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DETERMINATION OF THE FIRST-ORDER TRANSFER FUNCTIONS SENSITIVITIES OF A T-SHAPED FILTER

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Key words: *sensitivity analysis, transfer functions,T-filter*

Abstract: *Determining the sensitivity of the circuit functions of electrical circuits is a task, the solution of which is essential in the synthesis of different in type and purpose electrical and electronic devices. Finding this indicator is especially necessary in the design of frequency-selective devices, because the requirements for the accuracy of reproduction of their frequency characteristics are too high. Sensitivity theory is an universal power tool for solving a number of problems related to the analysis and diagnostics of electrical circuits (EC). Sensitivity functions are used as a criterion for comparing different configurations of frequency-selective circuits and represent one of the most important indicators in their analysis.*

The report defines the sensitivities of the first order of the transmission function of a T-shaped filter by means of signal graphs in symbolic form. The filter in question is part of the transmission path of a radio communication system and is used to compensate for some nonlinear effects of the real communication channel.

The simulation results of the transfer function sensitivity magnitude with respect to the parameters and frequency changing are presented.

INTRODUCTION

In the synthesis of electric filters, strict requirements are set both for their frequency and time characteristics, and for the permissible deviations from their nominal values, i.e. stability to these characteristics. This problem is solved by the methods of the theory of sensitivity of electric circuits. Sensitivity functions are used as a criterion for comparing different configurations of frequency-selective circuits and represent one of the most important indicators in their analysis.

Sensitivity analysis plays an important role in determining the critical design variables in analog circuit analysis and synthesis [1], [2]. The Modified Coates Flow Graph (MCFG) [3] allows to simplify the analysis of first-order sensitivity on the base of some network partial transfer functions [4], [5]. According to the classical formulae, the calculation of the first-order transfer function sensitivities needs in the first place to find the corresponding derivatives. This is the main problem sensitivity analysis and its investigation is an object of some special methods, described in the literature [4], [6]. The main drawback of some methods based on the adjoint graph is the necessity to analyze the corresponding graph twice and the method based on the Coates flow graph (CFG) gets over it. CFG is useful and often used in the network theory and in the linear system theory [7].

In the paper the first-order sensitivities of the transfer function of a T-shaped filter are determined by means of signal graphs in symbolic form. The filter in question is a part of the transmission path of a radio communication system and is used to compensate for some nonlinear effects of the real communication channel.

The dependence of the sensitivity of the transfer function on the parameters and frequency changing is studied and the corresponding characteristic is obtained using the software product Mathcad.

SYMBOLIC SENSITIVITY ANALUSIS BASED ON COATES FLOW GRAPH

We suppose that the voltage transfer function T_{qk} is under consideration. From a practical point of view it is necessary to find first-order transfer function sensitivity $S_{Y(s)}^{T_{kq}(s)}$ with respect to the value of passive network element $Y(s)$

$$S_{Y_i(s)}^{T_{kq}(s)} = \frac{Y_i(s)}{T_{kq}(s)}\frac{\partial T_{kq}(s)}{\partial Y_i(s)} = \frac{Y_i(s)}{T_{kq}(s)}\sum_{j,i}\frac{\partial T_{kq}(s)}{\partial Y_{ji}(s)}\frac{dY_{ji}(s)}{dY_i(s)} = \frac{Y_i(s)}{T_{kq}(s)}\sum_{j,i}T_{iq}T_{kj}\frac{dY_{ji}(s)}{dY_i(s)}, \tag{1}$$

where $Y_{ji}(s) = a_{ji}(s) + Y_i(s)$ is an element of the matrix **Y**(s) or an edge between vertex *i* and vertex *j* in the modified Coates flow graph;

Y_i - an admittance of parameter p_i;

$a_{ji}(s)$ - contains other network parameters.

