

## A NEW APPROACH IN ANALYTICAL DETERMINATION OF TORSIONAL STIFFNESS IN RAILWAY WAGONS

Nebojša BOGOJEVIĆ, Ranko RAKANOVIĆ, Dragan PETROVIĆ, Zlatan ŠOŠKIĆ

bogojevic.n@maskv.edu.yu, rakanovic.r@maskv.edu.yu, petrovic.d@maskv.edu.yu, soskic.z@maskv.edu.yu

Nebojša Bogojević, Prof. Dr Ranko Rakanović, Prof. Dr Dragan Petrović, Dr Zlatan Šoškić Faculty of mechanical engineering Kraljevo, Dositejeva 19, Kraljevo, SERBIA

**Abstract:** This paper presents a new approach in studying and calculation of torsional stiffness of wagons. The bases of calculation were established by L. A. Sadur, and then they were accepted by the consortium ORE. This paper treats the creation of a simple model of the two-axle wagon which served as a basis for derivation of an analytical expression for calucation of torsional stiffness of wagons which is in compliance with the existing regulations. The model created can further be very easily used as the starting point for determination of torsional stiffness of multi-axle wagons.

Key words: wagon, railway, calculation, torsion

## **INTRODUCTION**

The bases of calculation of torsional stiffness of wagons were established by L. A. Sadur, and then they were adjusted and accepted by the Association of European Railways, UIC and ORE (ERRI) [1], [7]. The regulations ORE B 55 [1] made in the period from 1980-1983 still represent the valid methodology of checking and calculation of torsional stiffness of railway vehicles.

## STIFF FRAME IN THE SPACE

In the period after passing the set of ORE B 55 regulations for calculation of torsional stiffness, various models for calculation, both analytical and numerical ones, have been developed.

Let us observe a stiff frame in the space shown in Figure 1.



Figure 1. Stiff frame in the space Oxyz

Let the frame be described by the coordinates of the end points 1, 2, 3 and 4 in the following way:  $x_1(0,0,z_1)$ 

$$x_{2}(2a,0,z_{2})$$

$$x_{3}(2a,2b,z_{3})$$

$$x_{4}(0,2b,z_{4})$$
(1)

Let us consider that it is a stiff frame, i.e. there is no elastic deformation of the frame. Let the frame occupy an arbitrary position in the space, as it is shown in Figure 2.



Figure 2 Arbitrary position of the frame in the space

Let us form three arbitrary vectors as it is shown in Figure 3.



Figure 3. Definition of the frame position in the space

The necessary and sufficient condition for having all points of the frame within the same plane is that the mixed product of the vectors formed should be equal to zero:

$$\overline{12}(\overline{13}\overline{X14}) = \mathbf{0}, \tag{2}$$

That is, when we replace the coordinates of the points:

$$(2a, 0, z_2 - z_1)[(2a, 2b, z_3 - z_1)X(0, 2b, z_4 - z_1)] = 0$$
After rearranging of the expression, we obtain
$$z_1 - z_2 + z_3 - z_4 = 0$$
(4)

## SUPPORTS

Let us now observe an elastic frame in space. Let us define the stiffness Cs as the relation between the force acting on one end of the frame (F) and the displacement of the same point (d).

$$C_s = \frac{F}{\delta} \tag{5}$$

If the frame has elastic supports – springs, then, according to [1], [8], obliquely symmetrical forces of equal magnitudes arise in the supports. According to [8], in thus created model it is not important at which point disturbance is introduced because force systems are always formed as it is presented in Figure 4.



Figure 4. Elastic frame on elastic supports

Displacement of the frame ends can be expressed in the following way:

$$z_{1} = z_{10} + \delta z_{2} = z_{20} - \delta z_{3} = z_{30} + \delta z_{4} = z_{40} - \delta$$
(6)

As  $z_{10}$ ,  $z_{20}$ ,  $z_{30}$ ,  $z_{40}$  are the points lying in the same plane, the condition (4) must be fulfilled. Now the coordinates of the deformed wagon body satisfy the equation:

$$z_1 - z_2 + z_3 - z_4 = 4\delta \tag{7}$$

# APPLICATION OF THE MODEL IN A RAILWAY WAGON

If we know the total torsional stiffness of a type of vehicle, then, on the basis of the chosen running gear, it is possible to give the designers of wagons the main guidelines for design or redesign of the wagon body, or vice versa, if the torsional stiffness of the body is known, then, on the basis of the total necessary torsional stiffness, it is possible to choose the corresponding running gear [6,9,10].

