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**COMPARATIVE ANALYSIS OF PUNCHING STRENGTH OF FLAT  
SLABS REINFORCED WITH SHEAR STUDS ON THE EXAMPLE OF  
CORNER COLUMN**

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**Key words:** *punching shear resistance, double headed studs, column, slab, connection, comparative analysis*

**Abstract:** *Problem of punching represents the biggest obstacle to use of flat concrete slabs. Traditional types of shear reinforcement like stirrups or the use of drop panels have been the solution, but they have their deficiencies. Stirrups are difficult to install and cannot be fully effective because of the way they need to be placed around the bending reinforcement. Hence, it is almost impossible for them to achieve required yield strength. The use of column capitols and drop panels complicates placement of installations and therefore additionally increases the height of the structures.*

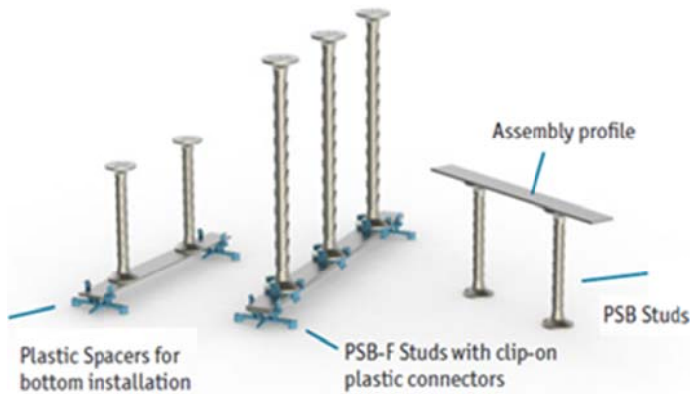
*This article analyses the shear reinforcement which is becoming very popular thanks to its ease of use and the ability to successfully deal with mentioned problems. After some thirty years of research, double-headed studs have become standard type of shear reinforcement for concrete slabs. Because of the different approaches to the problem of punching in the slab, for this work three different standards are chosen for comparative analysis: ACI 318-11, ETA -13/0151 and SIA 262 (2013).*

*Here are provided some standards' provisions and some calculation explanations. Numerical examples are done according to each of the presented codes for the case of corner column and the results are presented comparatively.*

*Authors' desire is that this comparative analysis provides basis for future application in our surrounding, because the available literature on double-headed studs is sparse.*

## **INTRODUCTION**

The slabs directly supported by columns offer interesting and simple solutions for variety of objects including buildings and smaller span bridges. These solutions are being adopted in residential, administrative, industrial types of objects and garage spaces. Their construction is more economic, they allow easier execution of mechanical and electrical installations as well as bigger storey heights.



**Fig. 1 Double headed shear studs mounted on the rail element** Source: [2]

The thickness of the slab is the function of the deflection and punching shear. Punching is usually related to concentrated load or columns. The shear stress which is produced in these cases is a result of gravitational load, but its value can be increased significantly due to lateral wind and seismic loads. The increase in temperature can also have an impact on how columns interact with a slab [1]. Therefore the problem of punching represents the biggest obstacle to use of flat concrete slabs.

Shear leads to brittle and sudden failure which is characterized by separation of truncated cone surface at the place of the column-slab joint. This failure happens quickly and is especially disastrous because it can lead to progressive collapse of adjacent columns and eventually entire structure. This can be prevented by adopting a thicker slab, by the use of higher resistance concretes or by the use of shear reinforcement.

The most effective way to deal with this problem is to use the shear reinforcement. It treats the punching directly and locally without the influence on the rest of the structure and it prevents the brittle failure. The biggest obstacle to use of this solution is its placing and anchoring. Traditional types of shear reinforcement like stirrups are difficult to install, because in order to be completely effective it is necessary that they enclose both upper and lower longitudinal flexural reinforcement. This is almost impossible for thinner slabs and when steel rebar meshes are used, which makes it impossible for them to achieve required yield strength. Because of this the designers turn to drop panels which have their own deficiencies like reduction of usable storey height, more complicated construction and bigger weight. All of this leads to increase of costs. Alternative solution which is becoming more popular in the world thanks to its ease of use and the ability to successfully deal with mentioned problems is the use of shear studs as the punching shear reinforcement. One example of this element is given on the Figure 1 [2].