The transfer function $T_{kq}(s)$ is as follows:

$$T_{kq}(s) = \frac{\sum_{Q=1}^{R}(-1)^{N_Q} P_Q}{\sum_{K=1}^{L}(-1)^{N_K} P_K}, \tag{2}$$

where Q is the separation of loops of $\mathbf{G}_{kq}^{MC}$,

K - the separation of loops of $\mathbf{G}_0^{MC}$,

N_Q - the number of the loops in the Q^{th} separation of loops of $\mathbf{G}_{kq}^{MC}$,

N_K - the number of the loops in the K^{th} separation of loops of $\mathbf{G}_0^{MC}$,

R - the number of the one factors in $\mathbf{G}_{kq}^{MC}$,

L - the number of the one factors in $\mathbf{G}_0^{MC}$,

P_Q -the product of the loop transmission coefficients in Q^{th} separation of loops of $\mathbf{G}_{kq}^{MC}$,

P_K - the product of the loop transmission coefficients in K^{th} separation of loops of $\mathbf{G}_0^{MC}$.

Let us find the sensitivity of voltage transfer function $T_{qk}(s) = T_{31}(s) = U_o(s)/U_i(s) = U_3/U_1$ for the filter shown in Fig. 1 for $R_0 = 600\Omega = const$, $R_2 = 66\Omega = const$, $L_1 = 7.5mH$, $C_1 = 0.06\mu F$, $L_2 = 20mH$ and $C_2 = 0.02\mu F$.

The modified Coates flow-graph is presented in Fig. 2, where $Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + sL_2 + \frac{1}{sC_2}}$

Then taking into account (1) the sensitivities with respect to the $sC_2(s)$ and $1/sL_2$ are, respectively:

(3) $$S_{sC_2}^{T_{31}} = \frac{sC_2}{T_{31}}\frac{\partial T_{31}}{\partial sC_2} = \frac{sC_2}{T_{31}}\frac{\partial T_{31}}{\partial Y_{22}}\frac{dY_{22}}{dsC_2} = \frac{sC_2}{T_{31}}T_{21}T_{32}\frac{dY_{22}}{dsC_2} = \frac{sC_2}{\Delta_{31}}\frac{\Delta_{21}\Delta_{32}}{\Delta}$$

(4) $$S_{1/sL_2}^{T_{31}} = \frac{1/sL_2}{T_{31}}\frac{\partial T_{31}}{\partial 1/sL_2} = \frac{1/sL_2}{T_{31}}\frac{\partial T_{31}}{\partial Y_{22}}\frac{dY_{22}}{d\,1/sL_2} = \frac{1/sL_2}{T_{31}}T_{21}T_{32}\frac{dY_{22}}{d\,1/sL_2} = \frac{1/sL_2}{\Delta_{31}}\frac{\Delta_{21}\Delta_{32}}{\Delta}.$$

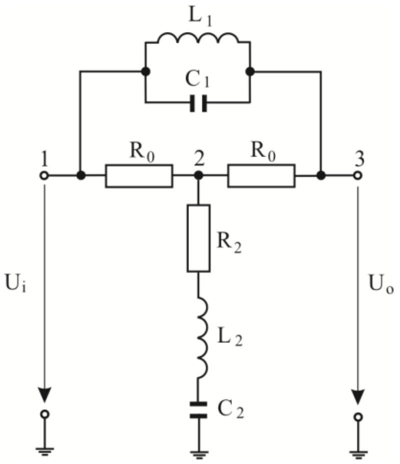

Fig. 1 T-filter

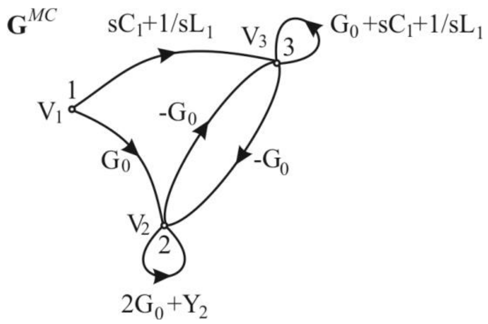

Fig. 2 Modified Coates flow-graph

The partial transfer functions T_{21}, T_{31} and T_{32} are obtained by modified Coates flow-graph $\mathbf{G}^{MC}$, using sub-graphs G_{21}^{MC}, G_{31}^{MC}, G_{32}^{MC} and G_0^{MC} respectively.

