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EQUATIONS FOR MOTION ON A TWO-PHASE FLOW IN CYLINDRICAL COORDINATES

Ivan Antonov, Svetlin Antonov, Rositsa Velichkova, Ivan Denev
mfantonov@abv.bg; rvelichkova@tu-sofia.bg;

Technical University of Sofia, Sofia 1000, 8 Kl. Ohridski Ave, BULGARIA

Key words: *turbulent flow, two-phase flow, cylindrical coordinates*
Abstract: *In investigation of number of turbulent flows with axial symmetry, it is imperative to use equations for movement, heat transfer and impurities and for continuity in cylindrical coordinates. In presented work is made an analysis of the current equations for motion in cylindrical coordinates by Reynolds type, mentioning a number of developments in Bulgaria. The solution for flow with axial symmetry in cylindrical coordinates is, however, more appropriate using the Navies-Stokes equations using the so called "effective viscosity".*

INTRODUCTION

In investigation of number of turbulent flows with axial symmetry, it is imperative to use equations for movement, heat transfer and impurities and for continuity in cylindrical coordinates. In presented work is made an analysis of the current equations for motion in cylindrical coordinates by Reynolds type, mentioning a number of developments in Bulgaria. The solution for flow with axial symmetry in cylindrical coordinates is, however, more appropriate using the Navier-Stokes equations using the so called "effective viscosity".
The use of motion equations in cylindrical coordinates is related to the investigation of asymmetric flows, such as the so-called swirling jets. The first research on these issues is related to [1], [2].In Bulgaria works on experimental and mathematical modeling of swirling turbulent currents starts in the early 70s [3, 4, 5]. Subsequently, in a gas burner study, successful mathematical and numerical modeling experiments were made in [7, 8, 9, 10]. This work deals with the issue of gas burners. Numerical simulations are confirmed by conducting experimental studies [11,12], which prove the applicability of the mathematical models which are created.
An important milestone in Reynolds' equation use in cylindrical coordinate systems is the modeling and simulation of two-phase turbulent flow which are described in [13,14]. Tasks are solved using an integral method [] and numerical with the application of a $k-\varepsilon$ turbulence model. The main contribution in these works is the application of so-called a two-fluid turbulence model in which each phase has its own velocity, temperature and density and is described in a separate system of partial differential equations.
Regardless of the very good correlation of the models and simulations with the experiment as a significant disadvantage, it is the binding of the solutions, respectively the model equations

with a specific model of turbulence. This is the $k-\omega$ model [15], and at the two-fluid $k-\omega$ model of turbulence.[16]

In the presented work, a new approach of investigation of turbulent flows is given in cylindrical coordinates, valid for two-phase flows. The equations for movement in stresses are used by making the equations of the Navie-Stokes type into which they are replaced by $\tau_{ef}=\tau+\tau_t$, respectively $\mu_{ef}=\mu+\mu_t$. The equations were obtained in a cylindrical coordinate for one-phase gas flow and for two-phase using a two-fluid model. This approach give possibility for the numerical solution not to be linked to a specific model of turbulence. This makes the mathematical model and numerical simulations universal and allows the choice of the most appropriate model of turbulence after its corresponding validation.

EQUATION IN CYLINDRICAL COORDINATE SYSTEM

The equations in cylindrical coordinates are derived from the equations for the dynamics of the fluid flows and represent a type of the equations of Navie-Stocks. In conclusion, the following are assumed: the dynamic and kinematic viscosity coefficients are replaced by μ_{ef}, respectively υ_{ef}, as follows:

$$\mu_{ef}=\mu+\mu_t;\upsilon_{ef}=\upsilon+\upsilon_t \tag{1}$$

Where μ_t,υ_t are coefficients of dynamic and kinematic viscosity of the turbulent flow. A cylindrical coordinate system is used - $x,r.\theta$ in which:

u,v - axial and radial velocity components, w is the component of rotation at θ. For the velocity vector:

$$\vec{V}=u_{\vec{i}}+v_{\vec{j}}+w_{\vec{e\theta}} \tag{2}$$

Where $\vec{i},\vec{j},\vec{e_\theta}$ are the single vectors on the axes and have the form:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+\frac{w}{r}\frac{\partial u}{\partial\theta}+u\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2u-\upsilon_t\nabla^2u \tag{3}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial r}+\frac{w}{r}\frac{\partial v}{\partial\theta}-\frac{w^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left(\nabla^2v-\frac{v}{r^2}-\frac{2}{r^2}\frac{\partial w}{\partial\theta}\right)-\upsilon_t\left(\nabla^2v-\frac{v}{r^2}-\frac{2}{r^2}\frac{\partial w}{\partial\theta}\right) \tag{4}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial r}+\frac{w}{r}\frac{\partial w}{\partial\theta}-\frac{vw}{r}=-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial\theta}+\upsilon\left(\nabla^2w-\frac{w}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta}\right)-\upsilon_t\left(\nabla^2w-\frac{w}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta}\right) \tag{5}$$