This article analyses this type of shear reinforcement that has after some thirty years of research become standard type of shear reinforcement for concrete slabs. Because of the different approaches to the problem of punching in the slab, for this work three different standards are chosen for comparative analysis: ACI 318-11, ETA -13/0151 and SIA 262 (2013).

Here are provided some standards' provisions and some calculation explanations. Analytical examples are done according to each of the presented codes for the case of corner column and the results are presented comparatively.

## STANDARDS OVERVIEW

According to ACI 318-11 punching shear resistance should be verified at critical sections according to Figure 2 and Figure 3 [3], and should satisfy the following condition:

$$(1) \quad v_u \leq \phi v_n$$

Where  $v_u$  is maximum shear stress at a critical section produced by the combination of factored shear force  $V_u$  and unbalanced moments  $M_{ux}$  and  $M_{uy}$ , and  $v_n$  is nominal shear strength. Nominal shear strengths of concrete and steel are given respectively as  $v_c$  and  $v_s$ .

$$(2) \quad v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} \cdot M_{ux} \cdot y}{J_x} + \frac{\gamma_{vy} \cdot M_{uy} \cdot x}{J_y}$$

$$(3) \quad v_n = v_c + v_s$$

$$(4) \quad v_c = \lambda \cdot \frac{\sqrt{f'_c}}{4}$$

$$(5) \quad v_s = \frac{A_v \cdot f_{yt}}{b_0 \cdot s} \geq \frac{1}{6} \cdot \sqrt{f'_c}$$

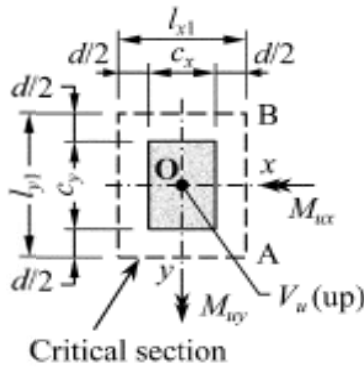


Fig. 2 Critical section inside of the reinforced zone  
Source: [3]

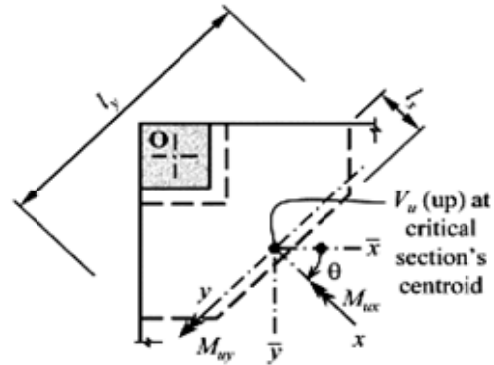


Fig. 3 Critical section outside of the reinforced zone  
Source: [3]

The number of studs on one element should be such that the shear stress at the section  $d/2$  ( $d$ -effective height of the slab) from the last row of studs is not greater than  $\frac{1}{6} \cdot \phi \cdot \lambda \cdot \sqrt{f'_c}$ .

$s$  – spacing between peripheral lines of shear reinforcement

$f_{yt}$  – specified yield strength of shear reinforcement

$b_0$  – length of perimeter of critical section

$A_v$  – cross-sectional area of shear reinforcement on one peripheral line parallel to perimeter of column section

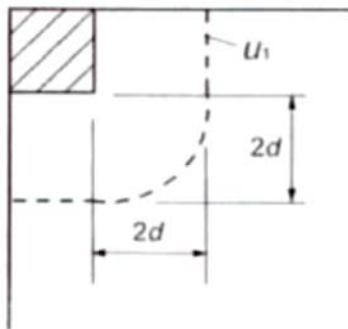


Fig. 4 Critical section inside of the reinforced zone

Source: [5]

$\gamma_{vx}, \gamma_{vy}$  – factors used to determine unbalanced moment about axes  $x$  and  $y$

$\phi$  – strength reduction factor

$\lambda$  – modification factor reflecting the reduced mechanical properties of lightweight concrete

$J_x, J_y$  – property of assumed critical section of any shape, equal to  $d$  multiplied by second moment of perimeter

$l_x, l_y$  – projections of assumed critical section on principal axes  $x$  and  $y$

More detailed explanations can be found in [4] and [3].

The use of double headed studs in Europe is regulated by European Technical Approvals (ETA), because they are not predicted by EN 1992-1-1. For this work ETA-13/0151 is chosen. According to this legislation punching shear should be verified at critical sections according to Figure 4 [5]. Critical section outside the reinforced zone is located  $1,5d$  from the last row of studs.

