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INFLUENCE OF MULTILAYERED DIELECTRIC ON CHARACTERISTIC PARAMETERS OF PARALLEL PLATE CAPACITOR

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Key words: *Capacitance, Characteristic impedance, Finite element method, Hybrid boundary element method, Multilayered dielectric, Parallel plate capacitor.*

Abstract: *In this paper a parallel plate capacitor with multilayered dielectric is analysed. Influence of the boundary surface position as well as relative permittivities on the characteristic parameters values are considered and showed as numerical results, presented graphically and in table. The hybrid boundary element method is applied for this quasi-static TEM analysis. The validity of obtained results is confirmed using the FEMM software and well known expression for the capacitance of parallel plate capacitor.*

INTRODUCTION

Hybrid boundary element method (HBEM) is developed at the Department of Theoretical Electrical Engineering at the Faculty of Electronic Engineering of Nis during 2010. The method has wide area of application. Up to now, it is applied for solving large scale of electrostatic and magnetostatic problems, [1]-[5].

The method presents a combination of the boundary element method [6] and the equivalent electrodes method [7]. His main advantage is possibility to analyse multilayered electromagnetics problems without using complex calculations. After method is applied, it is necessary to form and solve only a system of linear equations and to calculate parameters of interest.

In order to present a possibility of the method for multilayered planar problems solving, a parallel plate capacitor with angled boundary surface is analysed, Fig. 1. The position of the boundary surface is determined with a parameter α. One plate is on potential U and the other one is on the zero potential. The permittivities of the corresponding layers are denoted with ε_1, ε_2 and ε_3. The thickness of the plates is negligible.

An influence of parameter α on the characteristic impedance as well as the capacitance per unit length of this system will be considered in this paper. All results will be shown graphically and in the table.

To prove the method accuracy, it was necessary to verify obtained results. The geometry from Fig. 1 is modelled using FEMM software [8]. Also, well known expressions for the capacitance of parallel-plate capacitor is used, [9].

The quasi-static TEM approach is applied for the analysis.

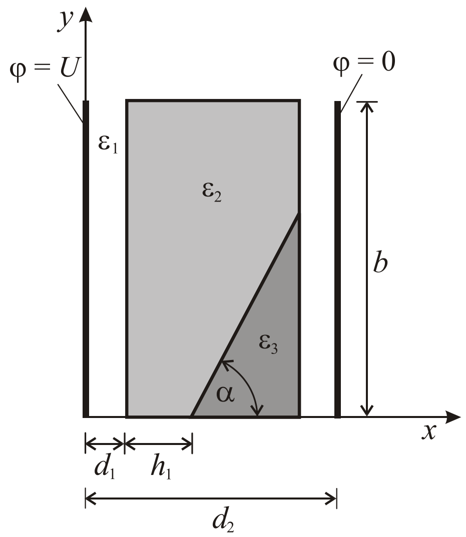

Fig.1 Parallel plate capacitor with angled boundary surface.

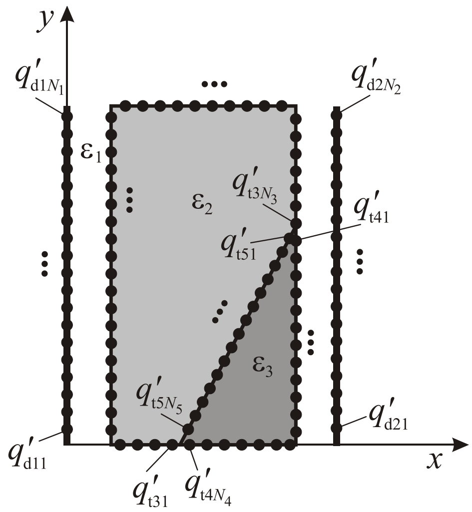

Fig.2 HBEM model.

HBEM APPLICATION

The application of the HBEM is described in [1] and [2]. Using that procedure, the equivalent HBEM model is formed, Fig. 2. Unknown values are line charges because the geometry from Fig. 2 is planar.

