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VIBRATIONS OF ELLIPTICAL DISC FROM INERTIAL EXCITATIONS

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Abstract: *The article examines the three-dimensional vibrations of an elliptical disk. It is rigidly connected to a horizontal cylindrical shaft. The normal axis to the plane of the disk makes an acute angle with the axis of the shaft. The shaft is considered to be a cylindrical perfectly elastic and weightless rod with a stiffness of bending and twisting. The supporting joints have linear elastic characteristics set by appropriate coefficients. The three-dimensional vibrations of the disk are caused by an inertial excitation that are developed in its plane. The study includes determination of the matrices of mass and inertial properties, the stiffness matrix, as well as determination of the generalized force. A specific program in the area of MatLab-Simulink has been created for the study of small three-dimensional vibrations. The laws of motion, velocities, and accelerations for all generalized coordinates are obtained. Numerically calculations by varying the angle between the plane of the disk and the shaft are performed. Relevant conclusions are created regarding to this influence.*

INTRODUCTION

The discrete dynamic models, which are studied in the discipline "Structural Mechanics of Machine Structures", consist of perfectly elastic and weightless rod elements and point concentrated masses, [1, 2]. Such models are also studied in the discipline "Dynamics of building structures", [3], as well as in the study of various structures such as cranes, lifting and transport equipment, and others, [4, 5, 6].

Very often, models with concentrated point masses cannot adequately describe the actual mechanical system. This requires the use of three-dimensional models of rigid bodies with their real dimensions, connected to perfectly elastic and weightless rod elements, [7].

The purpose of this article is to show a matrix approach for the study of specific discrete mechanical systems that are not studied in the above-mentioned courses, using the MatLab software package (Matrix Laboratory).

DYNAMICAL MODEL

The three-dimensional vibrations of a steel elliptical disk, which is rigidly connected at a point O for a perfectly elastic and weightless rod with a dense circular cross-section, are studied, (Fig. 1).

Two coordinate systems, $Ox_1y_1z_1$ and $Oxyz$, are defined. The axis Oz_1 is perpendicular to the plane of the disk. The axes Ox_1 and Oy_1 are lying in the disk's plane. The axis Ox is considered vertical. The axis Oz is considered horizontal along the central axis of the rod. The axes Oz_1 и Oz form an acute angle α with each other, (Fig. 1).

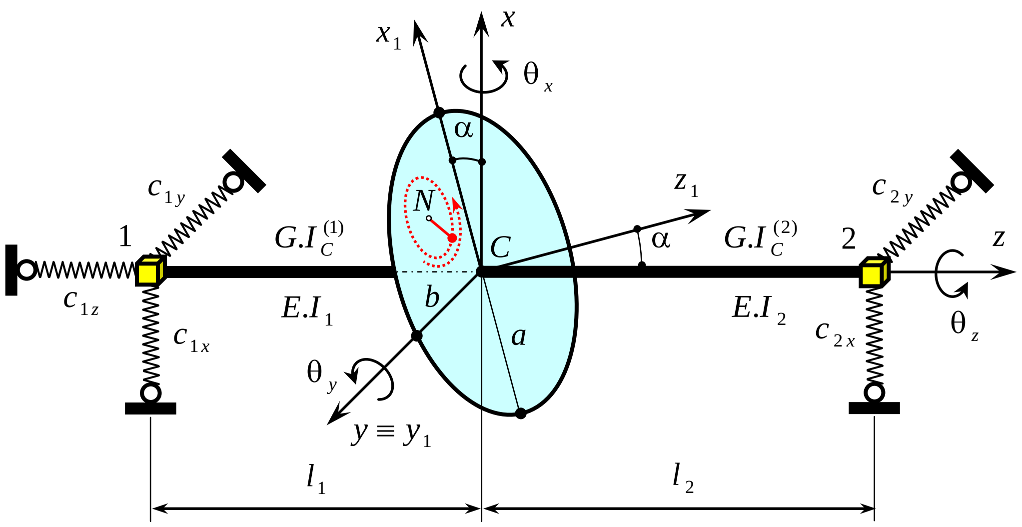

Fig. 1 Dynamical model.

The elliptical disk has a large half-axis a, a small half-axis b, and a thickness h. It is connected to the horizontal beam in its center of mass C.

The beam has two sections of length l_k, with bending stiffness $E.I_k$ and torsional stiffness $G.I_C^{(k)}$, $(k=1,2)$. It is connected in both nodes 1 and 2 with perfectly elastic and weightless elements with coefficients of elasticity c_{1x}, c_{1y}, c_{1z}, c_{2x} and c_{2y}, (Fig. 1).

