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SELF-RECOVERING TELECOMMUNICATION NETWORK ELEMENT TOPOLOGICAL STRUCTURE OPTIMIZATION BY COST CRITERION

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Abstract**:** *The main reason for overloading most telecommunication networks is the finite number of buffers in the switching nodes and the limited channel resource associated with the cost of the channels. Within the limits of the article the algorithm of optimization the self-recovering link of a telecommunication network is developed and possibility of reception technically realizable analytical decision of a problem is provided, using as the limiting condition of cost transfer quantity of the information falling for unit of channel capacity. The scientific novelty of solving this problem consists in using combined switching methods, in particular, in a packet network to transport long messages by hybrid channel switching. The identity of using cost functions of one or another kind, which simplifies the choice of the cost function that most fully corresponds to the conditions of a particular problem is described in the article*

INTRODUCTION

Formulation of the problem. In modern telecommunication networks, the buffer memory volume limitation reduces the average delay time by reducing the packet holding time in the waiting line, but it limits the possibility of transmitting long messages by methods with intermediate accumulation, when the message length can exceed the buffer memory volume required to provide the optimal solution. Thus, there is an actual problem of optimization the telecommunication network element structure and the necessity to obtain the analytical solution of the problem with the limiting criterion of the information quantity transmission cost.

Analysis of research and publications. The method proposed in [1] with unlimited node resources gives overestimated values of average delay time. The known method [2] of limiting the buffer memory size reduces the average delay time by reducing the packet holding time in the waiting line, but it limits the possibility of transmission of long messages by methods with intermediate accumulation [3], when the message length can exceed the buffer memory size, necessary to ensure an optimal solution. Thus, to date, all known methods for solving this problem for a limited number of places in the waiting line do not enable to obtain a strict analytical solution in the cost constraints of the communication channels lease because of obtained optimization functional complexity for the known types of cost function.

Purpose of the article – self-recovering element of the telecommunication network topological structure Optimization and ensuring the possibility of obtaining a technically feasible analytical solution to the problem with the limiting condition of the cost of transmitting the amount of information per channel capacity unit, which will enable to determine the payback period of the link in question.

THE MAIN PART

Based on Little's formula [4], the waiting lines of packets at the input to each communication channel are represented by a mass-maintenance system (M/M/1) of type M/M/1 with waiting. That is, the i-th waiting line receives a Poisson flow of packets with packet intensity per second and average service time per second, distributed according to the exponential law [5].

Taking into account the channel utilization rate (network load), the relative channel capacity (the average share of received applications served by the system) is

(1) $\overline{g}_i = \dfrac{1-\rho_i^{m_i+1}}{1-\rho_i^{m_i+2}}$, where m_i is the number of units in the waiting line.

The average number of applications in the following is determined by the following expression:

(2) $$\overline{r}_i = \frac{\rho_i^2\,[1-\rho_i^{m_i}\,(m_i+1-m_i\cdot\rho_i)]}{(1-\rho_i^{m_i+2})\,(1-\rho_i)}.$$

The average delay time $\overline{T_i}$, equal to the time spent by the request in the system, is expressed by the general formula:

(3) $\overline{T}_i = \overline{r}_i/\lambda_i + \overline{g}_i/\mu_i$.

Taking into account (1) and (2) expression (3) after transformation takes the form:

(4) $$\overline{T}_i = \frac{1}{\mu_i}\,\frac{1-\rho_i^{m_i+1}\,[(m_i+2)-\rho_i\,(m_i+1)]}{(1-\rho_i^{m_i+2})\,(1-\rho_i)}.$$

Let us denote $\sum\limits_{k=0}^{m_i+1}\rho_{\mathrm{i}}^k = \dfrac{1\text{-}\rho_i^{m_i+2}}{1-\rho_i} = \Sigma_{m_i}$,

where Σ_{m_i} is the sum of a geometric progression.