The potential at any point with coordinates (x, y) is

$$\begin{aligned}\varphi = \varphi_0 &- \sum_{i=1}^{N_1} \frac{q'_{\mathrm{d}1i}}{2\pi\varepsilon_1} \ln \sqrt{(x - x_{\mathrm{d}1i})^2 + (y - y_{\mathrm{d}1i})^2} - \sum_{j=1}^{N_2} \frac{q'_{\mathrm{d}2j}}{2\pi\varepsilon_1} \ln \sqrt{(x - x_{\mathrm{d}2i})^2 + (y - y_{\mathrm{d}2i})^2} - \\ &- \sum_{k=1}^{N_3} \frac{q'_{\mathrm{t}3k}}{2\pi\varepsilon_0} \ln \sqrt{(x - x_{\mathrm{t}3k})^2 + (y - y_{\mathrm{t}3k})^2} - \sum_{l=1}^{N_4} \frac{q'_{\mathrm{t}4l}}{2\pi\varepsilon_0} \ln \sqrt{(x - x_{\mathrm{t}4l})^2 + (y - y_{\mathrm{t}4l})^2} - \\ &- \sum_{m=1}^{N_5} \frac{q'_{\mathrm{t}5m}}{2\pi\varepsilon_0} \ln \sqrt{(x - x_{\mathrm{t}5m})^2 + (y - y_{\mathrm{t}5m})^2}, \end{aligned} \tag{1}$$

where $q'_{\mathrm{d}1i}$ $(i = 1,..., N_1)$ and $q'_{\mathrm{d}2j}$ $(j = 1,..., N_2)$ are unknown line free charges on plates. With $q'_{\mathrm{t}3k}$ $(k = 1,..., N_3)$, $q'_{\mathrm{t}4l}$ $(l = 1,..., N_4)$ and $q'_{\mathrm{t}5m}$ $(m = 1,..., N_5)$ are denoted total charges on boundary surface between any two layers. Index "d" denotes line charges placed in dielectric ("d") and "t" stands for total line charges placed in the air. With $(x_{\mathrm{d}1i}, y_{\mathrm{d}1i})$, $(x_{\mathrm{d}2j}, y_{\mathrm{d}2j})$, $(x_{\mathrm{t}3k}, y_{\mathrm{t}3k})$, $(x_{\mathrm{t}4l}, y_{\mathrm{t}4l})$ and $(x_{\mathrm{t}5m}, y_{\mathrm{t}5m})$ are presented the positions of the equivalent electrodes. φ_0 is unknown additive constant, that depends on the chosen referent point for the electric scalar potential. In this case, the free charges do not exist on dielectric boundary surfaces (free surface charges exist only on plates), so the total line charges between dielectric layers are equal to polarized line charges.

The total number of unknowns N_{tot}, is denoted by:

$$N_{\mathrm{tot}} = \sum_{i=1}^{5} N_i + 1. \tag{2}$$

Using the point matching method for the potential on the plates and the relation for the normal component of the electric field on the boundary surfaces as well as the electrical neutrality condition, the system of linear equations is formed. The solutions of this system are unknowns line charges and the additive constant. Now, it is possible to calculate the characteristic parameters of the geometry from Fig. 1.

The characteristic impedance is calculated using the expression

$$Z_c = \frac{1}{c\sqrt{C'C'_0}}, \tag{3}$$

where $c = 3 \cdot 10^8$ m/s, C' is the capacitance per unit length of the system and C'_0 is the capacitance per unit length of capacitor with air-filled dielectric.

NUMERICAL RESULTS AND COMPARISONS

The convergence study has been carried out by calculating the variations of characteristic impedance as a function of the number of unknowns. The dimensions of the geometry from Fig. 1 are:

$$\varepsilon_{r1} = 1,\ \varepsilon_{r2} = 3,\ \varepsilon_{r3} = 11,\ b/a = 2.0,\ d_1/a = 0.2,\ h_1/a = 0.1 \text{ and } \alpha = 85^\circ.$$

Those results are shown in Fig. 3 with solid black line. The HBEM results are compared with that obtained using FEMM [8]. The FEMM value is denoted with red line. It can be noticed that good convergence of the results is obtained.

On the same figure the computation time is given. This time includes the times necessary to: determine the position of the equivalent electrodes, form the system of linear equations, solve that system and calculate the characteristic parameters. Increasing the number of unknowns, the computation time increases.