For vehicle verification, in accordance with ERRI regulations, it is necessary to check the torsional stiffness of the complete railway vehicle. A railway vehicle possesses, besides its body, an appropriate running gear with its elastic elements, which is schematically shown in Figure 5. The model will assume that stiffnesses of the elastic elements of the running gear are equal and that they are connected with the body by the points 1, 2, 3 and 4 (Figure 4).

Let the disturbance x be introduced in one of the wheels as it is shown in Figure 6. Now it is possible to express the frame position:

$$z_1 = h; z_2 = 0; z_3 = 0; z_4 = 0;$$
 (8)

Thus defined disturbance of the track causes deformation of the body:

$$\delta = \frac{z_1 - z_2 + z_3 - z_4}{4} = \frac{h}{4}$$
(9)  
Now the force acting at the wagon ends is:  
$$\Delta F = C_s \frac{h}{4}$$
(10)

By the very structure of the two-axle wagon, it is possible to see that the wagon body resting against the axle assemblies is supported by the corresponding elastic elements – springs. Taking this connection into account, it is necessary to include the spring stiffness and their design parameters in the wagon model for the purpose of defining the increase in force due to the obliquely symmetrical disturbance.



Figure 5. Schematical presentation of the wagon with forces acting at the points of contact between the wagon body and the springs in the suspension system.

Figure 5 schematically presents the wagon which rests, through the springs, with the length  $l_i$  and stiffness  $c_i$ , against the axle assemblies that are, through the wheels, in contact with the track. Now the coordinates of the wagon ends can be defined by including both the spring lengths and their stiffnesses. Due to their dead weights Q, the wagon springs will be deformed by a certain value in the equilibrium position.

Let us take that the disturbance from the rail denoted by h has been introduced at point 1. Now:

$$z_1 = h + x_1; z_2 = x_2;$$
(11)

$$z_3 = x_3;$$
  
 $z_3 = x_3;$ 

 $z_4 = x_4;$ 

where  $x_i$  denotes the spring lengths.

The spring lengths can be expressed in the following way:

$$x_{1} = l_{1} + x_{1Q} + x_{1h},$$

$$x_{2} = l_{2} + x_{2Q} + x_{2h},$$

$$x_{3} = l_{3} + x_{3Q} + x_{3h},$$

$$x_{4} = l_{4} + x_{4Q} + x_{4h},$$
where:  
(12)

- *l<sub>i</sub>* the lengths of the springs in a nondeformed state,
- *x<sub>iQ</sub>* deformation of the springs due to the weigth of suspended masses;
- x<sub>ih</sub> deformation of the springs due to torsion of the wagon body.

$$x_{iQ} = \frac{Q}{4k_i};$$
(13)

where:

- i=1, ..., 4;
- and k<sub>i</sub> denotes stiffnesses of the springs, respectively.

If the wagon body is in the equilibrium position and if it is considered to be an elastic body, then the equilibrium of forces must exist in the contact between the springs and the wagon body:

$$-k_{1}x_{1} = C_{k}\delta; -k_{2}x_{2} = -C_{k}\delta; -k_{3}x_{3} = C_{k}\delta; -k_{4}x_{4} = -C_{k}\delta;$$
(14)

If thus obtained coordinates are now replaced in the equation (73), it is obtained that:

$$\delta = \frac{z_1 - z_2 + z_3 - z_4}{4} \tag{15}$$

i.e. after rearranging of the expression,

$$C_{k}\delta = \frac{h + (l_{4} - l_{2} + l_{5} - l_{4}) + \frac{Q}{4}(\frac{1}{k_{4}} - \frac{1}{k_{5}} + \frac{1}{k_{4}} - \frac{1}{k_{4}})}{\frac{4}{C_{k}} + \frac{1}{k_{4}} + \frac{1}{k_{2}} + \frac{1}{k_{5}} + \frac{1}{k_{4}}}$$
(16)

i.e.,

$$\Delta F = \frac{\mathbf{h} + (\mathbf{l}_1 - \mathbf{l}_2 + \mathbf{l}_3 - \mathbf{l}_4) + \frac{\mathbf{Q}}{4} \left(\frac{1}{\mathbf{k}_1} - \frac{1}{\mathbf{k}_2} + \frac{1}{\mathbf{k}_3} - \frac{1}{\mathbf{k}_4}\right)}{\frac{4}{\mathbf{C}_k} + \frac{1}{\mathbf{k}_1} + \frac{1}{\mathbf{k}_2} + \frac{1}{\mathbf{k}_3} + \frac{1}{\mathbf{k}_4}}$$
(17)

