Abstract: Braking at the extreme limit of adherence is a technical necessity determined by the exploitation safety, especially on high-speed traffic. Adherence constitutes a complex phenomenon, particularly for rail vehicles, and depends on numerous variables. Mainly, in this study, we want to analyze several parameters which might lead to influence elliptic dimensions to the wheel-rail contact.

Key words: adherence, wheel-rail contact surface, rail, profile wear.

INTRODUCTION

According the UIC 540 standard, rail vehicles are mandatory supposed to be equipped with auto-pneumatically basic brake systems, presuming shoes and disk brakes, mandatory for speeds higher than 160 km/h [1]. These braking systems are based on the adherence that develops between the rail and the wheel.

Consequently, previous acknowledgement and knowledge of the complex phenomenon of adherence is a must in order to a correct design of braking installations, also avoiding blocking the wheel system while braking and using the available adherence in the best way possible.

A series of determining factors that especially influence the wheel-rail adherence are: the contact surface’s dimensions, the nominal wheel diameter, wheel and rail design, wheel charge and contact surface temperature.

ELLIPSE CONTACT DIMENSIONS

Due to distortions of the metallic materials that the wheels and rails are fabricated from, the contact usually occurs elliptically. Generally, for this case, according the specific technical procedure, the contact dimensions of the ellipse are determined according to Hertz’s theory [1].

The theory is valid should the following hypotheses be followed:

- contact ellipse dimensions are much smaller than dimensions of frames coming into contact;
- the contact surface presumes only compression tensions, not tangential ones;
- the contact area material stays on proportional elastic distortion limits.

According Hertz’s theory, should we consider a and b as contact ellipse semi-axes (see fig.1), their controlled condition could be determined by the relation

\[(\alpha / \mu)^3 = (b / n)^3 = \left[\frac{3N(1 - \nu^2)}{E(A + B)}\right][E(A + B)] (1)\]

Where N[kN] represents the steady load on the contact surface; E[kN/mm²] the springiness module, \(\nu\) – the Poisson coefficient, while \(m\) and \(n\) are coefficients depending on \((A-B)/(A+B)\), defined by \(\cos\beta = (A-B)/(A+B)\).
Constants A and B, as set by Hertz, depend on the main curves of the contact surfaces.

As for wheel-rail system, if we consider $\rho_r$ and $\rho_s$ as crosswise compass of wheel design, according to those of rail, and counting r as wheel rolling compass, the $(A-B)$ and $(A+B)$ will be determined by

$$A + B = (1/r) + (1/\rho_r) + (1/\rho_s);$$

$$A - B = (1/r) - (1/\rho_r) - (1/\rho_s).$$

Since rail administrations prefers the wear designs, usually in concave shape, relations (2) are

$$A + B = (\rho_s + r)/\rho_p - 1/\rho_r;$$

$$A - B = (\rho_s - r)/\rho_p + 1/\rho_r.$$

**ESTABLISHING S-78 ROLLING CONTACT SURFACE**

**S-78 wear profile**

At Romanian Railway National Society (SNCFR), for tracked vehicles, S-78 rolling wheels profiles is mandatory (see fig. 2) [1], as set by professor engineer Stefan Sebesan, who established the geometrical characteristics required.

The study underlined the fact that using this wheel profile the contact with UIC 60 rail type (1:20 slope) follows the curve represented in fig. 3.

Calculations presumed a central positioning of the axis and a 1.500 gap between rated rolling circles plans.

This study reaches for finding a calculation program desired to determine the variables influencing the contact surface dimension; functions that define the dependence between the size of the angle $\beta$ and the coefficients m and n [1].

Using the Cebisev interpolation method, we decided that the following polynomials approximates good enough the dependence of the variables mentioned previously [2].

$$m = 0.363 \cdot 10^{-9} \cdot \beta^6 - 0.125 \cdot 10^{-6} \cdot \beta^5 + + 0.174 \cdot 10^{-4} \cdot \beta^4 + 0.126 \cdot 10^{-3} \cdot \beta^3 + + 0.05 \cdot \beta^2 - 11.6 \cdot \beta + 14.187$$

$$n = 0.65 \cdot 10^{-6} \cdot \beta^5 - 0.86 \cdot 10^{-4} \cdot \beta^3 + + 0.0112 \cdot \beta + 0.214$$

In this analyse it was study the next situations: the wheel diameter from the rolling rated circle plan is between 1000..760 mm and the load on the wheel from 100...40 kN.