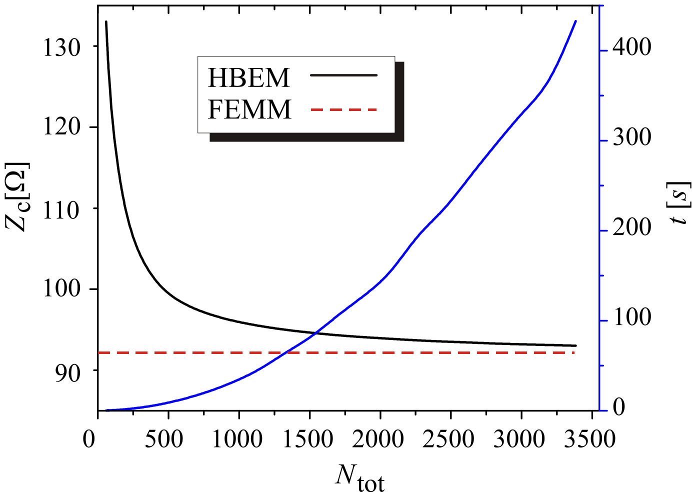

Fig.3 Convergence of results and computation time.

In Fig. 4 the equipotential lines for parallel plate capacitor with air-filled dielectric are shown.

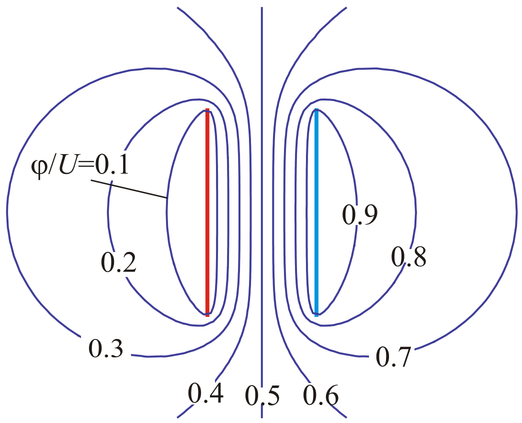

Fig.4 Equipotential lines for parallel plate capacitor with air-filled dielectric.

The dimensions of the system are: $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1$ and $b/a = 2.0$.

Using the developed HBEM program code, for $N_{tot} = 2500$, the characteristic parameters of the air-filled parallel plate capacitor is calculated for different values of parameter b/a. Those results are given in Table 1.

The HBEM results are verify with the FEMM results. Also, in this table the values obtained using analytical expression for parallel plate air-filled capacitor is shown [9],

$$C' = \varepsilon_0 \frac{b}{a}. \tag{4}$$

Very large results disagreement can be noticed between the HBEM and FEMM results on one side and analytical expression on the other side for $1 \le b/a \le 20$. The main reason for this disagreement is in the fact that the analytical expression is valid only for $b >> a$ [9]. That condition is satisfied for $b/a > 20$. The error rate is less than 2%.

Table 1. Characteristic parameters of air-filled parallel plate capacitor

$\frac{b}{a}$	HBEM		FEMM		Analytical expression	
	Z_c [Ω]	C'[pF/m]	Z_c [Ω]	C'[pF/m]	Z_c [Ω]	C'[pF/m]
1	182.19	18.3	178.76	18.6	376.65	8.85
2	118.16	28.2	117.20	28.4	188.32	17.7
5	59.72	55.8	60.20	55.3	75.33	44.2
10	33.21	100.3	32.75	101.8	37.66	88.5
15	23.25	143.4	22.56	147.7	25.11	133.3
20	17.96	185.6	17.84	186.8	18.83	177.4
30	12.40	268.8	12.35	269.9	12.55	265.2
40	9.44	352.9	9.35	356.5	9.42	354.3
50	7.68	433.7	7.53	442.7	7.53	442.7

An influence of the position of boundary surface between layers 2 and 3 is shown in Fig. 5. The equipotential lines for two different angles are shown.