Contrary to ACI 318-11, here the shear strength  $V_{Rd,sy}$  does not take into account the strength of concrete. Therefore the design value of shear stress  $v_{Ed}$  should not be greater than the minimum value of concrete strength  $v_{Rd,max}$  and than shear strength of studs.

$$(6) \quad v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_1 \cdot d}$$

$$(7) \quad v_{Rd,max} = 1,96 \cdot v_{Rd,c}$$

$$(8) \quad v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \geq (v_{min} + k_1 \cdot \sigma_{cp})$$

$$(9) \quad \beta \cdot V_{Ed} \leq V_{Rd,sy} = m_c \cdot n_c \cdot \frac{d_A^2 \cdot \pi \cdot f_{yk}}{4 \cdot \gamma_s \cdot \eta}$$

$u_l$  – length of critical perimeter

$\beta$  – coefficient which takes into account the load eccentricity

$C_{Rd,c}$  – empirical factor

$k$  – coefficient which takes into account the size of the effective height of the slab,  $d$

[mm]

$\rho_l$  – mean reinforcement ratio

$f_{ck}$  – characteristic value of concrete strength

$k_1$  – empirical factor

$\sigma_{cp}$  – normal concrete stresses in critical section

$m_c$  – number of elements in the zone C, refer to the reference [6]

$n_c$  – number of studs on each element in the zone C, refer to the reference [6]

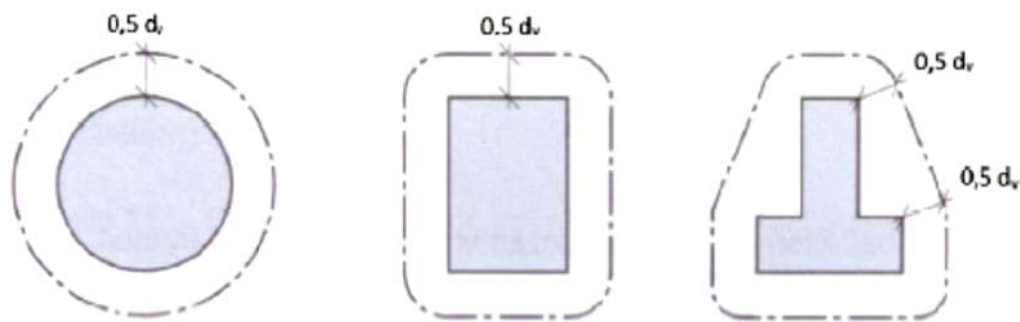


Fig. 5 Critical sections inside of the reinforced zone for different shapes of columns [7]

$f_{yk}$  – characteristic yield strength of steel

$\gamma_s$  – partial safety coefficient for steel

$d_A$  – diameter of shear stud

$\eta$  – factor which takes into account the effective height of the slab

The number of studs in the element is chosen so that the stress in the critical section outside of the reinforced area is not greater than the concrete strength. In order to accomplish that, the required outer critical perimeter should be calculated as:

$$(10) \quad u_{out} \geq \frac{\beta_{red} \cdot V_{Ed}}{v_{Rd,c} \cdot d}$$

$\beta_{red}$  – reduced factor for taking in account the effects of eccentricity in perimeter  $u_{out}$

SIA 262 (2013) also considers punching in critical sections. The shape of critical perimeter  $u$  and its position is given by the Figure 5 [7].

Design value of axial force in the column  $V_d$  is given as the column reaction  $R_d$  reduced for the actions of opposite direction  $\Delta V$  inside of critical section and divided by reduction coefficient of critical section  $k_e$ . Punching shear strength takes into account both the contribution of concrete and steel studs. It is necessary that the steel strength is higher than the difference between the design shear stress and concrete strength.

$$(11) \quad V_{Rd} = V_{Rd,c} + V_{Rd,s}$$

$$(12) \quad v_{Rd,c} = k_r \cdot \tau_{cd} \cdot d_v \cdot u$$

$$(13) \quad V_{Rd,s} = \sum A_{sw} \cdot \sigma_{sd} \cdot \sin \beta$$

$\tau_{cd}$  – design value shear stress

$k_r$  – coefficient which takes into account the dimension of the slab, its rotation as well as maximum aggregate size

$\beta$  – angle that shear reinforcement forms with slab plane

$\Sigma A_{sw}$  – cross section area of all shear reinforcement between  $0,35d_v$  and  $d_v$  from column edge