The vector that describes the small three-dimensional vibrations of the disk is:

(1) $$\mathbf{q} = \langle x \quad y \quad z \quad \theta_x \quad \theta_x \quad \theta_x \rangle^T .$$

These vibrations are caused by an inertial excitations due to a small unbalanced mass $\overline{m}$. It rotates in the plane of the disk at an angular velocity p around a center N with coordinates x_{1N} and y_{1N} with an eccentricity e, (Fig.1).

DIFFERENTIAL EQUATIONS

The derivation of the system of differential equations, which describes the small three-dimensional vibrations of the rigid elliptical disk around its equilibrium position, is performed using the Theorem for the change of the generalized impulse of a rigid body or using the Condensed Lagrange equations, [8].

The following system of differential equations is obtained:

(2) $$\mathbf{A}.\ddot{\mathbf{q}} + \mathbf{K}.\mathbf{q} = \mathbf{Q} \ ,$$

where $\mathbf{A}$ is the matrix of mass and inertial properties, $\mathbf{K}$ is the stiffness matrix, and $\mathbf{Q}$ is the vector of generalized forces.

The formation of the matrix $\mathbf{A}$ is performed on the basis of the following formulas:

(3) $$\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_C \end{bmatrix},$$

(4) $$\mathbf{M} = \mathbf{diag}\left[m \quad m \quad m\right],$$

(5) $$\mathbf{J}_C = \mathbf{diag}\left[J_{x_1} \quad J_{y_1} \quad J_{z_1}\right],$$

(6) $$J_{x_1} = \frac{m}{12}.\left(3.b^2 + 4.h^2\right),$$

(7) $$J_{y_1} = \frac{m}{12}.\left(3.a^2 + 4.h^2\right),$$

(8) $$J_{z_1} = \frac{m}{4}.\left(a^2 + b^2\right),$$

(9) $$\mathbf{A} = \mathbf{W}.\overline{\mathbf{A}}.\mathbf{W}^T,$$

(10) $$\mathbf{W} = \begin{bmatrix} \mathbf{U} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix},$$

(11) $$\mathbf{0} = \mathbf{diag}\left[0 \quad 0 \quad 0\right],$$

(12) $$\mathbf{U} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}.$$

The matrix $\mathbf{K}$ is determined using the flexibility matrix $\mathbf{D}$:

(13) $$\mathbf{K} = \mathbf{D}^{-1}.$$

The non-zero elements of the flexibility matrix $\mathbf{D}$ are found by the formulas:

(14) $$d_{11} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3.l_2^2}{3.E.I_1} + \frac{l_1^2.l_2^3}{3.E.I_2} + \frac{l_2^2}{c_{1x}} + \frac{l_1^2}{c_{2x}}\right),$$

(15) $$d_{55} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3}{3.E.I_1} + \frac{l_2^3}{3.E.I_2} + \frac{1}{c_{1x}} + \frac{1}{c_{2x}}\right),$$

(16) $$d_{15} = d_{51} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3.l_2}{3.E.I_1} - \frac{l_1.l_2^3}{3.E.I_2} + \frac{l_2}{c_{1x}} - \frac{l_1}{c_{2x}}\right),$$

(17) $$d_{22} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3.l_2^2}{3.E.I_1} + \frac{l_1^2.l_2^3}{3.E.I_2} + \frac{l_2^2}{c_{1y}} + \frac{l_1^2}{c_{2y}}\right),$$

(18) $$d_{44} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3}{3.E.I_1} + \frac{l_2^3}{3.E.I_2} + \frac{1}{c_{1y}} + \frac{1}{c_{2y}}\right),$$

(19) $$d_{24} = d_{42} = \frac{1}{\left(l_1+l_2\right)^2}.\left(\frac{l_1^3.l_2}{3.E.I_1} - \frac{l_1.l_2^3}{3.E.I_2} + \frac{l_2}{c_{1y}} - \frac{l_1}{c_{2y}}\right),$$

(20) $$d_{33} = 1/c_{1z},$$

(21) $$d_{66} = \frac{l_1}{G.I_{C1}} \cdot \frac{l_2^2}{\left(\frac{I_{C2}}{I_{C1}}.l_1 + l_2 \right)^2} + \frac{l_2}{G.I_{C2}} \cdot \frac{l_1^2}{\left(l_1 + \frac{I_{C1}}{I_{C2}}.l_2 \right)^2} .$$

The vector of generalized forces caused by the inertial excitation is determined by the following formulas:

(22) $$\mathbf{Q} = \mathbf{W}.\mathbf{Q}_1 ,$$

(23) $$\mathbf{Q}_1 = \begin{bmatrix} F_{x_1} = \overline{m}.e.p^2.\cos p.t \\ F_{y_1} = \overline{m}.e.p^2.\sin p.t \\ F_{z_1} = 0 \\ M_{x_1} = 0 \\ M_{y_1} = 0 \\ M_{z_1} = F_{x_1}.y_{1N} - F_{y_1}.x_{1N} \end{bmatrix} .$$

The system of differential equations (2) is linear, non-homogeneous, from the second order and it is composed by constant coefficients. It could be integrated analytically. But when multiple calculations are performed with variations of many parameters, it is advisable to solve it numerically by suitable program. Such of that program is MatLab.