**Wheel diameter rated influence**

After rolling the calculation program which was elaborated, the main results are synthesized in fig. 4 that outline, as we were expecting, that the wheel-rail contact surface is directly
proportional with the wheel rated diameter increase.

Fig. 4. The wheel rated diameter influence on contact surface for S-78 wear profile

Analysing the percentage variation of the value

\[ \text{var}_{\text{proc}} = \left( \frac{S_{\text{max}} - S_{\text{min}}}{S_{\text{max}}} \right) \times 100\% \]  \hspace{1cm} (6)

it had been noticed that, independent of the vertical weight on wheel, it is about 9.508% on the field of rated diameters analysed.

This proves that the wheel diameter has a sensible influence regarding the value of the contact surface; leading to a changing of the value up to 10%.

An analytical survey of the results shows that for the situations analysed we can consider as precisely enough the dependence:

\[ S_{\text{cont}} \approx c_1 D_o^{1/3}, \]  \hspace{1cm} (7)

were \( c_1 \) is a constant that depends on the effective value of the rated rolling diameter of the wheel.

Using this relation leads to deviations up to 1.4% compared to classical calculation methods presented in the science literature (see the relations (1) – (3)), but the application of this is possible only in the situations that respect the domains of applicability established earlier and specified.

**Vertical weight on wheel influence**

As in the previous study presented, the main results obtained from processing the data with computing program were figured in the diagram from fig. 5.

Making a study based on the relation (6) we can see that, irrespective of the rated diameter wheel value, the percentage variation for the specific domain is 45.712%. This proves that vertical weight on wheel has a major influence on the contact surface size.

An analytical survey of the results shows that for the situations analyse it can be considered as accurate enough the dependance

\[ S_{\text{cont}} \approx c_2 Q_o^{1/3}, \]  \hspace{1cm} (8)

were \( c_2 \) is a constant which depends of the effective value of the vertical weight on wheel.

Using this latter empirical relation, we can obtain deviations smaller than 0.5%, compared to the classical calculation methods presented in the science literature [1]. The condition is to respect the specific domains of applicability established earlier and the study can be made only for S-78 wear profile.

**Establishing the relation between the contact surface, rated diameter and vertical weight on wheel**

From relations (7) and (8) we have followed the establishment of a general calculus relation for the contact surface size of the following form

\[ S_{\text{cont}} = \text{const} \cdot D^{1/3} \cdot Q^{2/3}. \]  \hspace{1cm} (9)

In this way, using mathematical regression method, the following calculus relation was obtained
applicable to S-78 profile, where D is in mm, and Q in kN, the final result being counted in mm².

For the applicability domains previously mentioned, and only if we consider the S-78 profile (fig. 2), the outcome of the relation (10) can lead to no more than 1.2% difference compared with the classical calculation presented in the science literature, if we base on (1) – (3) relations [1].

Temperature influence on contact surface size

If, while braking, significant sustained slipperiness occurs, such as second generation electronically anti-blocking devices [3], then high temperatures could be reach on wheel rolling surface, up to 400 or 600°C [4].

As we know [1], at high temperatures, the elastic modulus decreases, comparing to its value at 20°C(E20).

Considering that, and following the two constants k₁ and k₂ defined by Hertz, and considering the Poisson coefficients equal, we get

\[ k_{1,2} = \left(1 - \nu^2\right)/2E_{s,r}, \]  

where:
- \( E_s \) - elasticity module on ambient temperature;
- \( E_r \) - elasticity module on the temperature achieved on the surface of rolling, due to kept slides.

Taking only the influence of temperature over the elasticity module of the wheel into account and in view of the fact that at a same temperature it is equal with the rail elasticity module then, we can write that \( E_r = E_k \) where \( k = E/E_{20} \).

The size of the contact ellipse in this situation can be determined with the relation:

\[ S_{cont} = S_{20}\left[\frac{k(t_s) + k(t_r)}{2k(t_s)k(t_r)}\right]^{1/3}, \]  

were:

\[ k(t) = -0,145 \cdot 10^{-10} \cdot t^4 + 0,105 \cdot 10^{-7} \cdot t^3 + 0,28 \cdot 10^{-5} \cdot t^2 - 0,768 \cdot 10^{-5} \cdot t + 1 \]  

With the purpose of establishing a general relation we have analysed the factor dependence

\[ k = \left[\frac{k(t_s) + k(t_r)}{2k(t_s)k(t_r)}\right]^{1/3} \]

of proportion \( k(t_r)/k(t_s) \).