$\sigma_{sd}$  – design value of normal stress in vertical shear reinforcement

Punching shear resistance is limited by fracture of the concrete strut in the proximity of the support zone which represents the maximum punching resistance of concrete. This resistance is given with following relation:

$$(14) \quad V_{Rd,c,max} = 3 \cdot k_r \cdot \tau_{cd} \cdot d_v \cdot u \leq 3,5 \cdot \tau_{cd} \cdot d_v \cdot u$$

The required number of studs on one element is determined so that the shear stress outside of the zone reinforced by shear studs is not greater than shear strength of concrete along the outer critical perimeter  $u_{out}$ :

$$(15) \quad V_d \leq V_{Rd,c,out} = k_r \cdot \tau_{cd} \cdot d_v \cdot u_{out}$$

For more details regarding these codes the reader is advised to consult [5], [6] and [7].

### COMPARATIVE ANALYSIS ON THE EXAMPLE OF THE CORNER COLUMN

In this chapter the design examples for corner column-slab joint together with the results will be presented. The idea was to find the combination of force and moments that lead to the maximum shear stress for adopted dimensions and properties of the slab and the column and for the adopted number of shear studs. Since the codes predict different sets of rules regarding the placement of reinforcement, two different examples are made with only difference between them being the number of shear elements and the quality of steel used for the double headed studs. The total number of shear studs remained the same. One example given by Figure 6 [8] covers the ACI 318-11 and the other presented on the Figure 7 [8] covers ETA-13/0151 and SIA 262 (2013).

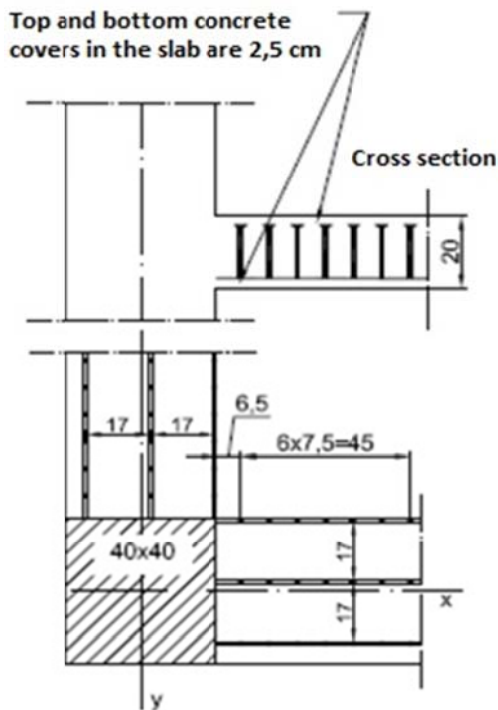


Fig. 6 Arrangement of studs for the case of ACI 318-11 Source: [8]

