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# THEORETICAL AND EXPERIMENTAL ANALYSIS OF WAGONS IMPACT

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***Key words:*** *Theoretical, Experimental, Analysis, Wagon, Impact*

***Abstract:*** *According to the international regulations, the experimental testing of wagons impact is one of the mandatory testing in the phase of certification. During the wagons impact, there is an intensive change of dynamic parameters such as forces, deformations, speeds, accelerations, etc. These parameters are also very important when it comes to the design and verification of wagon structure quality. The purpose of this paper is to point to a very complex problems of the theoretical and experimental determination of dynamic parameters which appear at the wagons impact. In theoretical sense, there are two approaches for solution of this problem. One is related to the formation of differential equations of motion, and the second one is related to the application of the law of conservation of energy and the quantity of motion. In defining the mathematical models, there are usually the certain restrictions: wagon structure, bogie and cargo are considered to be absolutely rigid; railway track is horizontal; the centre of mass of cargo and wagons are moving in parallel and the gaps in subassemblies of wagons are ignored. Such conducted theoretical research are basis for preparation and realization of experimental research which should lead to the proper and satisfying results. In that sense, the moving of cargo during the impact is particularly significant. It is concluded that only mutual theoretical and experimental research can lead to the reliable results of dynamic parameters which occur at the wagons impact.*

## 1. INTRODUCTION

The desire for permanent increasing of productivity of rail transport initiate the development of wagons with optimized supporting structures. In solving of many development problems related to the aforementioned objective, significant role belongs to the theoretical and experimental research, with special attention paid to the analysis of dynamic behaviour of the supporting structure of wagon [1, 2]. This paper presents the results of theoretical and experimental research of behaviour of the wagon supporting structure at the effect of impulse loads. When the train moves, due to the gap between the wagons and during the change of movement regime (starting, braking, stopping), especially in the formation of trains, there are very intensive longitudinal forces on the buffers, that significantly affect the stress-strain state of the supporting structure of the wagon [3]. These forces are rising and falling in a very short period of time.

The process of change in energy of the impact of deformable bodies is very complex and leads to great difficulties in mathematical problem solving. Consequently, the theoretical investigation is based on modelling of physical systems, which is usually idealized to a large degree [4, 5]. From this, it follows that the exact theoretical solution can be obtained only for the impact of bodies with a relatively simple geometries. To make a model that with more quality representing the behaviour of real object, theoretical research can be supplemented with the appropriate knowledge obtained by the experimental tests.

## 2. DIFERENTIAL EQUATIONS OF WAGONS IMPACT

Consider the case of two wagons impact with buffers stiffness $c_1$ and $c_2$, and stiffness of underframes $c_{ns1}$ and $c_{ns2}$. Wagon whose mass is $m_1$ is moving with the speed $\vec{v}_1$ and collides with wagon of mass $m_2$ which moves with the speed $\vec{v}_2$. Wagons are loaded with cargoes whose masses are $m_3$ and $m_4$. This is resulted with the relative movement of masses over the wagons for values $\vec{x}_3$ and $\vec{x}_4$. In addition, between the wagons and cargoes there are elastic connections with stiffness $c_3$ and $c_4$. Apart from that, the relative movements of cargoes with masses $m_3$ and $m_4$ are opposed by forces of dry friction and forces of viscous friction. These forces are proportional to the first degree of speeds of relative movements of cargoes $\vec{\dot{x}}_3$ and $\vec{\dot{x}}_4$. Also, the movements of the first and the second wagon are opposed by forces of rolling friction $\mu_1 \cdot g \cdot (m_1 + m_3)$ and $\mu_2 \cdot g \cdot (m_2 + m_4)$.

