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## NUMERICAL SOLUTION FOR THE DEFLECTION OF THE COLUMN-MOUNTED JIB CRANE STRUCTURE

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**Abstract:** Column-mounted jib crane is one of the most frequently utilized crane types within the production facilities. Hence, there are a wide variety of designs, according to many different requirements about lifting height, span, payload capacity, drive options, etc. One of the most considered design criteria is the deflection, where the crane structure is subjected to the full payload capacity at the maximum span. The paper presents the numerical solution for the deflection, obtained by the finite difference method (FDM) and MATLAB code, and its comparison with the analytical models proposed by several authors and the finite element analyses (FEA). The comparison between presented numerical solution and other approaches is made for the deflection of the jib tip. Obtained results are very close to some analytical models and especially to the FEA results. The advantages and shortcomings of the presented approach are given in the conclusion.

### 1. INTRODUCTION

Jib cranes are used for load handling in a circular working space, with a frequent transfer of load from one place to another. They are suitable for applications requiring repetitive lifting and transferring of loads within a fixed arc of rotation. Column-mounted jib crane is highly efficient hoisting equipment, with low space requirement for installation, very safe, energy efficient and easy to control during operation. The most commonly used jib cranes have the payload capacity up to 5 tons and span up to 6 meters.

A properly designed jib crane should increase the material handling efficiency and speed up the work flow. Modern industry requires versatile, flexible and cost effective material handling equipment, with increased productivity. The requirements are usually related to payload capacity, span, lifting height, drive options, etc. Therefore, jib cranes are available with many designs, according to specific requirements. Nevertheless, the main parts of the typical column-mounted jib crane are the column, the boom or the jib, the boom support leg and the hoist with trolley, which moves along the boom of crane. However, a significant feature of the jib crane is its static and dynamic stability, dependent on the deflection of the structure [1, 2].

Since the jib cranes are the most widespread cranes in industrial facilities, many researchers tried to simplify the model and quickly define the basic parameters of these types of the cranes. FEA models of the jib cranes certainly give the results relevant for calculation and dimensioning, but also require longer time to set up the calculation model.

Very often during the designing, the maximum deflection of the jib crane boom tip is a critical criterion. Hence, it is necessary to determine a simple expression for the boom deflection, based on which it is possible to define the basic parameters of the crane. In such way, the basic geometric parameters can be calculated quickly, which shortens the design time.

The analysis of stress and deformation state for the jib crane with different cross-sections was performed in paper [3]. FEA results and experimental tests were used to validate the results.

Besides the 3D model analysis, the research can be successfully conducted upon simplified line models [4]. The static and dynamic analyses of different design solutions of the jib cranes were performed in [5], using several software packages. The goal of the analysis was to reduce the mass and the stresses and to increase natural frequencies.

The paper [6] presented the analytical verification of the lateral–torsional behaviour of the steel web tapered tee-section cantilevers. The check of the model and the obtained results was performed through FEA.

The elastic lateral-torsional behaviour of the cantilever is analysed in [7]. The obtained results in this research can be easily applied in the design of the boom of the jib crane.

The analysis of stresses and deflections of the jib crane structure for different web thickness and web height was performed in [8]. Structure locations with maximum deflection and regions with maximum stresses were determined.

The optimization of the jib crane by evolutionary algorithm was carried out in [9]. Which resulted in significant material savings.

Further on, the optimization of the length of boom support leg, which connects the jib with the column, was carried out in [10]. This analysis was performed by FEA. The paper showed the influence of the height  $H_l$  on the deflection of the structure.

In [11], the analysis was performed with different types of finite elements in order to obtain reliable results.

Stress and deflection analysis was performed through FEA in [12]. Firstly, after using analytical expressions, the density of mesh was corrected. In the second phase of the calculation, the height and thickness of the web were varied.

In the first part of [13], the analysis of stresses and deflections was performed by using FEA. In the second part, the analysis of individual parts for the connection with the fundament was carried out.

Using FEA and the direct integration method, the dynamic responses of the jib crane structure in vertical and horizontal directions were considered in [14]. Forced vibration responses of the jib structure due to the action of equivalent moving forces were determined, where the mass matrix was time-dependent.

In the paper [15], integrated FEA of whole jib crane model was established and arm-side deflection formula for this type of crane structure was derived.

The paper [16] gave refined expression for the deflection of the jib crane boom tip, taking into account the actual load position at the very end of the boom.

