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MODIFIED EQUATIONS OF LAGRANGE OF II ORDER

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Abstract: *When it is necessary to describe the movement of an ideal holonom mechanical system with its dynamic analysis using summarized coordinates the number of which exceeds its degrees of freedom, the dependents can be expressed by the summarized coordinates chosen to be independent and the equations of Lagrange of II order can be applied for a holonom system or it can be applied with factors. That creates certain difficulties as the coordinates expressed take part in the energy functions being on second power and the factors are determined for each problem.*

Having in mind the difficulties with using two approaches already known, the paper presents a new form of "the general equation of dynamics" in summarized coordinates that is convenient to apply. This form is called "modified equations of Lagrange of II order".

1.Introduction

When it is necessary to describe the movement of an ideal holonom mechanical system with summarized coordinates the number of which exceeds its degrees of freedom with its dynamic analysis, one can proceed in two ways:

- to express the dependents by the summarized coordinates chosen to be independent and to apply the equation of Lagrange of II order to a holonom system. It creates certain difficulties as the coordinates expressed take part in the energy functions being on second power;
- to apply the equation of Lagrange of II order with factors as the factors are determined for each problem.

Having in mind the difficulties with using the two approaches already known, the paper presents a new convenient to apply form of "the general equation of dynamics" in summarized coordinates. This form is called "modified equations of Lagrange of II order".

2. Deduction of equations

Let the movement of a holonom mechanical system subordinated to ideal and bilateral connections with **k** degrees of freedom is described with **n** dependent summarized coordinates $q_1, q_{2,\ldots\ldots\ldots} q_n$ $(n > k)$. They are subordinated to holonom connections $s = n - k$.

(2.14) $\Gamma_r(q_1, q_2, \ldots\ldots\ldots, q_n)$ $(r = 1,2,\ldots\ldots, s)$

Consequently, from all variations δq_j $(j = 1,2,\ldots, n)$ only **k** are independent.

Writing down the general equation of dynamics in summarized coordinates

(2.15) $$\sum_{j=1}^{n}\left[Q_j - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial q_j}\right]\delta q_j = 0,$$

the equality of the expressions in parenthesis to zero cannot be ascertained due to the dependency of variations.

From system (2.14) we can express **s** summarized coordinates. To make the deduction of modified equations convenient, we choose the first **s** coordinates

(2.16) $q_r = f_r(q_{s+1}, q_{s+2}, \ldots\ldots\ldots q_n)$ $(r + 1,2,\ldots\ldots\ldots, s)$

and determine their variations

(2.17) $$\delta q_r = \sum_{j=s+1}^{n} \frac{\partial f_r}{\partial q_j}\delta q_j \qquad (r = 1,2,\ldots\ldots\ldots, s)$$

Equation (2.15) can be presented in the kind of

(2.18) $$\sum_{r=1}^{s}(Q_r + \Phi_r)\,\delta q_r + \sum_{j=s+1}^{n}(Q_j + \Phi_j)\,\delta q_j = 0$$

where Φ_r and Φ_j are the summarized inertia forces.

Substituting (2.17) in (2.18) and after brief transformation, we obtain:

(2.19) $$\sum_{j=s+1}^{n}\left(Q_j + \Phi_j + \sum_{r=1}^{s}(Q_r + \Phi_r)\frac{\partial f_r}{\partial q_j}\right)\delta q_j = 0$$

These $n - s = k$ variations of summarized coordinates are independent and equation (2.19) is satisfied by the condition for equality of the expressions in parenthesis to zero. Thus the modified equations that are convenient to compose differential equations of the movement of a holonom mechanical system subordinated to ideal and bilateral connections with dependent summarized coordinates in the form of Lagrange are obtained:

(2.20)
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j+\sum(Q_r+\Phi_r)\frac{\partial f_r}{\partial q_j}$$

where

(2.21)
$$\Phi_r=-\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right)-\frac{\partial T}{\partial q_r}\right]\qquad (r=1,2,........,s)$$

Q_r => summarized forces of the dependent coordinates.

3.Application

Let examine a discrete manipulation system with its movement in a vertical plane. The driving module is body 1 on which driving force $F_Д$ has been applied. The interaction between the regional movement and its accompanying oscillations with a limited power of driving has been modeled.