It is added the continuity equation

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)+\frac{1}{r}\frac{\partial}{\partial\theta}(rw)=0 \tag{6}$$

With ∇^2 is denoted Laplace operator, which is given by:

$$\nabla^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{7}$$

TWO-FLUID MODEL OF TWO-PHASE TURBULENT FLOWS

The two - fluid model of two - phase turbulent flows were developed on the basis of Lactu's research in the 1940s. In experiment with a liquid helium, he finds that in the same volume there is a "normal" helium and possesses the property of superconducting liquid helium. Based on these results, then in [17] apply this at two - fluids flows. According to the created theory, in the same volume there are two fluid media of the carrier (gas or liquid) phase and the impurity phase. At impurity phase is accepted so called incomplete set of particles. This excludes the reading of the impacts between particulate impurities, respectively. The amount of movement in this process. In this case, it is assumed that the impurity phase does not have its own stress tensor.

For both mediums - the carrier phase and phase of impurities are recorded corresponding equations of Navie-Stokes type:

$$\frac{\partial u_g}{\partial t}+v_g\frac{\partial u_g}{\partial r}+\frac{w_g}{r}\frac{\partial u_g}{\partial \theta}+u_g\frac{\partial u_g}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u_g-\upsilon_{tg}\nabla^2 u_g-F_x \qquad (8)$$

$$\frac{\partial v_g}{\partial t}+v_g\frac{\partial u_g}{\partial x}+v_g\frac{\partial v_g}{\partial r}+\frac{w_g}{r}\frac{\partial v_g}{\partial \theta}-\frac{w_g^{\,2}}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left(\nabla^2 v_g-\frac{v_g}{r^2}-\frac{2}{r^2}\frac{\partial w_g}{\partial \theta}\right)$$
$$-\upsilon_{tg}\left(\nabla^2 v_g-\frac{v_g}{r^2}-\frac{2}{r^2}\frac{\partial w_g}{\partial \theta}\right)-F_y \qquad (9)$$

$$\frac{\partial w_g}{\partial t}+u_g\frac{\partial w_g}{\partial x}+v_g\frac{\partial w_g}{\partial r}+\frac{w_g}{r}\frac{\partial w_g}{\partial \theta}-\frac{v_g w_g}{r}=-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta}+\upsilon\left(\nabla^2 w_g-\frac{w_g}{r^2}-\frac{2}{r^2}\frac{\partial v_g}{\partial \theta}\right)$$
$$-\upsilon_{tg}\left(\nabla^2 w_g-\frac{w_g}{r^2}-\frac{2}{r^2}\frac{\partial v_g}{\partial \theta}\right)-F_z \qquad (10)$$

$$\frac{\partial}{\partial x}\left(ru_g\right)+\frac{\partial}{\partial r}\left(rv_g\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(rw_g\right)=0 \qquad (11)$$

$$\frac{\partial u_p}{\partial t}+v_p\frac{\partial u_p}{\partial r}+\frac{w_p}{r}\frac{\partial u_p}{\partial \theta}+u_p\frac{\partial u_p}{\partial x}=\upsilon_{tp}\nabla^2 u_p+F_x \qquad (12)$$

$$\frac{\partial v_p}{\partial t}+v_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial v_p}{\partial r}+\frac{w_p}{r}\frac{\partial v_p}{\partial \theta}-\frac{w_p^{\,2}}{r}=-\upsilon_{tp}\left(\nabla^2 v_p-\frac{v_p}{r^2}-\frac{2}{r}\frac{\partial w_p}{\partial \theta}\right)+F_y \qquad (13)$$

$$\frac{\partial w_p}{\partial t}+u_p\frac{\partial w_p}{\partial x}+v_p\frac{\partial w_p}{\partial r}+\frac{w_p}{r}\frac{\partial w_p}{\partial \theta}-\frac{v_p w_p}{r}=\upsilon_{tp}\left(\nabla^2 w_p-\frac{w_p}{r}-\frac{2}{r^2}\frac{\partial v_p}{\partial \theta}\right)+F_z \qquad (14)$$

$$\frac{\partial}{\partial x}\left(ru_p\right)+\frac{\partial}{\partial r}\left(rv_p\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(rw_p\right)=0 \qquad (15)$$