It can be seen from the previous equations that the variation of force due to the wagon torsion depends, in the general case, on the design parameters of the spring (free lengths of the springs  $l_i$  and stiffnesses of the springs  $k_i$ ), the weight of the suspended mass of the wagon and

#### XVII INTERNATIONAL SCIENTIFIC CONFERENCE "TRANSPORT 2007"

the stiffness of the wagon body  $C_k$ . If the springs have approximately the same characteristics, i.e. the same free lengths and same stiffnesses, than the equation is as follows:

$$\Delta \mathbf{F} = \frac{\mathbf{n}}{\frac{4}{C_{\rm k}} + \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_2} + \frac{1}{k_4}}$$
(18)

### CONCLUSION

The model developed represents a new approach in determination of torsional stiffness of railway vehicles. The advantages of this model are seen in the given analytical formula which represents a basis for development of more complex models for calculation. In this model, certain assemblies with an approximately linear characteristic of stiffness are decomposed.

The analytical expression for calculation of torsional stiffness is in accordance with the existing regulations ORE B 55 Rp:1 – 8, ERRI B 12 DT 135, as well as with the regulations 14363. The results obtained by calculation according to the model presented and the results from testing have a high degree of agreement.

#### **REFERENCES:**

[1] ORE B 55, «Moyens propres á assurer la circulation normale sur des voies présentant des gauches», ORE de UIC, Utrecht, 1983.

[2] V. Lučanin, G. Simić, D. Marinković. "Experimental Verification of Auto Carrier Car Stength Calculation" FME Transactions. - ISSN 1451-2092. - Vol. 32, no. 1 (2004), pp. 43-48.

[3] Iwnicki S.D, Bezin Y., "Simulation as a Tool for Assessing the Match between Track and Vehicle standards", Rail Technology Unit, Manchester Metropolitan University, Department of Engineering & Technology, Manchester, United Kingdom, January 2004.

[4] Australian Transport Safety Bureau, RAIL SAFETY INVESTIGATION REPORT, "Derailment of Train 4VM9-V, Benalla, Victoria", 23 September 2004, Commonwealth of Australia 2005.

[5] Rail Technology Unit, "The Manchester Benchmarks for Rail Vehicle Simulation", Manchester Metropolitan University, March 1998.

[6] Atmadžova D., Penchev Ts. "A geometrical model of determining obliquely symmetrical load of biaxial (two-bogie) rolling stock" Mehanika transport komunikaciji, no. 1/2007, Sofia 2007, page BG-2.11-BG-2.18.

[7] ERRI B12 / DT 135, "Allgemein verwendbare Berechnungsmethoden für die Entwicklung neuer Güterwagenbauarten oder neuer Güterwagenderhgestelle" European Rail Research Institute, Utrecht, 1995.

[8] Л.А. Шадур "Вагоны конструкция, теория и расчет", Москва Транспорт, 1980.

[9] Пенчев, Ц., Д. Атмаджова, Л. Паскалев Методи за определяне при деповски (заводски) и експлоатационни условия на сумарната хлабина по диагоналите между страничните плъзгалки на талиговия подвижен железопътен състав. София, XI НК с международно участие "ТЕМПТ 2001 – ТРАНСПОРТЪТ НА XXI ВЕК" на ВТУ "Т. Каблешков", 2001.

[10] Пенчев Ц. и Д. Атмаджова Въпроси от експлоатация, ремонта и рециклирането на пътнически вагони от парка на БДЖ. София, Сборник от методични материали за курс от следдипломна специализация ВТУ "Т. Каблешков", 2003.

## НОВ ПОДХОД ЗА АНАЛИТИЧНО ОПРЕДЕЛЯНЕ НА КОРАВИНАТА НА УСУКВАНЕ НА ЖЕЛЕЗОПЪТНИ ВАГОНИ

#### Небойша Богойевич, Ранко Раканович, Драган Петрович, Златан Шошкич

Небойша Богойевич, проф. д-р Ранко Раканович, проф. д-р Драган Петрович, д-р Златан Шошкич Машинен факултет в Кралево, ул. Доситеева 1, Кралево,

Сърбия

**Резюме:** Този доклад представя нов подход за изследване и изчисление на коравината на усукване на вагоните.. Основите на изчисленията са поставени от Л. А. Садур и после са приети от консорциума ORE. Докладът разглежда създаването на прост модел на вагон с две оси, който е основа за извеждането на аналитичен израз за изчисление на коравината на усукване на вагоните в съответствие със съществуващите правила. Създаденият модел може лесно да бъде използван като отправна точка за определяне на коравината на усукване на вагони с много оси.

Ключови думи: вагон, железница, изчисление, усукване.