The dimensions of the system from Fig. 1 used for this calculation are:

$$\varepsilon_{r1} = 1,\ \varepsilon_{r2} = 3,\ \varepsilon_{r3} = 11,\ b/a = 2.0,\ d_1/a = 0.2,\ h_1/a = 0.1.$$

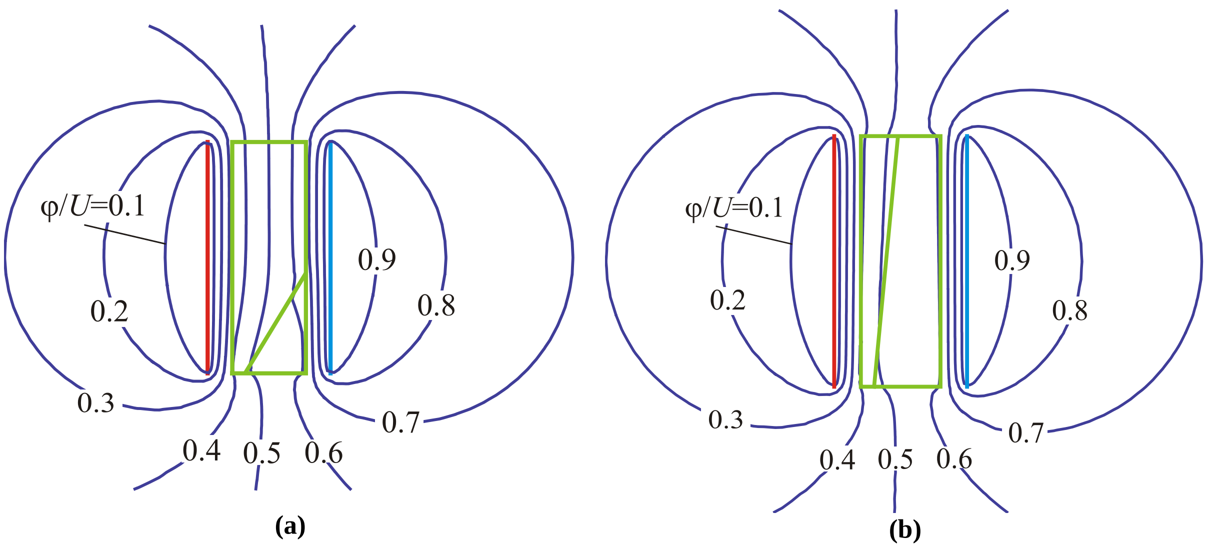

Fig.5 Equipotential lines for: (a) $\alpha = 60^\circ$ **and (b)** $\alpha = 85^\circ$**.**

The effect of the boundary surface position on the characteristic impedance is presented in Fig. 6 for following dimensions:

$\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 3$, $\varepsilon_{r3} = 11$, $b/a = 2.0$, $d_1/a = 0.2$, $h_1/a = 0.1$.

Increasing the parameter α, the characteristic impedance decreases. For $\alpha > 100^\circ$ the characteristic impedance becomes almost constant. Namely, the parameter h_1/a is small, so the boundary surface between layers 2 and 3 for $\alpha > 100^\circ$ is very close to the left side of the layer 2. Increasing the parameter α, the area of layer 2 stays almost constant.

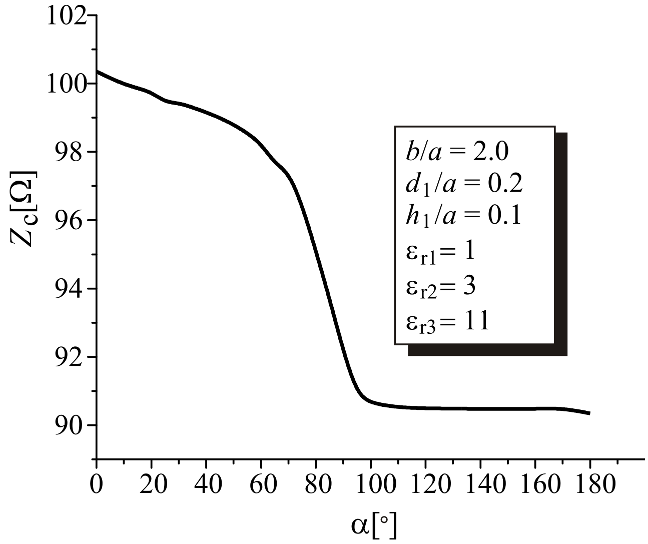

Fig.6 Characteristic impedance distribution versus boundary surface position.

A case when the boundary surface is not so close to one of the dielectric sides is shown in Fig. 7. Dimensions of the capacitor are:

$\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 3$, $\varepsilon_{r3} = 11$, $b/a = 0.5$, $d_1/a = 0.1$, $h_1/a = 0.5$.