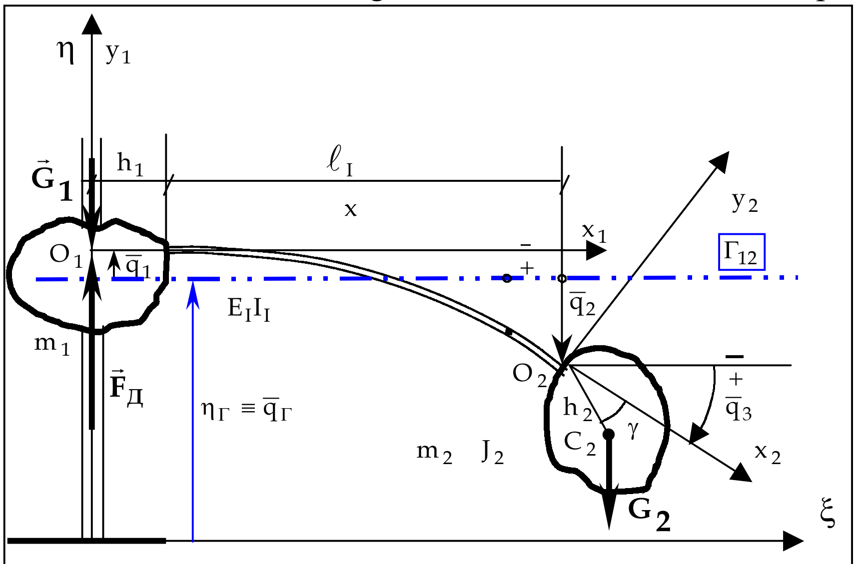

Fig. 1 Modeling of disturbed movement.

The axis of centers of the system examined is determined for the initial conditions corresponding to the static equilibrium where $\theta=k_4\delta$ is the rotation and δ is the static slack of the cross section at point O_2 of an elastic hand fixed to body 1.

For the parameter of the axis of centers Γ_{12} it is obtained that

(3.1)
$$\overset{*}{q_1}=q_{1,0}-\frac{1}{m_1+m_2}\left(m_2\delta+m_2\theta h_{C_2}\cos\gamma\right),$$

and, having introduced the summarized coordinates of centers, for the geometry condition of centers it is obtained that:

(3.2)
$$m_1\bar{q}_1+m_2\bar{q}_2+m_2\bar{q}_3h_{C_2}\cos\gamma=0$$

Using the expression of potential energy in the summarized coordinates, for the equilibrium position the following conditions are valid:

(3.3) $$m_2 g - \left(\frac{\partial \Pi}{\partial q_2}\right)_{\substack{q_2=\delta \\ q_3=\theta}} = 0 \qquad m_2 g h_{C_2} \cos\gamma - \left(\frac{\partial \Pi}{\partial q_3}\right)_{\substack{q_2=\delta \\ q_3=\theta}} = 0$$

Hence, from the system of equations

(3.4) $$\begin{aligned} m_2 g &= c_{22}\delta + c_{23}\theta \\ m_2 g h_{C_2} \cos\gamma &= c_{23}\delta + c_{33}\theta \end{aligned}$$

δ and θ are determined unambiguously.

Kinetic and potential energy

The kinetic and potential energies expressed by the summarized coordinates of centers are represented by expressions:

(3.5) $$\begin{aligned} T &= \frac{1}{2}\left[m_1(\dot{\bar{q}}_\Gamma - \dot{\bar{q}}_1)^2 + m_2(\dot{\bar{q}}_\Gamma - \dot{\bar{q}}_2)^2 + J_{O_2}\dot{\bar{q}}_3^2\right] - m_2(\dot{\bar{q}}_\Gamma - \dot{\bar{q}}_2)\dot{\bar{q}}_3 h_{C_2} \cos\gamma \\ \Pi_{ел} &= \frac{1}{2}\left[c_{22}(\bar{q}_1 + \bar{q}_2)^2 + 2c_{23}(\bar{q}_1 + \bar{q}_2)\bar{q}_3 + c_{33}\bar{q}_3^2\right] \end{aligned}$$