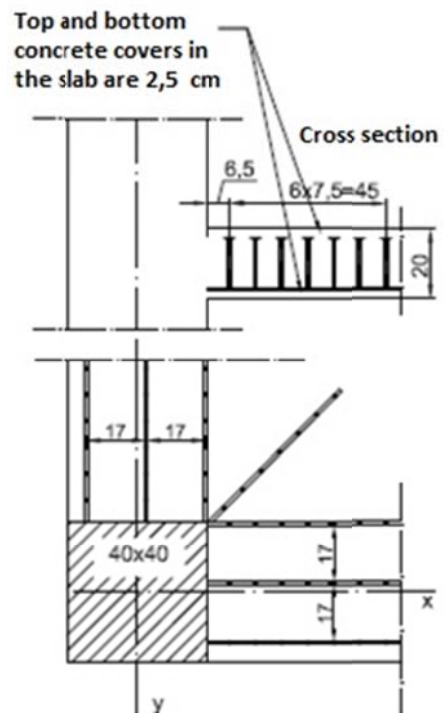
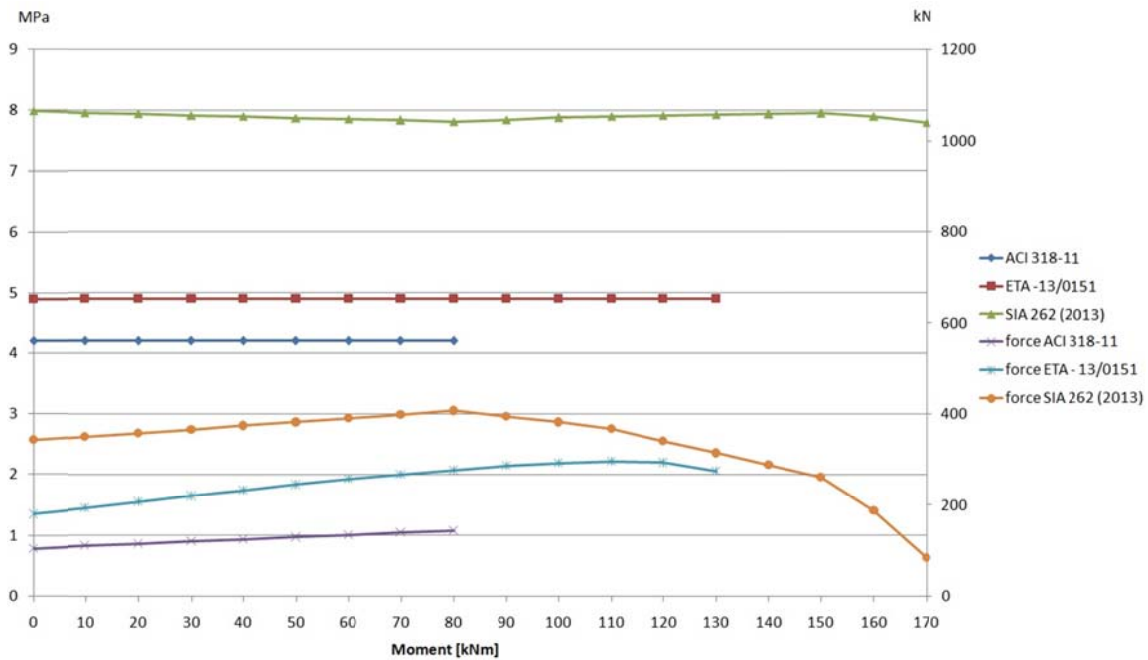
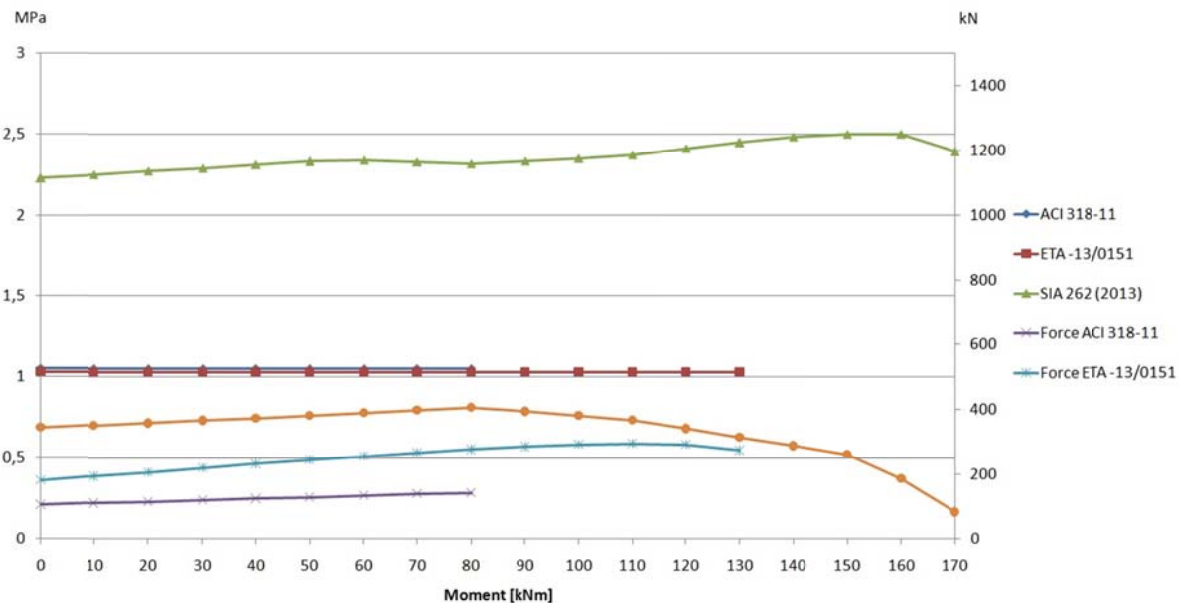


