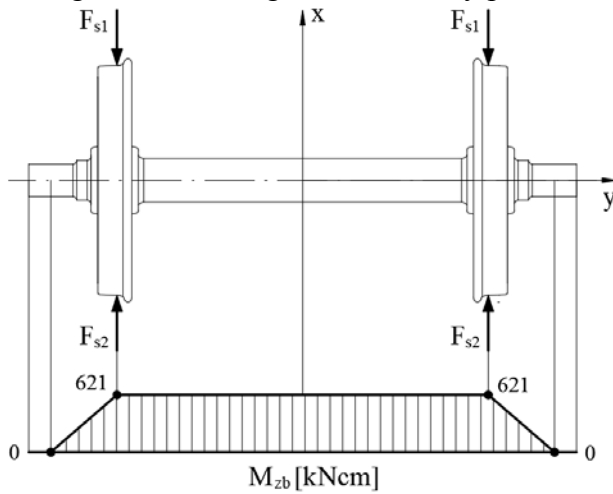


Fig. 4. Diagram of bending moments of axle in xy plane due to braking

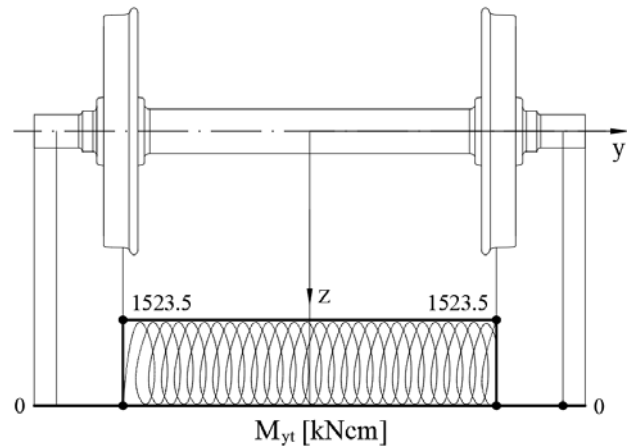


Fig. 5. Diagram of torsion moment of axle around y axis due to braking

The torsion moment of axle around y axis due to braking can be determined from the following expression:

$$M_{yt} = 0.3 \cdot P_{wheel} \cdot R \quad (6)$$

By substituting known values, the value of moment  $M_{yt}=1523.5$  kNcm is obtained. The diagram of torsion moment around y axis due to braking is shown in Fig. 5.

Four characteristic sections S1÷S4 are selected for calculation of axle strength, as shown in Fig. 6. They are in places of transition radii where stress concentration is expected, as well as at the middle of the axle span.

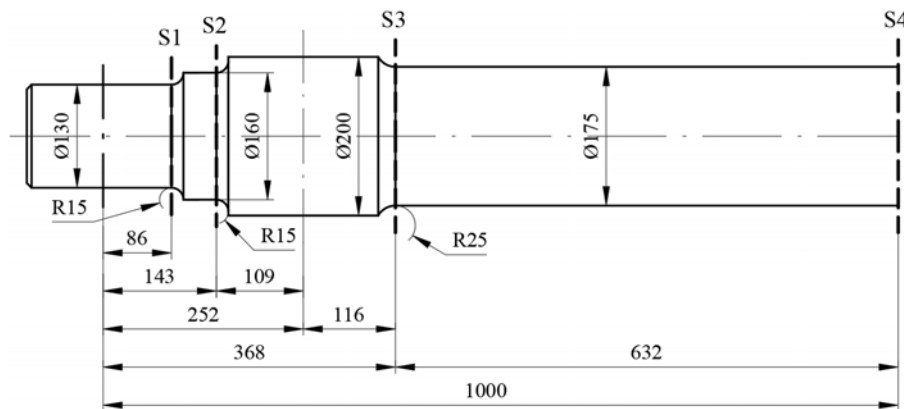


Fig. 6. Characteristic sections for calculation of axle strength

The equivalent stress at the any point of the axle can be calculated in accordance with the following expression:

$$\sigma_e = \sqrt{\sigma^2 + 4\tau^2} \quad (7)$$

Normal and tangential stresses in the arbitrary section at the axle surface are:

$$\sigma = \frac{32M_b}{\pi d^3} \quad (8)$$

$$\tau = \frac{16M_{yt}}{\pi d^3} \quad (9)$$

By substituting expressions (8) and (9) into expression (7), and taking into account the stress concentration factor, the following expression for equivalent stress is obtained:

$$\sigma_e = S_k \cdot \frac{32M_e}{\pi d^3} \quad (10)$$

where  $M_e$  is equivalent moment which is defined as:

$$M_e = \sqrt{M_x^2 + M_y^2 + M_z^2} \quad (11)$$

Stress concentration factor can be determined from the following expression:

$$S_k = A + 1; \quad A = \frac{(4-Y)(Y-1)}{5 \cdot (10X)^{(2.5X+1.5-0.5Y)}}; \quad X = \frac{r}{D}; \quad Y = \frac{D}{d} \quad (12)$$

Interpretation of sizes  $r$ ,  $d$  and  $D$ , in cases where section is in the zone of pressed joint between wheel and axle and outside of that joint, is shown in Fig. 7.

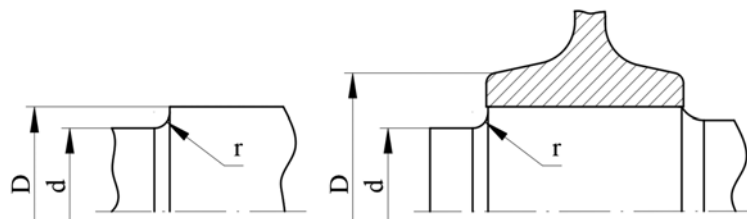


Fig. 7. Interpretation of sizes  $r$ ,  $d$  and  $D$  in expressions for calculation of stress concentration factor

On the basis of the expression (12), the values of stress concentration factors for defined sections S1÷S4 are calculated and they amount:  $S_{k,S1}=1.11$ ;  $S_{k,S2}=1.36$ ;  $S_{k,S3}=1.19$ ;  $S_{k,S4}=1.0$ . Finally, on the basis of previously defined expressions, the values of equivalent stresses at the axle surface in the defined sections S1÷S4 are calculated. The obtained results are shown in comparative Table 1 in the next section.

#### 4. VERIFICATION OF OBTAINED RESULTS

In order to verify the obtained results, FEM model of the wheelset has been formed in Ansys software package (Fig. 8). It is composed of 233962 nodes and 140359 finite elements. Results of equivalent stress obtained by FEM calculation are shown in Fig. 9, while it's numerical values in analyzed sections are given in the Table 1, which simultaneously contains comparison with analytical results. The analysis of the obtained FEM results has shown the high degree of matching with results of analytical calculation.

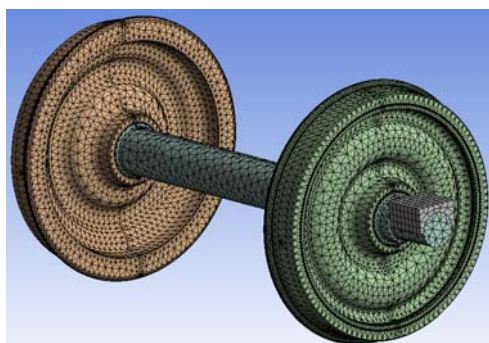
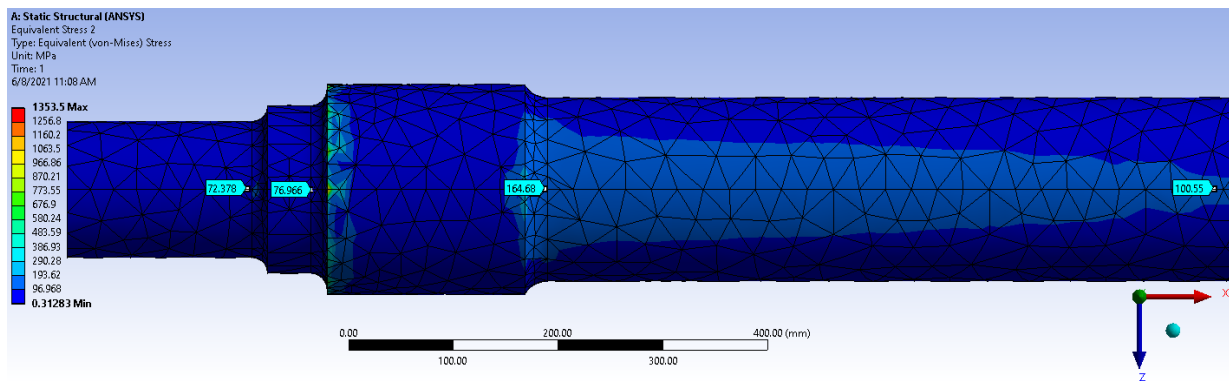