## 2. EXISTING ANALYTICAL SOLUTIONS FOR THE DEFLECTION OF THE COLUMN-MOUNTED JIB CRANE STRUCTURE

Fig.1 shows a typical design of the column-mounted jib crane structure.

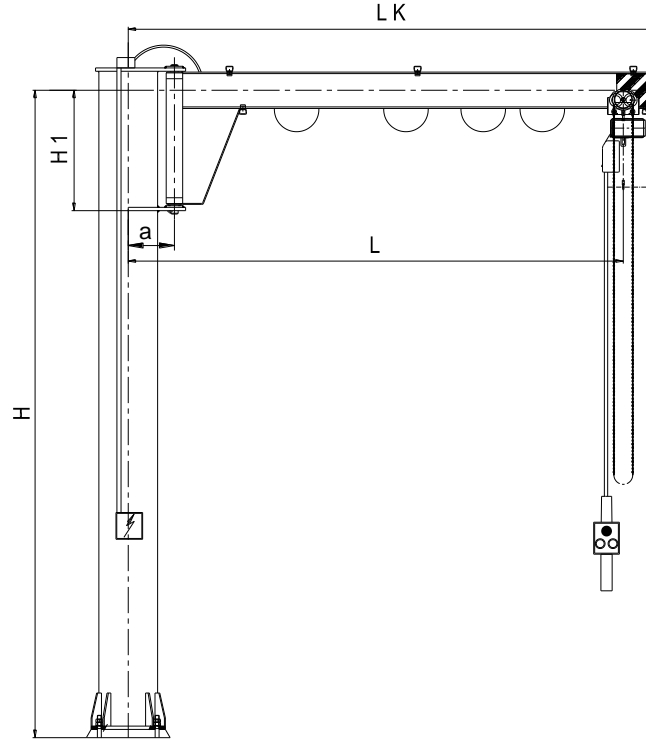


Fig.1 - The column-mounted jib crane

According to one approach from [12], the deflection of the boom can be calculated as the superposition of the deflection caused by the force (concentrated load at the end of the boom) and the deflection from the boom self-weight, while neglecting the influence of the column deformation:

$$f_u = f_K + f_q = \frac{F_1 \cdot L_k^3}{3 \cdot E \cdot I_{x,K}} + \frac{q_K \cdot L_k^4}{8 \cdot E \cdot I_{x,K}} \quad (1)$$

where:

$$F_1 = \gamma \cdot (\psi \cdot Q + m_k) \cdot g$$

$\gamma$  - the classification class coefficient,

$\psi$  - the dynamic coefficient which represents the influence of load oscillation in the vertical plane,

$Q$  - the payload capacity of the crane,

$m_k$  - the mass of the trolley with the hoist,

$I_{x,K}$  - moment of inertia of the boom,

$q_K$  - weight per unit length of the boom girder,

Next analytical solution, presented in [15], took into account the effect of the column deformation on the deflection of whole structure, while the self-weight of the boom was neglected:

$$f_u = f_S + f_K = tg \left( \frac{M \cdot (H - H_1/2)^2}{E \cdot I_{x,S}} \right) \cdot L_K + \frac{F_1 \cdot L_k^3}{3 \cdot E \cdot I_{x,K}} \quad (2)$$

where:

$I_{x,S}$  - moment of inertia for the boom,

$$M = F_1 \cdot L_K.$$

Also, the following expressions for the deflection, suitable for the use in engineering practice, can be found in the technical literature and many projects:

$$f_u = f_s + f_k = tg \left( \frac{M \cdot (H - H_1/2)^2}{E \cdot I_{x,S}} \right) \cdot L + \frac{F_1 \cdot L^3}{3 \cdot E \cdot I_{x,K}} \quad (3)$$

or

$$f_u = f_s + f_k = \frac{F_1 \cdot L^2}{E \cdot I_{x,S}} \cdot (H - 2 \cdot H_1/3) + \frac{F_1}{3 \cdot E \cdot I_{x,K}} \cdot [(L - a)^3 + L^2 \cdot H_1] \quad (4)$$

where

$$M = F_1 \cdot L + \frac{q_K \cdot L^2}{2}$$

It can be noticed that the self-weight of the boom is not taken as a member in the expression for the deflection, but only its influence on the moment of bending.

The fact that the actual load position is not at the very end of the boom (Fig.2) was considered in [16].