Expression (4) is represented in the transformed form:

(5) $$\overline{T}_i = \frac{1}{\mu_i}\cdot\frac{\sum\limits_{\alpha=0}^{m_i}(1+\alpha)\rho_i^{\alpha}}{\sum\limits_{\alpha=0}^{m_i+1}\rho_i^{\alpha}} = \frac{1}{\mu_i}\cdot\frac{\left(\sum\limits_{\alpha=0}^{m_i+1}\rho_i^{\alpha}\right)'}{\sum\limits_{\alpha=0}^{m_i+1}\rho_i^{\alpha}} = \frac{1}{\mu_i}\,\frac{\Sigma'_{m_i}}{\Sigma_{m_i}},$$ where Σ' means $\dfrac{\partial\Sigma}{\partial\rho}$.

The average delay time in the whole network is given with (5):

(6) $$\overline{T} = \frac{1}{\gamma}\sum_{i=1}^{k}\rho_i \frac{\left(\sum\limits_{\alpha=0}^{m_i+1}\rho_i^{\alpha}\right)'}{\sum\limits_{\alpha=0}^{m_i+1}\rho_i^{\alpha}} = \frac{1}{\gamma}\sum_{i=1}^{k}\rho_i \frac{\Sigma'_{m_i}}{\Sigma_{m_i}}.$$

Dependence (6) with $m_i \to \infty$ converted to the well-known formula $\overline{T} = \frac{1}{\gamma}\sum_{i=1}^{k}\frac{\rho_i}{1-\rho_i}$, which corresponds to the network model with unlimited queue, and with $m_i = 0$; $\overline{T} = \frac{1}{\gamma}\sum_{i=1}^{k}\frac{\rho_i}{1+\rho_i}$, which corresponds to the network model in the form of an LMS with failures. Function (6) is a convex function, but does not contain extremes, which makes it impossible to find the minimum of the average delay time by calculating the partial derivatives $\frac{\partial \overline{T}}{\partial \rho_i} = 0$. Thus, this problem is a problem of conditional optimization. The analytical solution of the problem is possible if the cost function is chosen as the bounding condition. Numerical calculations show that usually there is no big difference between cases of using cost functions of one or another kind, that is, we should choose the cost function that best corresponds to the conditions of a particular problem. Let us consider a cost function of the form

(7) $D = \nu\sum_{i=1}^{k}\frac{F_i}{V_i}$, where in packet transmission of messages $F_i = L\lambda_i$, $V_i = L\mu_i$; L – the fixed packet length (bits). Then the cost function (7) takes the form $D = \nu\sum_{i=1}^{k}\rho_i$ and is expressed in units of the cost of transmission of a unit amount of information, i.e. information flow density, which corresponds to the accepted principles of payment for the use of communications. Thus, the optimization problem can be formulated in the following form: to determine the optimal values of information flow density, minimizing the average delay

(8) $\overline{T} = \frac{1}{\gamma}\sum_{i=1}^{k}\rho_i\frac{\Sigma'_{m_i}}{\Sigma_{m_i}} \to \min$, with a cost limitation of the total amount transmission of information per unit of communication line capacity

(9) $$D = \nu\sum_{i=1}^{k}\rho_i \le D_{set}.$$

To solve this problem, the method of indefinite Lagrange multipliers is applied. Let us compose the optimization functional:

(10) $D = \nu\sum_{i=1}^{k}\rho_i \le D_{set}$, where *p* is an indefinite Lagrange. Multiplier calculating the partial derivatives $\frac{\partial \Phi}{\partial \rho_i} = 0$, we obtain a system of *k* equations of the form

(11) $$\left(\rho_i\frac{\Sigma'_{m_i}}{\Sigma_{m_i}}\right)' + \gamma P\nu = 0,\ \ \mathrm{i} = \overline{1,\mathrm{k}}.$$

Analysis of expression (11) shows that each equation of this system depends on the variable ρ_i and parameters m_i, γ, P, ν. If we put $m_i = m$ same for all nodes, then these parameters do not depend on the index i, i.e. $\rho_i = F(m, \gamma, P, \nu)$. This allows us to conclude that $\rho_i^{onm} = \rho = const$, i.e. the optimal values of the information flow densities are the same for all branches and do not depend on the number of the communication branch.