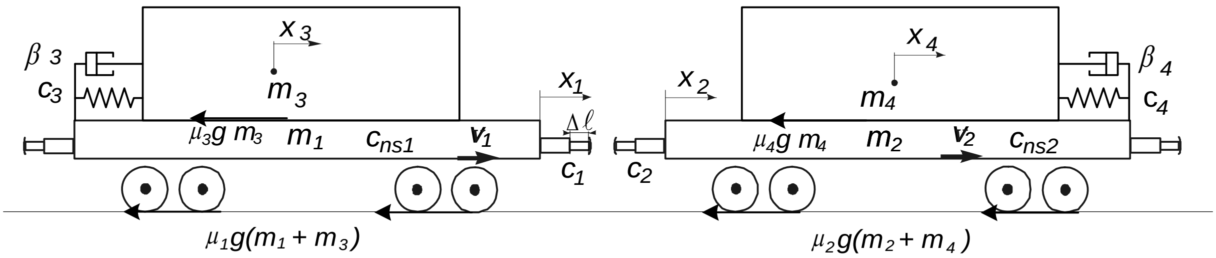



**Fig. 1. The impact of two wagons when the movement of cargo is present**

Equations for kinetic energy $E_k$, potential energy $E_p$ and dissipation function $\Phi_r$ of the system on the Fig.1 are [6]:

$$(1) \quad E_k = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\left(\dot{x}_3 + \dot{x}_1\right)^2 + \frac{1}{2}m_4\left(\dot{x}_4 + \dot{x}_2\right)^2$$

$$(2) \quad E_p = \frac{1}{2}c_s\left(x_2 - x_1\right)^2 + \frac{1}{2}c_3x_3^2 + \frac{1}{2}c_4x_4^2$$

$$(3) \quad \Phi_r = \mu_1(m_1 + m_3)g\left|\dot{x}_1\right| + \mu_2(m_2 + m_4)g\left|\dot{x}_2\right| + \mu_3 m_3 g\left|\dot{x}_3\right| + \mu_4 m_4 g\left|\dot{x}_4\right| + \frac{1}{2}\beta_3\dot{x}_3^2 + \frac{1}{2}\beta_4\dot{x}_4^2$$

where:

$c_s$ – equivalent stiffness of the system at the moment of impact,

$\mu_1$, $\mu_2$, $\mu_3$ and $\mu_4$ –coefficients of dry friction,

$\beta_3$ and $\beta_4$ –dynamic viscosities that define the environment resistance,

$g$ –acceleration due to gravity.

As the system is influenced by the conservative and dissipative forces, the Lagrange equations of second kind have the following form:

$$(4) \quad \frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}_i} + \frac{\partial \Phi_r}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial q_i} = 0$$

By changing the equations for kinetic and potential energy into the previous equations, as well as changing the equation for dissipation function, the following differential equations are obtained:

$$
\begin{aligned}
&\left(m_1+m_3\right)\ddot{x}_1+m_3\ddot{x}_3+c_s x_1-c_s x_2+\mu_1(m_1+m_3)g\cdot sign\dot{x}_1=0\\
&\left(m_2+m_4\right)\ddot{x}_2+m_4\ddot{x}_4+c_s x_2-c_s x_1+\mu_2(m_2+m_4)g\cdot sign\dot{x}_2=0\\
(5)\quad &m_3\ddot{x}_3+m_3\ddot{x}_1+\beta_3\dot{x}_3+c_3x_3+\mu_3m_3g\cdot sign\dot{x}_3=0\\
&m_4\ddot{x}_4+m_4\ddot{x}_2+\beta_4\dot{x}_4+c_4x_4+\mu_4m_4g\cdot sign\dot{x}_4=0
\end{aligned}
$$