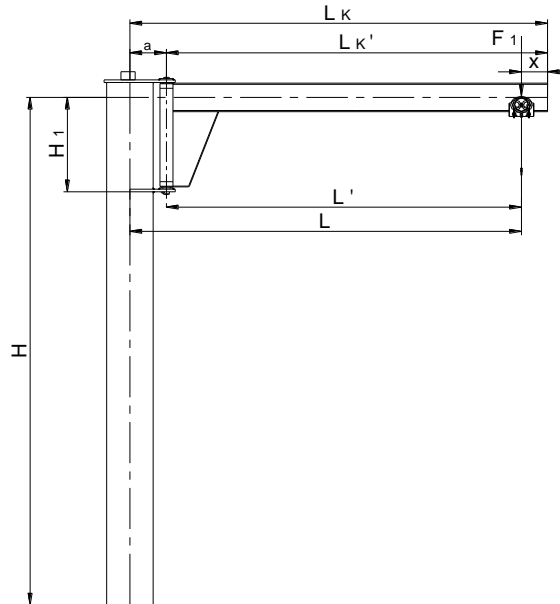


Fig.2 – Calculation model with a load offset

Hence, the following expression for the deflection was proposed:

$$f_u = tg \left( \frac{M \cdot (H - H_1/2)^2}{E \cdot I_{x,S}} \right) \cdot L_K + \frac{F_1 \cdot L_K'^3}{6 \cdot E \cdot I_{x,K}} \cdot \left( \frac{L'}{L_K'} \right)^2 \cdot \left[ 3 - \left( \frac{L'}{L_K'} \right) \right] + \frac{q_K \cdot L_K'^4}{8 \cdot E \cdot I_{x,K}} \quad (5)$$

In this expression, all members who have influence on the deflection of total structure are present.

### 3. NUMERICAL SOLUTION BY FDM

Numerical solution for the deflection of the column-mounted jib crane structure was derived by FDM and its model is presented in Fig. 3.

Static equilibrium yields the following expressions for reaction forces and moments:

$$R_A = F + q_2 L + q_1 H \quad M_A = FL + \frac{q_2 L^2}{2} = M_C \quad R_C = F + q_2 L \quad (6)$$

where  $q_1$  and  $q_2$  are the weights per unit length of the column and the boom respectively.

In addition, the bending moment in point C can be written as follows:

$$M_C = F_1 H_1 \Rightarrow F_1 = \frac{1}{H_1} \left( FL + \frac{q_2 L^2}{2} \right) \quad (7)$$