- Sub-graph G_0^{MC}, shown in Fig. 3(a), is obtained from $\mathbf{G}^{MC}$ due to removing all outgoing edges from the vertex-source 1. Figures 3(b) and 3(c) show the sub-graph's 1Fs.

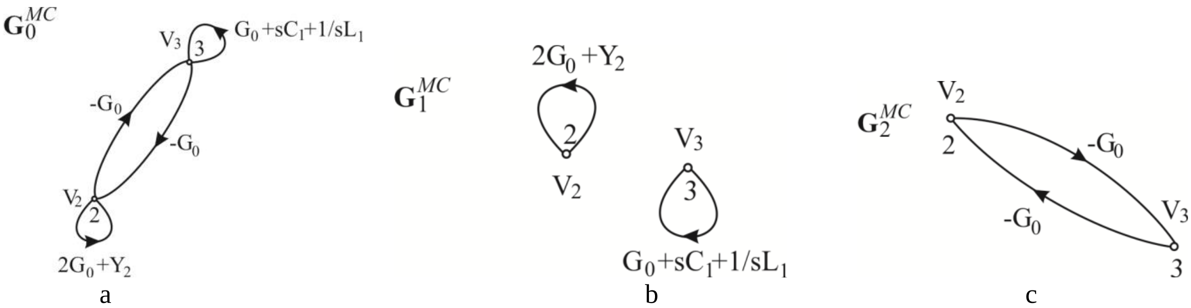

Fig. 3 Sub-graph G_0^{MC} and its 1Fs.

Then for $L = 2$, $N_1 = 2$, $N_2 = 1$ it is obtained

(5) $$\sum_{K=1}^{L}(-1)^{N_k}P_K = \Delta = (-1)^2(2G_0 + Y_2)(G_0 + sC_1 + 1/sL_1) + (-1)^1(-G_0)(-G_0) = Y_{22}Y_{33} - G_0^2$$

- Sub-graph G_{31}^{MC}, shown in Fig. 4(a), is obtained from $\mathbf{G}^{MC}$ due to removing all outgoing edges, including the self-loop in vertex 3 with a signal $V_3(s)$, and moving the vertex-source into vertex 3. Figures 4(b) and 4(c) show the sub-graph's 1Fs.

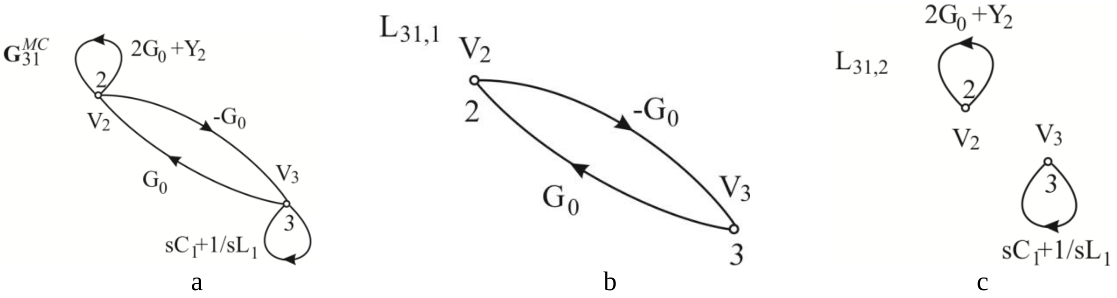

Fig. 4 Sub-graph G_{31}^{MC} and its 1Fs.