Summarized forces along the coordinates of centers

(3.6) $$\begin{aligned} Q_{\bar{\Gamma}} &= F_Д - (m_1 + m_2)g \\ Q_{\bar{1}} &= F_Д - m_1 g - c_{22}(\bar{q}_1 + \bar{q}_2) - c_{23}\bar{q}_3 \\ Q_{\bar{2}} &= m_2 g - c_{22}(\bar{q}_1 + \bar{q}_2) - c_{23}\bar{q}_3 \\ Q_{\bar{3}} &= m_2 g h_{C_2}(\cos\gamma - \bar{q}_3 \sin\gamma) - c_{23}(\bar{q}_1 + \bar{q}_2) - c_{33}\bar{q}_3 \end{aligned}$$

The geometry condition of centers (3.2) represents holonom connection Γ_1 between the summarized coordinates of centers due to which they are dependent. For that reason to deduce the differential equations of movement we will apply the modified equations of Lagrange of II order that have been deduced in this paper. Coordinate $\bar{q}_1 = f_1(\bar{q}_\Gamma, \bar{q}_2, \bar{q}_3)$ has been chosen to be independent from the rest, consequently:

(3.7) $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\bar{q}}_j}\right) - \frac{\partial T}{\partial \bar{q}_j} = Q_{\bar{j}} + \sum_{r=1}^{s}\left(Q_{\bar{1}} + \Phi_1\right)\frac{\partial f_1}{\partial \bar{q}_j} \qquad (j = \Gamma, 2, 3)$$

(3.8) $$\bar{q}_1 = f_1(\bar{q}_\Gamma, \bar{q}_2, \bar{q}_3) = \frac{m_2}{m_1}\bar{q}_2 + \frac{m_2}{m_1}\bar{q}_3 h_{C_2}\cos\gamma$$

(3.9) $$\Phi_1 = -\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\bar{q}}_1}\right) - \frac{\partial T}{\partial \bar{q}_1}\right] = -m_1(\ddot{\bar{q}}_\Gamma - \ddot{\bar{q}}_1)$$

$$\frac{\partial f_1}{\partial \bar{q}_\Gamma} = 0 \qquad \frac{\partial f_1}{\partial \bar{q}_2} = \frac{m_2}{m_1} \qquad \frac{\partial f_1}{\partial \bar{q}_3} = \frac{m_2}{m_1}h_{C_2}\cos\gamma$$

Substituting the kinetic energy and the summarized energy in equations (3.7) and considering (3.9), we obtain the mathematical model of disturbed motion:

(3.10)

$$(m_1 + m_2)\ddot{\bar{q}}_{\Gamma} = F_{\text{Д}} - (m_1 + m_2)g$$

$$\ddot{\bar{q}}_2 + \ddot{\bar{q}}_3 h_{C_2}\cos\gamma + \frac{c_{22}(m_1 + m_2)}{m_1 m_2}\bar{q}_2 + \frac{c_{23}m_1 + c_{22}m_2 h_{C_2}\cos\gamma}{m_1 m_2}\bar{q}_3 = \frac{F_{\text{Д}}}{m_1 + m_2}$$

$$\ddot{\bar{q}}_2 \frac{(m_1 + m_2)m_2 h_{C_2}\cos\gamma}{m_1} + \ddot{\bar{q}}_3\left[J_{O_2} + \frac{(m_2 h_{C_2}\cos\gamma)^2}{m_1}\right] +$$

$$+\bar{q}_2 \frac{m_1 + m_2}{m_1}\left(c_{23} + c_{22}\frac{m_2}{m_1}h_{C_2}\cos\gamma\right) +$$

$$\bar{q}_3\left[\begin{array}{l} 2c_{23}\dfrac{m_2}{m_1}h_{C_2}\cos\gamma + c_{22}\left(\dfrac{m_2}{m_1}h_{C_2}\cos\gamma\right)^2 + \\ + c_{33} + m_2 g h_{C_2}\sin\gamma \end{array}\right] = F_{\text{Д}}\frac{m_2}{m_1}h_{C_2}\cos\gamma$$

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