NUMERICAL SOLUTION

For the numerical solution of the system of differential equations (2) in the time area, the program using MatLab Simulink Toolbox is prepared.

The calculations are made using the following numerical parameters.

Data of the steel: $E = 2.10^{11}\ Pa$, $G = 8,077.10^{10}\ Pa$, $\rho = 7850\ kg/m^3$;

Dimensions of the disc: $a = 0,18\ m$, $b = 0,12\ m$, $h = 0,04\ m$;

Dimensions of the staff: $l_1 = 1,2\ m$, $l_2 = 1,8\ m$, $d_1 = 0,02\ m$, $d_2 = 0,03\ m$;

Coefficients of elasticity: $c_{1x} = 6.10^4\ N/m$, $c_{1y} = 5,8.10^4\ N/m$, $c_{1z} = 12.10^4\ N/m$, $c_{2x} = 8.10^4\ N/m$, $c_{2y} = 7,8.10^4\ N/m$; Unbalanced mass: $\overline{m} = 0,05\ kg$;

Eccentricity of the unbalanced mass: $e = 0,06\ m$;

Coordinates of the center N: $x_{1N} = 0,04\ m$, $y_{1N} = 0,03\ m$.

The first task, that has to be solved, is to determine the natural frequencies. This task is related to find the own numbers of the following matrix:

(24) $$\mathbf{G} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{A}^{-1}.\mathbf{K} & \mathbf{O} \end{bmatrix}_{12\times12} ,$$

where $\mathbf{O} = [\ 0\ ]_{6\times6}$ and $\mathbf{I} = [\ 1\ ]_{6\times6}$ are correspondingly zero matrix and unit matrix.

The eigen circular frequencies ω_k, $(k = 1,2,...,6)$, are derived by the own numbers n_k, which have the form:

(25) $$n_k = 0 \pm i.\omega_k , \qquad i = \sqrt{-1} .$$

The following values of the eigen circular frequencies are obtained: $\omega_1 = 13,615\ s^{-1}$, $\omega_2 = 13,684\ s^{-1}$, $\omega_3 = 23,159\ s^{-1}$, $\omega_4 = 141,65\ s^{-1}$, $\omega_5 = 250,05\ s^{-1}$, $\omega_6 = 337,96\ s^{-1}$.

The forced circular frequency has to be selected in the range $0,70.\omega_k \le p \le 1,30.\omega_k$, $(k = 1,2,...,6)$, in order to avoid the undesirable resonance phenomenon.

The forced circular frequency is accepted $p = 4\ s^{-1}$ in this study.

For the Simulink model file, the differential equations (2) are presented in the following matrix form, [9]:

$$\ddot{\mathbf{q}} = \left[-\mathbf{A}^{-1}.\mathbf{K} \quad \mathbf{O} \right] . \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} + \mathbf{A}^{-1}.\mathbf{Q} \ . \qquad (26)$$

This system is integrated with a variable step by the selected ode 113 (Adams) and maximum time duration $t = 2\ s$.

The displacements x_C, y_C, z_C, the velocities $\dot{x}_C$, $\dot{y}_C$, $\dot{z}_C$, the accelerations $\ddot{x}_C$, $\ddot{y}_C$, $\ddot{z}_C$ of the mass center C are determined by the composed program. The angular rotations θ_x, θ_y, θ_z, the angular velocities $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$, and the angular accelerations $\ddot{\theta}_x$, $\ddot{\theta}_y$, $\ddot{\theta}_z$ of the elliptical disk are also obtained.

The calculations were performed at seven values of angle α, namely, $\alpha = (\pi/12).k$, where $(k = 1, 2, ..., 6)$.