Then for $R = 2$ and $N_1 = 2$, $N_2 = 1$ we obtain

(6) $$\sum_{Q=1}^{R}(-1)^{N_Q} P_Q = \Delta_{31} = (-1)^2 Y_{22}(sC_1 + 1/sL_1) + (-1)^1(-G_0)G_0 = Y_{22}(sC_1 + 1/sL_1) + G_0^2$$

The transfer function $T_{31}(s)$ is as follows

(7) $$T_{kq}(s) = T_{31}(s) = \frac{\sum_{Q=1}^{R}(-1)^{N_Q} P_Q}{\sum_{K=1}^{L}(-1)^{N_K} P_K} = \frac{\Delta_{31}}{\Delta} = \frac{Y_{22}(sC_1 + 1/sL_1) + G_0^2}{Y_{22}Y_{33} - G_0^2}$$

- Sub-graph G_{21}^{MC}, shown in Fig. 5(a), is obtained from $\mathbf{G}^{MC}$ due to removing all outgoing edges, including the self-loop in vertex 2 with a signal $V_2(s)$, and moving the vertex-source into vertex 2. Figures 5(b) and 5(c) show the sub-graph's 1Fs.

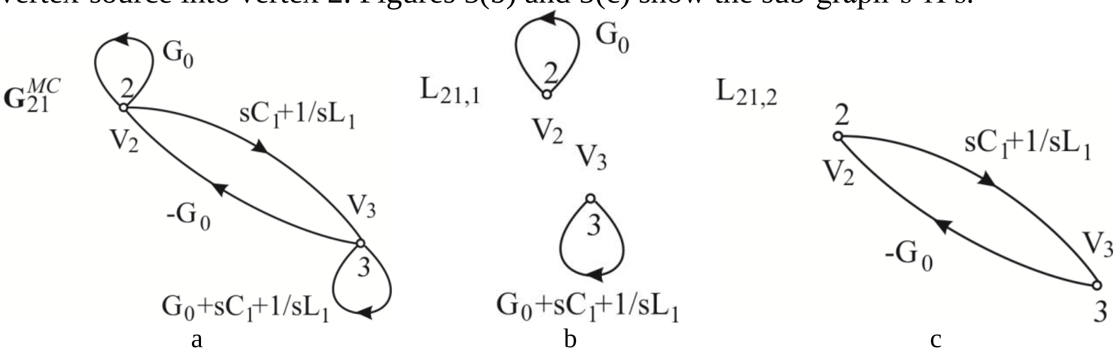

Fig. 5 Sub-graph G_{21}^{MC} and its 1Fs.

Then for $R = 2$ and $N_1 = 2$, $N_2 = 1$ it is gotten

(8)

$$\sum_{Q=1}^{R}(-1)^{N_Q} P_Q = (-1)^2 G_0(G_0 + sC_1 + 1/sL_1) + (-1)^1(-G_0)(sC_1 + 1/sL_1) = G_0(G_0 + 2sC_1 + 2.1/sL_1)$$

- Sub-graph G_{32}^{MC}, shown in Fig. 6(a), is obtained from G_0^{MC} due to removing all outgoing edges, including the self-loop in vertex 3 with a signal $V_3(s)$, as well as by removing all incoming edges, including the self-loop in vertex 2 with a signal $V_2(s)$ and must be added an edge $Y_{23} = -1$. Figure 6(b) shows the sub-graph's 1Fs.

Therefore for $R = 1$ and $N_1 = 1$ follows

(9) $\sum_{Q=1}^{R}(-1)^{N_Q} P_Q = \Delta_{32} = (-1)^1(-G_0)(-1) = -G_0$

The first-order transfer function sensitivity with respect to $sC_2(s)$ and $1/sL_2$ according to (3) and (4) is as follows

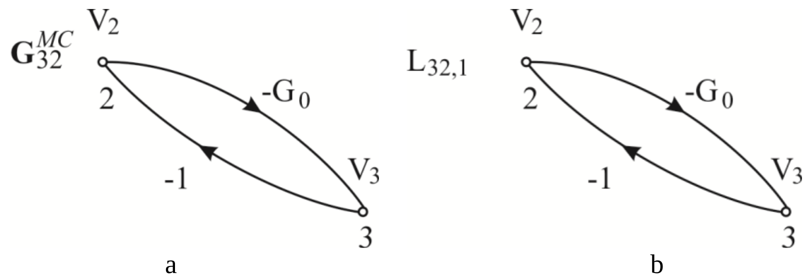

Fig. 6 Sub-graph G_{32}^{MC} and its 1F.