Fig. 8. FEM model of wheelset

Table 1. Comparative review of obtained results of equivalent stresses in characteristic sections of axle

Section	$\sigma_e$ [kN/cm <sup>2</sup> ]		Deviation [%]
	analytical calculation	numerical calculation by FEM	
1	7.04	7.24	2.76
2	7.60	7.70	1.30
3	15.60	16.47	5.28
4	10.83	10.06	7.10



**Fig. 9. Results of equivalent stress in considered sections obtained by FEM calculation**

## 5. CONCLUSION

The paper presents a methodology for analytical calculation of strength of railway axle of freight wagon according to European standards. The axle of standard freight wheelset for normal track gauge and axle-load of 22.5 t was considered. Obtained analytical results in four characteristic sections of axle have shown that operating equivalent stresses are smaller than allowed stresses – endurance limit which for considered axle material and considered sections amounts 16.6 kN/cm<sup>2</sup>. On the basis on results, it can be concluded than Section 3 is critical section of the axle in which operating stress is very close to allowed stresses. The obtained analytical results are verified by results of FEM calculation, whereby deviations between those results are less than 8%. Further research should be directed toward analysis of influence of pressed connection between wheel and axle on the axle strength and reliability.

## ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Serbian Ministry of Education, Science and Technological Development for supporting this research (contract no. 451-03-9/2021-14/200108).

## REFERENCES

- [1] W. Schütz, A history of fatigue, *Engineering Fracture Mechanics*, Volume 54, Issue 2, pp. 263-300, 1996.
- [2] M. Novosad, R. Fajkoš, B. Reha, R. Rezniček, *Fatigue tests of railway axles*, *Procedia Engineering*, 2, pp. 2259–2268, 2010.
- [3] L. Náhlík, P. Pokorný, M. Ševčík, R. Fajkoš, P. Matušek, P. Hutař, *Fatigue lifetime estimation of railway axles*, *Engineering Failure Analysis*, Volume 73, pp. 139-157, 2017.
- [4] F. Dikmen, M. Bayraktar, R. Guclu, *Determination of critical section of wagon axle by considering dynamic and safety factors*, *Alexandria Engineering Journal*, Volume 58, Issue 2, pp. 611-624, 2019.
- [5] M. Filippini, M. Luke, I. Varfolomeev, D. Regazzi, S. Beretta, *Fatigue strength assessment of railway axles considering small-scale tests and damage calculations*, *Procedia Structural Integrity*, 4, pp. 11-18, 2017.
- [6] B. Asngali, A. Susanto, M.F. Subkhan, I. Martinez, K. Yamada, F. Majedi, *Static Analysis of Railway Axle using Finite Element Method and Monitoring of Railway Bearing Based on Vibration Analysis*, *Journal of Physics: Conference Series*, 1845, 012037, pp. 1-7, 2021.
- [7] EN 13103, *Railway applications - Wheelsets and bogies - Non-powered axles - Design method 2010*.

[8] Ralev V., Atmadzhova D., STRENGTH CALCULATIONS OF WHEELS FOR PASSENGER BOGIES, International Scientific Conference "TRANSPORT - 2019" Scientific journal <http://www.mtc-aj.com> Mechanics Transport Communications, issue No 17 2367-6620 (online) VI-40 - VI-51, 2019.

[9] Atmadzhova D., Monograph, "STUDY OF WAGON WHEELS - FORCES AND DYNAMICS OF INTERACTION" WHEEL-RAILWAY ", Publishing House" Avangard Prima "ISBN 978-619-239-456-1, 2020.