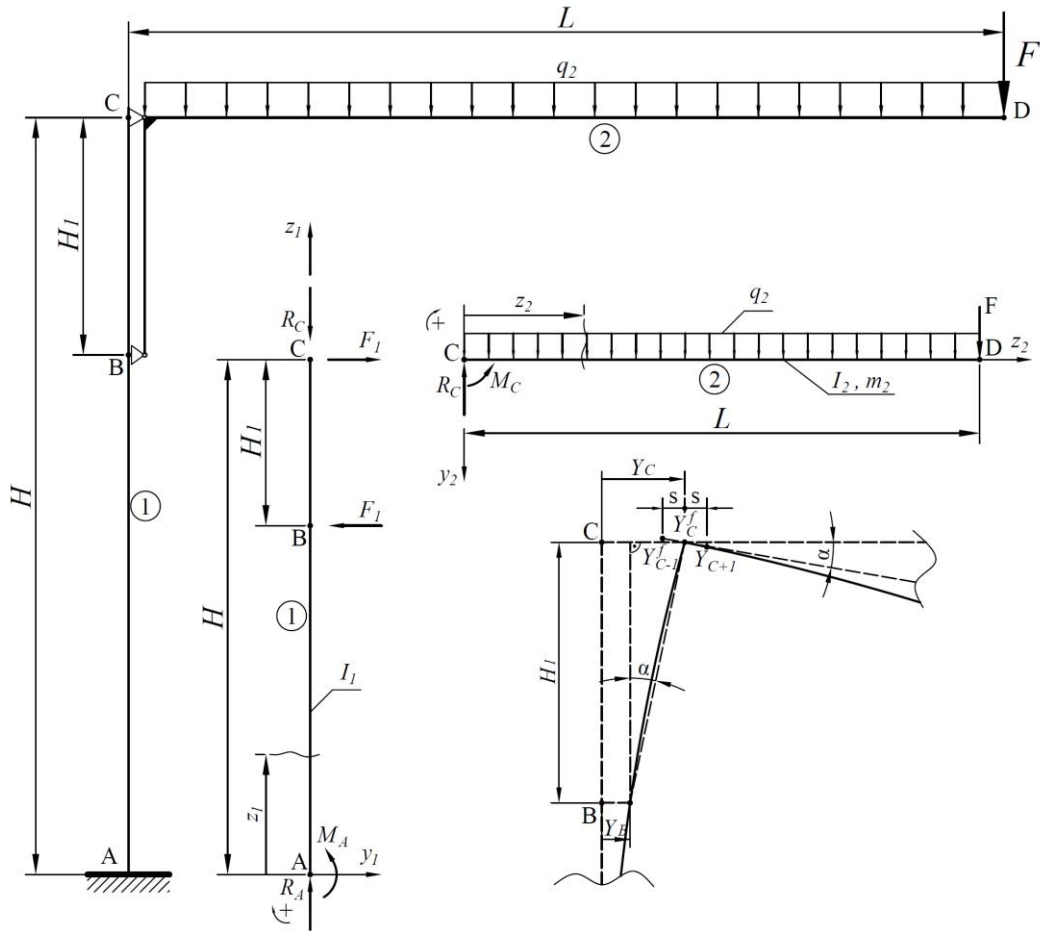


Fig.3 – The model of the structure for the numerical solution by FDM

Governing differential equation for bending of the column is:

$$\frac{d^2 y_1(z_1)}{dz_1^2} = -\frac{M_1(z_1)}{EI_1} = -\frac{M_1(z_1)}{B_1} \quad (8)$$

where  $B_1 = EI_1$  is column bending stiffness.

Bending moment along the column is:

$$M_1(z_1) = -M_A + F_1 [z_1 - (H - H_1)] \quad (9)$$

Using following transformations introduced by FDM

$$\left( \frac{d^2 y_1(z_1)}{dz_1^2} \right)_i \approx \frac{Y_{i-1} - 2Y_i + Y_{i+1}}{s^2} \quad (10)$$

and

$$z_1 = i \cdot s \quad (11)$$

eq. (8) gets the new form

$$\frac{Y_{i-1} - 2Y_i + Y_{i+1}}{s^2} = -\frac{1}{B_1} \{ -M_A + F_1 [is - (H - H_1)] \} \quad (12)$$

where  $i$  is a node number and  $s$  is the division distance of the elastic curve function domain.

Boundary conditions for the displacement and the slope in support point A

$$y_1(z_1 = 0) = 0 \quad (13)$$

$$y_1'(z_1 = 0) = 0 \quad (14)$$

after transformation in algebraic form yield the following expressions

$$Y_0 = 0 \quad (15)$$

$$\left. \frac{Y_{i-1} - 2Y_i + Y_{i+1}}{2s} \right|_0 = \frac{Y_{-1}^f - 2Y_1}{2s} = 0 \Rightarrow Y_{-1}^f = 2Y_1 \quad (16)$$

where upper index “f” means it is a displacement of a fictitious node (the node out of the curve domain). After introducing designations for characteristic nodes ordinals  $B=(H-H_1)/s$  and  $C=H/s$ , the elastic curve of the column can be represented by the following set of algebraic equations:

$$i = 0 \rightarrow Y_1 = \frac{M_A s^2}{2B_1} \quad (17)$$

$$i = 1 \rightarrow -2Y_1 + Y_2 = \frac{M_A s^2}{B_1} \quad (18)$$

$$i = 2 \div B - 1 \rightarrow Y_{i-1} - 2Y_i + Y_{i+1} = \frac{M_A s^2}{B_1} \quad (19)$$

$$i = B \div C - 2 \rightarrow Y_{i-1} - 2Y_i + Y_{i+1} = \frac{s^2}{B_1} \{M_A - F_1[is - (H - H_1)]\} \quad (20)$$

$$i = C - 1 \rightarrow Y_{C-2} - 2Y_{C-1} + Y_C = \frac{s^2}{B_1} [M_A - F_1(H_1 - s)] \quad (21)$$

Governing differential equation for bending of the boom is:

$$\frac{d^2 y_2(z_2)}{dz_2^2} = -\frac{M_2(z_2)}{EI_2} = -\frac{M_2(z_2)}{B_2} \quad (22)$$

where  $B_2=EI_2$  is boom bending stiffness.