The system of differential equations (5) defined in this way takes into account both the movement of cargoes and the influence of friction that occurs between the individual sub-assemblies when two wagons collide, and it is suitable for numerical solution. Also, the function signum (sign) allows the determination of the sign of friction forces that depends on the speeds of the movement of wagons and cargoes. In order to preparation for numerical solution of system of differential equations (5), by changing of the third equation in the first one, and the fourth equation in the second one, the following equations are obtained:

$$
(6)\quad
\begin{aligned}
&\ddot{x}_1=a_1(x_2-x_1)+a_2x_3+a_9\dot{x}_3-a_{10}sign\dot{x}_1+a_{13}sign\dot{x}_3\\
&\ddot{x}_2=a_3(x_1-x_2)+a_4x_4+a_5\dot{x}_4-a_{11}sign\dot{x}_2+a_{14}sign\dot{x}_4
\end{aligned}
$$

By changing the first equation (4) in the third equation (3) and second equation (4) in the fourth equation (3), the following equations are obtained:

$$
(7)\quad
\begin{aligned}
&\ddot{x}_3=a_1(x_1-x_2)-(a_2+a_{12})x_3-(a_9+a_7)\dot{x}_3+a_{10}sign\dot{x}_1-(a_{13}+a_{15})sign\dot{x}_3\\
&\ddot{x}_4=a_3(x_2-x_1)-(a_4+a_8)x_4-(a_5+a_6)\dot{x}_4+a_{11}sign\dot{x}_2-(a_{14}+a_{16})sign\dot{x}_4
\end{aligned}
$$

The constants $a_1 \div a_{16}$ are given in the following Table 1.

**Table 1. The values of the constants $a_1 \div a_{16}$**

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
|---|---|---|---|---|---|---|---|
| $c_s/m_1$ | $c_3/m_1$ | $c_s/m_2$ | $c_4/m_2$ | $\beta_4/m_2$ | $\beta_4/m_4$ | $\beta_3/m_3$ | $c_4/m_4$ |
| $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ |
| $\beta_3/m_1$ | $\mu_1\frac{m_1+m_3}{m_1}g$ | $\mu_2\frac{m_2+m_4}{m_2}g$ | $c_3/m_3$ | $\mu_3\frac{m_3}{m_1}g$ | $\mu_4\frac{m_4}{m_2}g$ | $\mu_3 g$ | $\mu_4 g$ |

This system, however, does not take into account the reduction of impact forces due to the loss of energy in oscillating structures of wagons during the impact. This effect can be taken into account by using Newton's coefficient of restitution. However, when it comes to the railway vehicles, the coefficient of restitution at impact can be determined only by the experimental testing [7, 8]. Defined system of differential equations takes into account the movement of cargoes during the impact, and is suitable for solution in numerical way. Based on the formed system of differential equations which describe the dynamic process of the impact of two wagons, a program for their solution is made. It is important to note that method of Runge-Kutta of IV level and programming language Fortran 77 were used [9, 10]. Based on the underlying methodology, the process of two wagons impact is simulated whereby model enables calculations with and without movement of cargo during the impact.

## 3. ANALYSIS OF WAGON IMPACT BY USING LAW OF CONVERSATION

According to the fact that during the impact other forces are smaller than the forces of mutual effect of two wagons, it can be said that the impact is a process in an isolated system where the formula about sustain are dominant (the quantity of movements, energy, etc.). General formulas about sustain are transformed shape of differentiated movement formulae.

With their application, the complex process of integration of differentiated formulae is avoided. Generally, the overall kinetic energy of wagon movement during the impact transforms in:

$$(8)\quad \frac{m_1 v_1^2}{2}+\frac{m_2 v_2^2}{2}=\int_{x=0}^{2\Delta\ell} F_{o(x)}dx+\frac{(m_1+m_2)v_{cm}^2}{2}+\sum_{i=1}^{n}\int_0^{x_i} b_i\dot{x}_i dx_i+E_{osc}$$