After differentiation and transformations, we obtain a second-order differential equation for each branch. Omitting the index i, we have

(12) $$\frac{\sum_m^{'}}{\sum_m} + \rho\frac{\sum_m^{''}}{\sum_m} - \rho\frac{(\sum_m^{'})^2}{\sum_m^2} + \gamma P \nu = 0.$$

By changing the variable $\sum_m^{''} / \sum_m = Z$ and $\sum_m^{''} = Z^{'} \sum_m + Z \sum_m^{'}$ equation (12) is transformed into an inhomogeneous linear equation of the 1st order

(13) $$Z^{'} + \frac{1}{\rho} Z = -\frac{1}{\rho}\gamma P\nu.$$

The general solution of equation (13) is found by the method of variation of an arbitrary constant. The corresponding homogeneous equation $Z^{'} + \frac{1}{\rho} Z = 0$ with separable variables has a general solution in the following form: $Z = a_1 / \rho$.

Let us put $a_1 = a_1(\rho)$ some continuously differentiable function of ρ, then

(14) $$Z = \frac{a_1(\rho)}{\rho}.$$

Let us choose a function $a_1(\rho)$ so that expression (14) satisfies equation (13). Substituting (14) into (13) after transformations, we will get

(15) $$a_1(\rho) = \ \gamma P\nu.$$

Integrating (15), we have $a_1(\rho) = -\gamma P\nu\rho + a_2$ and, therefore $Z = -\gamma P\nu + a_2/\rho$. Coming back to the old variable (13), we will get:

(16) $$\partial \sum_m^{'} / \sum_m = -\gamma P\nu + (a_2/\rho).$$

Separating the variables in expression (16), we will work with the following equation

$$\frac{d\sum_m}{\sum_m} = -\gamma P\nu\, d\rho + a_2 \frac{d\rho}{\rho},$$ integrate it and get.

(17) $$\textstyle\sum_m = a_3 \rho^{c_2} \cdot e^{\ \gamma P\nu\rho},$$ where a_3 is the constant of integration of the 2nd quadrature.

Using equation (17), we find arbitrary constants of integration and by solving the Cauchy problem for given initial conditions.

In further calculations, we restrict ourselves to dependence (17), from which the values for each branch of the considered network are determined:

(18) $$\rho = \frac{a_2}{\gamma P\nu + \left(\sum_m^{'} / \sum_m\right)}.$$

Let us determine the values a_2 from the initial condition $\rho_o = 1$ and equation (6)

(19) $\left(\Sigma'_m/\Sigma_m\right)\Big|_{\rho=1} = \dfrac{1+2+\cdots+(m+1)}{m+2} = \dfrac{(m+1)(m+2)}{2(m+2)} = \dfrac{m+1}{2}$.

Expression (19) takes into account that the numerator $1+2+...+(\mathrm{m_i}+1) = \dfrac{(m+1)(m+2)}{2}$ is the sum of an arithmetic progression. From equation (18) under condition (19) we determine the constant a2: $a_2 = \gamma P\nu + \dfrac{\mathrm{m}+1}{2}$. Finally, we get:

(20) $\rho = \left(\gamma P\nu + \dfrac{\mathrm{m}+1}{2}\right) \Big/ \left(\gamma \mathrm{P}\nu + \left(\dfrac{\Sigma'_\mathrm{m}}{\Sigma_\mathrm{m}}\right)\right)$.

To determine the indefinite Lagrange multiplier, we use the condition for the limiting value of the cost: $\nu \sum\limits_{\mathrm{i}=1}^{\mathrm{n}} \dfrac{\gamma P\nu + (\mathrm{m}+1)/2}{\gamma P\nu + \left(\Sigma'_\mathrm{m}/\Sigma_\mathrm{m}\right)} = \nu\mathrm{n}\, \dfrac{\gamma P\nu + (\mathrm{m}+1)/2}{\gamma P\nu + \left(\Sigma'_\mathrm{m}/\Sigma_\mathrm{m}\right)} = \mathrm{D_{sat}}$.

After transformations, we get the value of the Lagrange multiplier p:

(21) $p = \dfrac{(\mathrm{m}+1)/2 - \left(\Sigma'_\mathrm{m}/\Sigma_\mathrm{m}\right)\cdot\left(\mathrm{D_{sat}}/\nu\mathrm{n}\right)}{\gamma\nu\left(\left(\mathrm{D_{sat}}/\nu\mathrm{n}\right)-1\right)}$.