Bending moment along the boom is:

$$M_2(z_2) = -M_C + R_C z_2 - \frac{q_2 z_2^2}{2} \quad (23)$$

Using previous transformations introduced by FDM and

$$z_2 = (i - C) \cdot s \quad (24)$$

eq. (21) gets the form as follows

$$\frac{Y_{i-1} - 2Y_i + Y_{i+1}}{s^2} = -\frac{1}{B_2} \left\{ -M_C + R_C(i - C)s - \frac{q_2(i - C)^2 s^2}{2} \right\} \quad (25)$$

Boundary conditions for the displacement and the slope of the boom elastic curve in point C, depicted in Fig. 3,

$$y_2(z_2 = 0) = 0 \quad (26)$$

$$y_2'(z_2 = 0) = \alpha \quad (27)$$

were transformed into the following algebraic form

$$Y_C^f = 0 \quad (28)$$

$$\frac{-Y_{C-1}^f + Y_{C+1}}{2s} = \frac{Y_C - Y_B}{H_1} \Rightarrow Y_{C-1}^f = \frac{2s}{H_1} (Y_B - Y_C) + Y_{C+1} \quad (29)$$

After introducing designation for characteristic node ordinals  $N=D=(H+L)/s$ , the elastic curve of the boom can be represented by the following set of algebraic equations:

$$i = C \rightarrow \frac{2s}{H_1} Y_B - \frac{2s}{H_1} Y_C + 2Y_{C+1} = \frac{M_C s^2}{B_2} \quad (30)$$

$$i = C + 1 \rightarrow -2Y_{C+1} + Y_{C+2} = -\frac{s^2}{B_2} (-M_C + sR_C - \frac{q_2 s^2}{2}) \quad (31)$$

$$i = C + 2 \div D - 1 \rightarrow Y_{i-1} - 2Y_i + Y_{i+1} = -\frac{s^2}{B_2} [-M_C + R_C(i-C)s - \frac{q_2(i-C)^2 s^2}{2}] \quad (32)$$

Expressions (17÷21) and (30÷32) form the system of  $N=(L+H)/s$  algebraic equations, with the node displacements ( $Y_j, j=1, \dots, N$ ) as unknowns, where the displacement  $Y_N$  is actually the deflection of the boom tip of the column-mounted jib crane. Written in a matrix equation form, this system is the base for writing the MATLAB code [17].

#### 4. COMPARISON OF THE OBTAINED RESULTS AND THE RESULTS FROM EXISTING ANALYTICAL MODELS AND FEA

For the results comparison purpose, the example of the existing design solution of the column-mounted jib crane in [16] was considered. Its payload capacity is 500 kg and the span is  $L=3000mm$ , the total mass of the trolley and hoist is 40 kg and classification class is 2. The column height is  $H=3900mm$  and boom support leg height is  $H_1=720mm$ . The column is made of pipe with the diameter 355.6mm and the wall thickness 5.6mm, while the boom is standard IPE220 beam. Also, FEA using SAP2000 and CATIA software from [16] were used for the results comparison.

Table 1 shows the results of the boom tip deflection according to Eq. (1), (2), (3), (4), (5), FEA from [16] and the results obtained by presented approach.

**Table 1**

<i>Eq.(1)</i>	<i>Eq.(2)</i>	<i>Eq.(3)</i>	<i>Eq.(4)</i>	<i>Eq.(5)</i>	<i>SAP2000</i>	<i>CATIA</i>	<i>FDM solution</i>
12,60 mm	25,17 mm	22,13 mm	20,90 mm	21,74 mm	22,65 mm	21,40 mm	21,92 mm

#### 5. CONCLUSIONS

Based on the results shown in Table 1, it can be concluded that the presented approach using FDM yielded very accurate result for the boom tip deflection of the column-mounted jib crane. Also, as stated in [16], the deflection of the column must be taken into account, because it significantly affects the value of the deflection of the boom. The advantage of presented approach is that it can successfully handle the cases with non-uniform cross-sections of the structure elements. Also, it gives the displacements of points along the whole structure, not only the displacement of the boom tip. Further on, this solution can give the dependence of the deflection (or the slope) in any point of the structure on any input geometric or load parameters. On the other hand, the shortcoming of this method is that it takes some more effort and time to switch from differential to algebraic domain of the problem and to transform the mathematical model in a form suitable for programming.