(10) $S_{sC_2}^{T_{31}} = \frac{sC_2}{\left(Y_{22}\left(sC_1 + 1/sL_1\right) + G_0^2\right)} \frac{G_0\left(G_0 + 2sC_1 + 2.1/sL_1\right)\left(-G_0\right)}{\left(Y_{22}Y_{33} - G_0^2\right)}$

(11) $S_{1/sL_2}^{T_{31}} = \frac{1/sL_2}{\left(Y_{22}\left(sC_1 + 1/sL_1\right) + G_0^2\right)} \frac{G_0\left(G_0 + 2sC_1 + 2.1/sL_1\right)\left(-G_0\right)}{\left(Y_{22}Y_{33} - G_0^2\right)}$

SIMULATION RESULTS

Using Mathcad software the magnitude of sensitivity $S_{sC_2}^{T_{31}}$ versus frequency is shown in Fig.7.

$k := 1..15000 \quad f_k := k \quad j := \sqrt{-1} \quad \omega_k := 2.3{,}14.f_k \quad s_k := j.\omega_k$

$Z2_k := R_2 + s_k.L_2 + \frac{1}{s_k.C_2} \quad Y2_k := \frac{1}{Z2_k} \quad Y22_k := 2.G0 + Y2_k$

$Y33_k := G0 + s_k.C_1 + \frac{1}{s_k.L_1}$

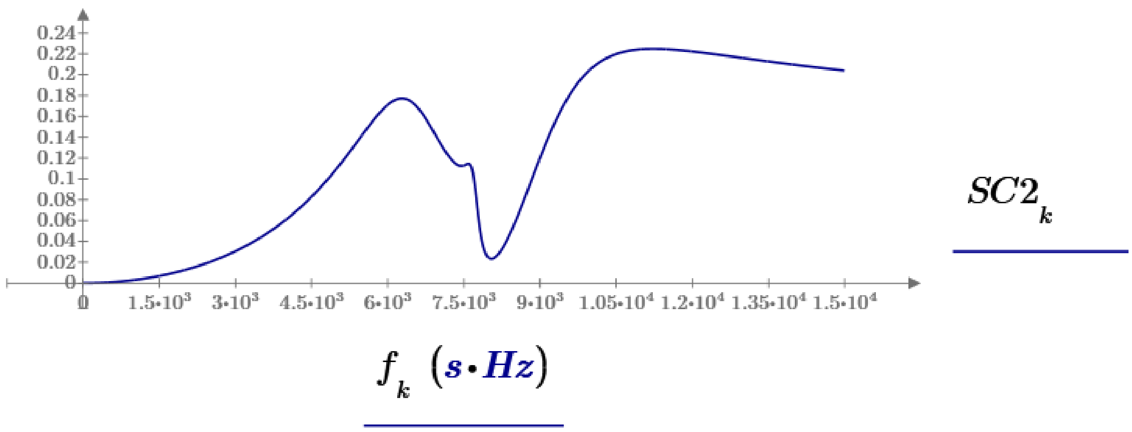

Fig. 7 Sensitivity $S_{sC_2}^{T_{31}}$ versus frequency.

As it is seen, the sensitivity has highest value at $f_1 = 6200$Hz and $f_2 = 10500$ Hz.