Substituting (21) into (20), we get the conditions for the extreme $\overline{T}_i$ of expression (6):

(22) $\left(\rho_\mathrm{opt} - \mathrm{D_{sat}}/\nu\mathrm{n}\right)\left((\mathrm{m}+1)/2 - \Sigma'_\mathrm{m}/\Sigma_\mathrm{m}\right) = 0$.

Conditions (22) are satisfied if any of their factors is zero, i.e.

(23) $\rho_{opt} - \mathrm{D_{sat}}/\mathrm{k}\nu = 0$, $\Sigma'_m/\Sigma_m - (\mathrm{m}+1)/2 = 0$

Condition (23) determines the optimal value of specific flow in the branches:

(24) $\rho_{opt} = \mathrm{D_{sat}}/\nu\mathrm{n}$,

which provides the minimum value of the average delay time:

(25) $\overline{T}^{\min} = \dfrac{1}{\gamma}\cdot\dfrac{\mathrm{D_{sat}}}{\nu}\cdot\left(\dfrac{\Sigma'_\mathrm{m}}{\Sigma_\mathrm{m}}\right)_{opt}$, $0 < \rho_{\text{опт}} < 1$.

The analysis of the obtained results (25) displays that network cost is determined mainly by the costs of data transmission, so it is necessary to use the resources of the network as efficiently as possible. According to expression (24) we can conclude that the communication network should be isotropic in the sense of constancy of values of the density of the flow of transmitted information in all communication lines ($\rho_{opt} = \mathrm{D_{sat}}/\mathrm{kn} < 1$ does not depend on the number of communication branches).

If flows in branches at network synthesis are set in the form of gravitation matrix $\|\lambda_i\|$, then at a fixed packet length L the throughput capacities of corresponding branches are directly proportional to the values of flows of these branches, that is

(26) $V_i = \left(\nu\mathrm{n}/\mathrm{D_{sat}}\right)\cdot \mathrm{F_i}$,

what is a necessary condition of elimination of network blocking (that is $V_i > \mathrm{F_i}$), and the degree of this excess is determined by the ratio of the number of branches of the network to their cost.

CONCLUSION

A new method for optimizing the topological structure of a self-recovering element in a telecommunications network based on the cost criterion is proposed in the article. Based on the proposed method, it was proved that the number of places in the waiting line does not depend on the number of the node or branch, which is fair, since according to relation (26), an increase in the information flow leads to the need for a proportional increase in the channel capacity, which, in turn, leads to a faster freeing buffers, so that the number of requests at the entrance to each channel remains unchanged and the required number of buffers remains constant.

Thus, the obtained analytical expressions (24, 25) enable, at a given cost of transferring an information unit of, to select the number of buffer memory elements and the optimal value of the information flow density that provides the minimum average delay in message transmission in the communication network.

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ОПТИМИЗАЦИЯ ТОПОЛОГИЧЕСКОЙ СТРУКТУРЫ САМОВОССТАНАВЛИВАЮЩЕГОСЯ ЗВЕНА ТЕЛЕКОММУНИКАЦИОННОЙ СЕТИ ПО КРИТЕРИЮ СТОИМОСТИ

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РЕПУБЛИКА БЪЛГАРИЯ

Ключевые слова: *телекоммуникационная сеть, время задержки, оптимизация, плотности информационного потока, самовосстановление, пропускная способность*

Аннотация**:** *Основной причиной перегрузок большинства телекоммуникационных сетей является конечное число буферов в узлах коммутации и ограниченность канального ресурса, связанная со стоимостью каналов. В рамках статьи разработан алгоритм оптимизации самовосстанавливающегося звена телекоммуникационной сети и обеспеченна возможность получения технически реализуемого аналитического решения задачи, за сет использования в качестве ограничивающего условия стоимости передачи количества информации, приходящуюся на единицу пропускной способности. Научная новизна решения данной задачи состоит в использовании комбинированных методов коммутации, в частности, в пакетной сети длинные сообщения транспортировать методом гибридной коммутации каналов. В статье показана тождественность использования стоимостных функций того или иного вида, что упрощает выбор стоимостной функции, которая наиболее полно соответствует условиям конкретной задачи.*