Equations 8 ÷ 10 are indicate to movement of the carrier gas phase, and equation 11 is for the continuity of the same phase. Equation 12 ÷ 14 describes the movement of the impurity phase, and the equation. 15 is for the continuity of the impurity phase. According to the two-fluid model, as mentioned in the impurity movement equations, there is no viscosity and pressure because there is no stress tensor. The inscriptions for the impurity phase "p" and "g" for the gas phase respectively are entered. Since it is assumed that the carrier phase generates impulse phase movement, the interfacial interaction forces on the respective axes have a "-" sign on the carrier phase and a "+" sign on the impurities.

The equation for the carrier phase equation can be added to the resulting system differential equations 8 ÷ 15:

$$p=\rho_g RT \qquad (16)$$

And equation for heat transfer

-for carrier phase

$$\rho_g c_{pg}\left(\frac{\partial T_g}{\partial t}+u_g\frac{\partial T_g}{\partial x}+v_g\frac{\partial T_g}{\partial r}+\frac{w_g}{r}\frac{\partial T_g}{\partial \theta}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_g r\frac{\partial T_g}{\partial r}+q_{rg}\right)$$
$$+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\lambda_g}{r}\frac{\partial T_g}{\partial r}+q_{\theta g}\right)+\frac{\partial}{\partial x}\left(\lambda_g r\frac{\partial T_g}{\partial r}+q_{xg}\right) \qquad (17)$$

-for phase of impurities

$$\rho_p c_{pp}\left(\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial r}+\frac{w_p}{r}\frac{\partial T_p}{\partial \theta}\right)=\frac{1}{r}\frac{\partial}{\partial r}q_{rg}+\frac{1}{r}\frac{\partial}{\partial t}q_{\theta p}+\frac{\partial}{\partial x}q_{xp} \tag{18}$$

Where

$$\begin{aligned} q_{xg}&=-\rho_g c_{pg}\overline{u_g^{'}T_g^{'}};q_{rg}=-\rho_g r c_{pg}\overline{v_g^{'}T_g^{'}};q_{\theta g}=-\rho_g c_{pg}\overline{w_g^{'}T_\theta^{'}}\\ q_{xp}&=-\rho_p c_{pp}\overline{u_p^{'}T_p^{'}};q_{rp}=-\rho_p r c_{pp}\overline{v_p^{'}T_p^{'}};q_{\theta p}=-\rho_p c_{pp}\overline{w_p^{'}T_\theta^{'}}\end{aligned} \tag{19}$$

Conclusions

The described equations 9 ÷ 19 represent a mathematical model of the two-phase flow, observing a two-fluid flow scheme in cylindrical coordinates. They can be represented by analogy to those in Cartesian coordinate by a common characteristic equation, whose numerical modeling is not a substantial difficulty.

Verification of the two-fluid model of biphasic and single-phase (gas) flow will be presented in a future publication of the team.

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УРАВНЕНИЯ ЗА ДВУГАЗОВ ПОТОК В ЦИЛИНДРИЧНИ КООРДИНАТИ

Иван Антонов, Светлин Антонов, Росица Величкова, Иван Денев
mfantonov@abv.bg; rvelichkova@tu-sofia.bg;

Технически университет – София
София 1000, бул. „Климент Охридски“ 8
БЪЛГАРИЯ

Ключови думи: *турбулентен поток, двуфазен поток, цилиндрични координати*

Резюме: *При изследване на броя на турбулентните потоци с аксиална симетрия е наложително да се използват уравнения за движение, пренос на топлина и примеси и за непрекъснатост в цилиндрични координати. В представената работа е направен анализ на текущите уравнения за движение в цилиндрични координати по метода на Рейнолдс, като се споменават редица разработки в България. Решението за поток с аксиална симетрия в цилиндрични координати, обаче, е по-подходящо при използване на уравнения на морските кораби, използвайки т.нар. „ефективен вискозитет“.*