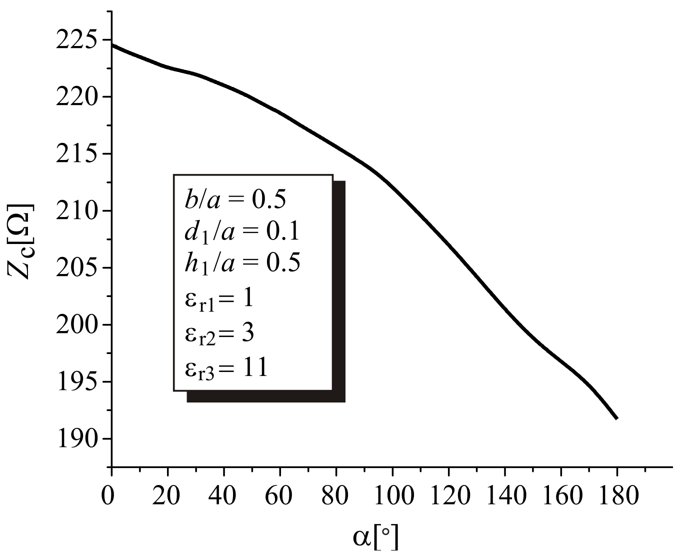

Fig.7 Characteristic impedance distribution versus boundary surface position.

From this figure it is evident that increasing the angle α, the characteristic impedance decreases.

An influence of layers permittivities on characteristic impedance is shown in Fig. 8. Increasing the relative permittivities ε_{r2} and ε_{r3}, the characteristic impedance decreases.

The capacitor dimensions are: $\alpha = 130^\circ$, $b/a = 0.5$, $d_1/a = 0.1$, $h_1/a = 0.5$.

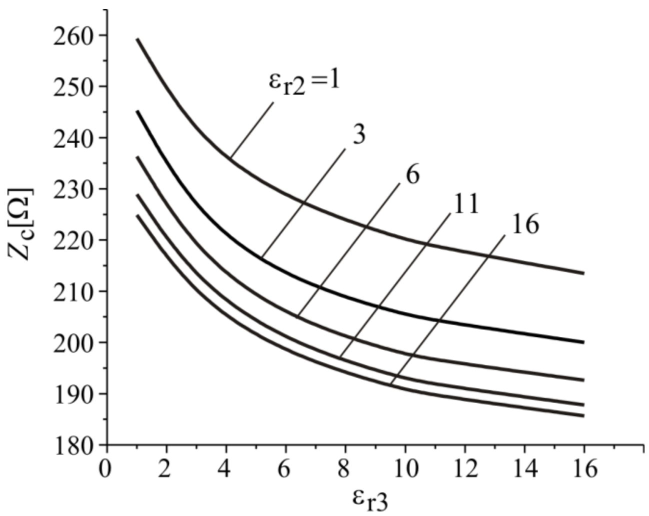

Fig.8 Characteristic impedance distribution versus different permittivities ε_{r2} and ε_{r3}.

CONCLUSION

The paper presents an application of the HBEM for analysis an influence of the multilayered dielectric with angled boundary surface on the characteristic parameters values. The obtained results of static parameters have been compared with software FEMM so the validity of the HBEM is confirmed. Effect of different permittivity values is also noticed.

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ВЛИЯНИЕ НА МНОГОПЛАСТОВИТЕ ДИЕЛЕКТРИЦИ ВЪРХУ ПАРАМЕТРИТЕ И КАПАЦИТЕТА НА ПАРАЛЕЛНИ ПЛАСТИНИ

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Ключови думи: *капацитет, пълно съпротивление, метод на крайните елементи, метод на хибридния граничен елемент, многопластов диелектрик, капацитет на паралелна пластина.*

Резюме: *В настоящият доклад се анализира капацитета на паралелна пластина с многопластов диелектрик. Също така се изследва влиянието на граничната позиция на повърхността, както и диелектричната проницаемост на стойностите на параметрите, които са представени като числа, под формата на таблици и графично. Методът на хибридния граничен елемент се прилага при този квази-статичен TEM анализ. Валидността на получените резултати е доказана с помощта на FEMM софтуер и добре познатия математически израз за капацитета на паралелни палстини.*