Fig. 7 Arrangement of studs for the case of ETA-13/0151 and SIA 262 (2013) Source: [8]



**Fig. 8 Punching resistance at inner critical perimeter for different combinations of moments and force**  
**Source: [8]**

As it could be seen, the slab of thickness 20 cm was in both cases supported by rectangular 40x40 cm column. The concrete strength was 40 MPa. The strength of shear studs in the case of ACI 318-11 was 400 MPa and for the case of ETA-13/0151 and SIA 262 (2013) it was 500 MPa. In the case of ACI 318-11, 7 studs were distributed on each of the 6 elements whereas in other case 6 studs were placed on each of the 7 elements. The diameter of shear studs was 10 mm and 16 mm for longitudinal flexural reinforcement.



**Fig. 9 Punching resistance at the critical section outside of reinforced zone for different combinations of moments and force**  
**Source: [8]**

According to brief presentation of calculation procedures it is clear that approaches differ what as a consequence provide different results. As aforementioned, the values of

effects on the structure are calculated for maximum values of stress in the joint. Figures 8 and 9 [8] show the values of punching resistances at the critical section inside and outside of the reinforced zone respectively. The resistances are presented together with corresponding combination of flexural moments in the slab and the axial force in the column which are given by the separate curves and which values could be seen on the right vertical axis.

Difference in the resistances is the result of the different lengths of critical perimeters and different corresponding force values come from the different resistances according to each of the standards considered. Limitation of the steel strength used for shear studs in ACI 318-11 represent an important difference compared to the other two codes.

## CONCLUSION

The aim of this article was to introduce the double headed shear studs to use in column-slab joints through comparison of three different standards. The basic equations are presented together with input data for calculation. The references are made to the literature where it is possible to find more detailed explanations. From the analysis of the results presented comparatively following conclusions are drawn:

- Calculation methods differ and it is impossible to do the analysis on the identical examples, so that the results of these examples are also different. However it was possible to determine and to compare different parameters of resistance capacity.
- ACI 318-11 gives the most conservative results
- Due to the resistance limitation of the shear studs, ACI 318-11 needs more studs along the critical perimeter to transfer the same stress as in other two cases
- SIA 262 (2013) requires less studs at the critical section, because contrary to ETA -13/0151 it takes into account the contribution of concrete to overall strength
- SIA 262 (2013) requires less studs per element
- For the same characteristics of column-slab joint and the identical arrangement of studs SIA 262 (2013) allows greater forces in the critical section than ETA -13/0151
- SIA 262 (2013) is the only one taking into account the effect of the slab deformation on the punching shear strength, what allows more realistic approach to the problem
- SIA 262 (2013) give the most economic solutions regarding the quantity of steel needed for reinforcement.
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## **СРАВНИТЕЛЕН АНАЛИЗ НА УДАРНАТА СИЛА НА ПЛОСКОСТИ, АРМИРАНИ С НАПРЕЧНИ ШИПОВЕ ПО ПРИМЕРА НА НОСЕЩИТЕ КОЛОНИ**

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**СЪРБИЯ**

**Ключови думи:** *устойчивост към напречни ударни сили, двуглави шипове, колона, връзка, сравнителен анализ*

**Резюме:** *Използването на бетонни плочи най-често е свързано с възникването на проблеми в резултат на ударни сили. Те могат да бъдат преодоляни чрез употребата на стремена или падащи панели, които обаче си имат своите особености. Изграждането на стремена обикновено е трудоемка дейност, като те не могат да бъдат използвани напълно ефективно, поради начина, по който се разполагата около основите на постройките. Ето защо почти е невъзможно чрез тях да се постигне изискуемата предавателна сила. От друга страна употребата на падащи панели затруднява построяването на сградите, тъй като е необходимо те да бъдат изградени с по-голяма височина.*

*Настоящият доклад анализира въздействието на напречните режещи ръбове, като съвременен метод за решаване на гореспоменатите проблеми. В резултат на дългогодишни научни изследвания, двуглавите шипове са определени като стандартен метод за преодоляване на ударните сили при изграждането на стоманени плоскости. Приложени са три различни стандарта за извършване на сравнителния анализ: ACI 318-11, ETA -13/0151 и SIA 262 (2013). Представени са математически изчисления и обяснения, като основната цел на авторите е да представят постановки за бъдещо приложение на постигнатите резултати в *nin our surrounding, because the available literature on double-headed studs is sparse.**