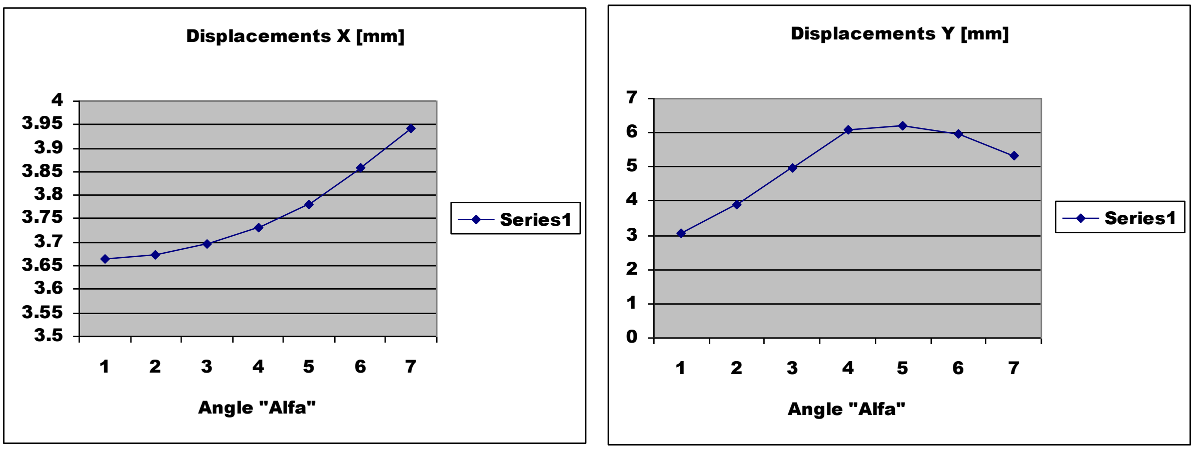

Fig.2 Displacements of the ellipsoid mass center.

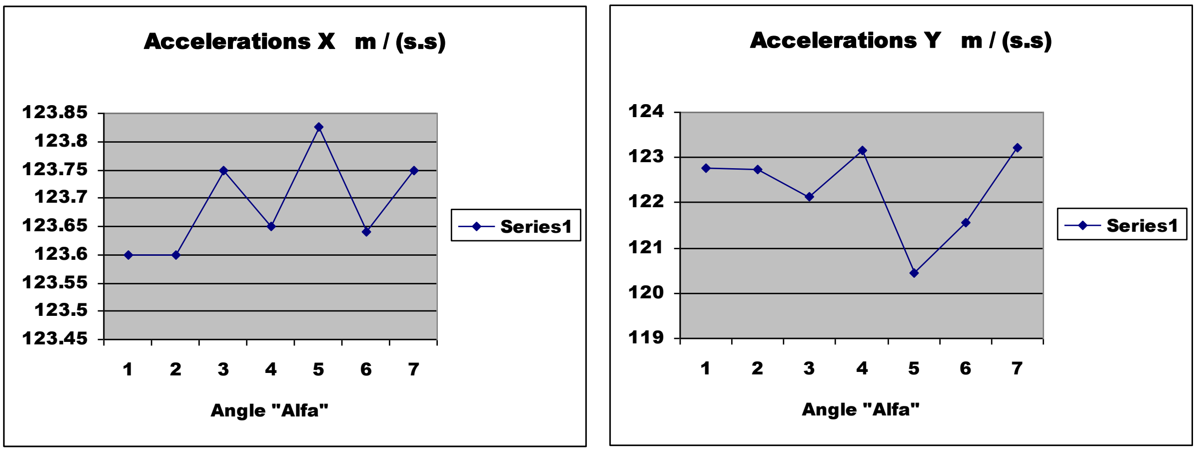

Fig.3 Accelerations of the ellipsoid mass center.

It can be seen from Fig. 2 that the displacements of the mass center C in the direction of the axis x are increasing by a nonlinear law with increasing of the angle α. The displacements of the mass center C in the direction of the axis y are reaching their maximum at the angle $\alpha = \pi/3$ and then they are slowly decreasing.

It can be seen from Fig. 3 that the accelerations of the mass center C in the direction of the axes x and y are approximately in the range between 120 m/s^2 and 124 m/s^2.

CONCLUSION

A matrix approach has been developed to study the dynamics of a specific discrete mechanical system using the MatLab software package.

The research may find a practical orientation in studying the dynamics and strength of railway axles in view of their optimal design, [10, 11].

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ТРЕПТЕНИЯ НА ЕЛИПТИЧЕН ДИСК ОТ ИНЕРЦИОННО СМУЩЕНИЕ

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Ключови думи: *принудени пространствени трептения, елиптичен диск, инерционни смущение, изолиране, числено решение, MatLab*

Резюме: *В статията се изследват пространствените трептения на елиптичен диск, свързан кораво с хоризонтален цилиндричен вал. Нормалата, към равнината на диска, сключва остър ъгъл с оста на вала. Той е приет за идеално еластичен и безтегловен с коравина на огъване и усукване. Опорните възли имат линейни еластични характеристики. Пространствените трептения на диска се пораждат от инерционно смущение, развиващо се в неговата равнина. Съставена е програма в средата на MatLab-Simulink. Получени са законите на движение, скоростите и ускоренията по всички обобщени координати. Извършени са изчисления с вариране на ъгъла между равнината на диска и вала. Направени са съответни изводи относно това влияние.*