On the other hand, a simulation result, isn't given in the paper, shows that the sensitivity $S^{T_{31}}_{1/sL_2}$ has highest value at the same frequencies. The magnitude of sensitivities $S^{T_{31}}_{sC_2}$ with respect to C_2 and $S^{T_{31}}_{1/sL_2}$ with respect to L_2 for the frequencies (f_1, f_2), given above shows, that the values of the parameters C_2 and L_2 for the T-filter are out of the range, where the sensitivities depend on these parameters and the simulation results aren't presented in the paper.

CONCLUSION

The experimental results obtained by the symbolic sensitivity analysis for the T-filter have shown which frequency values are important for the first-order sensitivity of the filter. All symbolic results for the voltage transfer function and for all partial transfer functions, their products and first-order transfer function sensitivity with respect to sC_2 and $1/sL_2$ are gotten.

Simulations are used to determine the magnitudes of the transfer function sensitivity versus the parameters and frequency changing.

Advantages of the suggested method are that it is not necessary to multiply analyze the corresponding graph and the modified node admittance matrix inversion is not required.

References

[1] M. Fakhfakh, E. Tlelo-Cuautle, and F.V. Fernández (Eds.), Design of Analog Circuits through Symbolic Analysis. Bentham Science Publishers Ltd., 2012.

[2] M. Fakhfakh and B. Rodanski (Eds.), Pathological Elements in Analog Circuit Design. Bentham Science Publishers Ltd., 2018.

[3] Coates, C. L.: "General topological formulas for linear networks ", IRE Trans. On Circuit Theory, vol. CT-5,1, 1958.

[4] G. A. Nenov and I. N. Georgieva, "Determination of signal-flow-graph transfer function sensitivity using first- and second-order derivatives and graphs," 5-th Electronic Devices and Systems Conference, Brno, Czech Republic, 1998, pp. 291-294.

[5] M. Fakhfakh, E. Tlelo-Cuautle, and F.V. Fernández (Eds.), Design of Analog Circuits through Symbolic Analysis. Bentham Science Publishers Ltd., 2012, ch. 5

[6] F. Balik and B. Rodanski, "Calculation of first- and second-order symbolic sensitivities in sequential form via the transimpedance method," Proc. ECCTD'99, 1999, pp. 70-73.

[7] M. Fakhfakh and M. Pierzchala, "Computing symbolic transfer functions of CC-based circuits using Coates flow-graph," 5th International Conference on Design and Technology of Integrated Systems in Nanoscale Era, 10.1109/DTIS.2010.5487579, 2018.

ОПРЕДЕЛЯНЕ НА ЧУВСТВИТЕЛНОСТИТЕ ОТ ПЪРВИ РЕД НА ПРЕДАВАТЕЛНИ ФУНКЦИИ НА Т-ОБРАЗЕН ФИЛТЪР

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РЕПУБЛИКА БЪЛГАРИЯ

Ключови думи: *анализ на чувствителността, предавателни функции, Т-филтър*

Резюме: *Определянето на чувствителността на схемните функции на електрически вериги е задача, чието решаване има съществено значение при синтеза на различни по вид и предназначение електрически и електронни устройства. Намирането на този показател е особено необходим при проектирането на честотно селективни устройства, тъй като при тях изискванията към точността на възпроизвеждане на честотните им характеристики са твърде високи. Теория на чувствителността е универсален апарат за решаване на редица задачи, свързани с анализ и диагностика на електрически вериги (ЕВ). Функциите на чувствителността се ползват като критерий за сравнение на различни конфигурации честотно-селективни вериги и представляват един от най-важните показатели при анализа им.*

В доклада са определени чувствителностите от първи ред на предавателната функция на Т –образен филтър чрез сигнални графи в символен вид. Разглежданият филтър е част от преносвателния тракт на радиокомуникационна система и се използва да компенсира някои нелинейни ефекти на реалния канал за връзка. Изследвана е зависимостта на чувствителността на предавателната функция от честотата и е построена съответната характеристика чрез използване на програмния продукт Mathcad.