Basing on the interpolation relation, determined by the dependence of temperature elasticity module, we have established that the power type function is the function that estimates the best the evolution of the phenomenon (see fig. 6):

\[ k = 1,005\left[\frac{k(t_r)}{k(t_s)}\right]^{0,3606} \]  

\[ k \approx 1,005\left[\frac{k(t_r)}{k(t_s)}\right]^{1/3} \]  

Fig. 6. K factor dependence on the proportion \( k(t_r)/k(t_s) \)

Taking the relation of dependence between the contact surface with the rated diameter of the wheel and with the vertical weight which works on it, into account and using the relation (10), we can write:

\[ S_{cont} \approx \text{const}\left[DQ^2 k(t_s)/k(t_r)\right]^{1/3}. \]

Thus, for \( D \) and \( Q \) domains specified previously and for the wear profile S-78 the constant value is 0.793.
DB II profile is also a wear profile, used by the German railway at engine and draw vehicle, adapted for UIC 60E and UIC 49 rail with 1:40 inclination.

In calculations it has been applied the same theoretic basis, procedures and calculus programs as in the S-78 roll profile.

In study we have considered the rated diameter for rolling of the wheels as 760…1000 mm, the vertical weight extents between 40..100 kN and the same temperature of the wheel and rail 20° C. We have also considered that the set axle moves on a railway which has the UIC 60 rail type with 1:20 inclination.

Analysing, as in the previous case, the influence of the rated diameter of the wheel on the contact surface dimensions we can see that a rise of it will determine an increase of the contact surface. The relative percentage growing value is 12.6%, independent of the vertical weight on wheel size, with approximately 3% bigger then in the S-78 profile case.

The main results obtained after the work of the calculus programme are synthesized in the diagram from fig. 7.

The analytic study of the obtained results points out the fact that for the analysed situations we can consider accurately enough the dependence

\[ S_{\text{com}} \approx c_3 D_0^{0.4322}, \quad (16) \]

in which \( c_3 \) is a constant that depends on the effective size of the rated diameter for rolling.

Regarding the influence of the vertical weight on wheel, the percentage growth value is 45.712 %, independent of the rated diameter of the wheel size, the same with the one established in the S-78 profile case, for the same parameters.

The results obtained after the analysed cases are materialized in the diagram from fig. 8.

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Спирането при екстремално ограничаване на сцеплението е техническа необходимост, обусловена от безопасността на експлоатация, специално при високоскоростно движение. Сцеплението представлява сложно явление, в частност за железопътните возила, и зависи от много променливи. В това изследване искаме да анализираме главно няколко параметри, които могат да доведат до въздействие върху елиптичните измерения на контактата колело-релса.

Резюме: Спирането при екстремално ограничаване на сцеплението е техническа необходимост, обусловена от безопасността на експлоатация, специално при високоскоростно движение. Сцеплението представлява сложно явление, в частност за железопътните возила, и зависи от много променливи. В това изследване искаме да анализираме главно няколко параметри, които могат да доведат до въздействие върху елиптичните измерения на контактата колело-релса.

Ключови думи: сцепление, поверхност на контакта колело-релса, релса, износване на профила.

\[
S_{\text{cont}} = 0.6 \cdot D_0^{0.422} Q^{2/3}, \quad (19)
\]

available for DB II profile considering the diameter and load mentioned previously.

CONCLUSIONS

The study made had as reference the importance of the adhesion force and its influence, decisive and lim itary, in the development of the braking force. So, knowing the fact that changing the dimension of the contact ellipse will determine a change of the adhesion force, it has been impose the necessity to determine the weight in which a parameter (vertical weight on wheel, rated diameter of rolling and temperature from the rolling surface) changes the contact surface size.

For diameters and vertical weight on wheel, usually for passenger charts, it has been established that the rolling diameter can influence with 10% the dimension of the contact ellipse and the vertical weight with 45%. A rise of temperature for the rolling surface of the wheel, can get to a rise of over than 10% for the dimensions of the contact surface.

For the wear profile S-78 type and for the DB II one, considering that it rolls on a UIC 60 type of rail with a 1:20 inclination, it had been established a series of empiric relations to calculate the contact surface values. The use of this method gives us a faster way to get better results which doesn’t differ with more than 1,2% then the ones obtained in a classic method.

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