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## REFERENCES

- [1] Krastanov K., Kotzev N. On the causes of accidents with cranes, MTC, Vol.10, 3/2 ISSN 1312-38-23, pp. BG-5.17 – 5.21, 2012
- [2] Petkov B., Process Control Systems for Railway Construction Machines, TEMPT, 2001
- [3] Chaudhary S. A., Khan N.S., A review paper on structural analysis of cantilever beam of jib crane, International Journal of Engineering Research and General Science, (2015), Vol 3, No 3, p.p. 612-615.
- [4] Kolev P., Dynamic Analysis Methods, T. Kableshkov University of Transport, 2015
- [5] Hadžikadunić F., Vukojević N., Huseinović S., An analysis of jib crane constructive solution in exploitation, 12th International Research/Expert Conference, "Trends in the Development of Machinery and Associated Technology" TMT 2008, Istanbul, Turkey, 26–30 August, (2008).
- [6] Yuan W., Kim B., Chen C., Lateral–torsional buckling of steel web tapered tee-section cantilevers, Journal of Constructional Steel Research, (2013), Vol 87, p.p. 31-37.
- [7] Andrade A., Providência P., Camotim D., Elastic lateral-torsional buckling of restrained web-tapered I-beams, Computers and Structures, (2010), Vol 88, p.p. 1179-1196.
- [8] Rajmane S.M., Jadhav A., Finite element analysis of jib crane, International journal of innovative research in technology , Volume 2, Issue 6, (2015), p.p. 404-407
- [9] Gandhare K., Thute V., Design Optimization of Jib Crane Boom Using Evolutionary Algorithm. International Journal of Scientific Engineering and Research (IJSER), Volume 3, Issue 4, (2015), p.p. 5-8
- [10] Yawale P., Khandare N., Effect of Length of Boom Support Leg on Free Standing Jib Crane, International Journal of Scientific Engineering and Research (IJSER), Volume 4, Issue 5, (2016), p.p. 19-23.
- [11] Vakani C.A., Jani S.S., Analysis and Optimization of 270° Jib Crane Deflection: A Review, IJSRD - International Journal for Scientific Research & Development, Volume 2, Issue 10, (2014), p.p. 471-473
- [12] Kiranalli S.S., Patil N.U., Jib Crane Analysis using FEM. IJSRD - International Journal for Scientific Research & Development, Volume 3, Issue 4, (2015), p.p. 185-189
- [13] Khetre S.N., Chaphalkar S. P., Meshram A., Modelling and stress analysis of column bracket for rotary jib crane, International Journal of Mechanical Engineering and Technology (IJMET), Volume 5, Issue 11, (2014), p.p. 130-139
- [14] Gašić V., Zrnić N., Rakin M., Consideration of a moving mass effect on dynamic behaviour of a jib crane structure, Technical Gazette, Vol. 19, Issue 1, (2012), pp. 115-121.
- [15] Ma X., Wang H., Column Jib Crane Based on ANSYS Analysis of Static Stiffness and Improving Measures, Progress in Advanced Manufacturing Technologies, Volume 522, (2012), pp. 485-489
- [16] Pavlović G., Gašić M., Savković M., Zdravković N., Marković G., Comparative analysis of the models for determination of deflection in the column-mounted jib crane structure, XXIII International Scientific Conference TRANSPORT 2017, Sofia, October 12-14, (2017).
- [17] Ivanov A., Modeling Dynamic Problems with MATLAB, ISBN 978-954-323-837-8, Avangard Prima Publishing House, Sofia, 2011, ISBN 978-954-323-837-8, 2011



## ЧИСЛЕНО РЕШЕНИЕ НА ОТКЛОНЕНИЕТО НА МОНТАЖНА КОЛОНА ОТ КОНСТРУКЦИЯТА НА СТРЕЛОВИ КРАН

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**Ключови думи:** монтажта колона, стрелови кран, отклонение (завъртане), метод на крайните разлики

**Резюме:** Кранът, монтиран на колона, е един от най-често използваните видове кран в производствените съоръжения. Следователно, има голямо разнообразие от дизайни, в съответствие с много различни изисквания за височина на повдигане, обхват, капацитет на полезно натоварване, опции за задвижване и т.н. капацитет при максималния обхват. В статията е представено численото решение за отклонението, получено чрез метода на крайните разлики (FDM) и MATLAB код, и неговото сравнение с аналитичните модели, предложени от няколко автори, и анализа на крайните елементи (FEA). Сравнението между представеното числено решение и други подходи е направено за отклонението на върха на стрелата. Получените резултати са много близки до някои аналитични модели и особено до резултатите от FEA. В заключение са дадени предимствата и недостатъците на представения подход.