Members on the right part of the formula are:

| | |
|---|---|
| $\int_{x=0}^{2\Delta\ell} F_o(x)dx$ | - absorbed energy of antagonistic springs of both wagons |
| $v_{cm}$ | - speed of the mass center, |
| $\frac{(m_1+m_2)v_{cm}^2}{2}$ | - kinetic energy of the systems during the impact, |
| $\sum_{i=1}^{n}\int_0^{x_i} b_i\dot{x}_i dx_i$ | - thermal energy, that is all the forces of fractions of both wagons (thermal energy appears during wagon movements, cargo movement, etc.), |
| $E_{osc}$ | - energy of oscillation of both wagons with cargo, |
| $x = x_1 - x_2 = 0\div 2\,\Delta\ell$ | - action when the buffers of both wagons are compacted in case that the buffers of both wagons of the same rigidity, generally: $x=\Delta\ell_1+\Delta\ell_2$, |
| $F_o(x)$ | - the force in the buffers, |
| $b_i$ | - coefficient of proportionality which characterizes resistance, |
| $x_i, \dot{x}_i$, i=1÷4 | - motion and speed of mass. |

### 3.1 The loss of kinetic energy during two wagons impact

If we mark kinetic energy of two wagons before impact with $E_{ko}$ and with $E_k$ the kinetic energy of the system after the impact, then the loss of kinetic energy $\Delta E_k = E_i = E_{osc}$, (during the non-elastic impact of two wagons when tha cargo is motionless) is:

$$(9)\quad E_i=\Delta E_k=E_{ko}-E_k=\frac{1}{2}\left[m_1(v_1^2-v_1'^2)+m_2(v_2^2-v_2'^2)\right]$$

Substituting in the previous formula the formula for the speed of wagons after the impact ($v_1'$, $v_2'$) gives loss of kinetic energy during the impact as:

$$(10)\quad E_i=(1-k_r^2)\frac{m_1 m_2 (v_1-v_2)^2}{2(m_1+m_2)}$$

It is easy to determine the value of lost energy from the formula (10) and also the maximum value of the force during the wagons impact. The coefficient of restitution $k_r$ can be determined in another way by using experimental results of wagons impact. During the impact the intensity of the force on the buffers is very easily changed from zero to its maximum value, and then goes back to zero again. If time interval of impact duration is marked with $\tau$ , the impact impulse is:

$$(11)\quad I=\int_{t_o}^{t_o+\tau} F_o dt$$

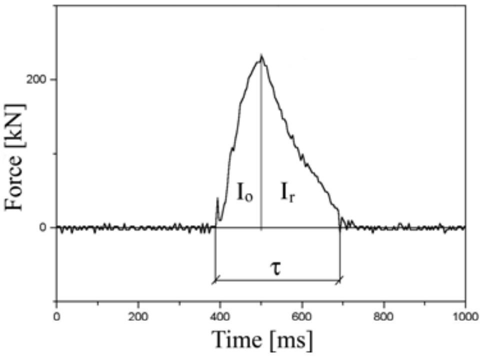
If impulse of loading is marked with $I_o$ and impulse of unloading with $I_r$, their values are:

$$(12)\quad I_o=m_1(v_1-v_{cm})=m_2(v_{cm}-v_2),\qquad I_r=m_1(v_{cm}-v_1')=m_2(v_2'-v_{cm})$$

The coefficient of restitution $k_r$ is equal to the ratio of these impulses:

$$k_r = I_r / I_o \qquad (13)$$

Impulses $I_o$ and $I_r$ are equal to the adequate areas on the diagram shown in Fig. 2.




$t$ [ms] – time

$t_o$ [ms] – initial moment of impact

$\tau$ [ms] – duration of impact

**Fig. 2. Dependence of the force on the buffers on the time during the impact of wagons**

## 4. EXPERIMENTAL DETERMINATION OF DYNAMIC PARAMETERS OF WAGONS IMPACT

Experimental testing of wagons impact is conducted on the special polygon in the Wagon Factory Kraljevo, Serbia (Fig. 3). Wagon which is running and hits the tested wagon is pulled along an inclined plane using a winch with rope, electromotor and gears. The test polygon is primarily made of metal, while the slope of the inclined plane is 12 % (6°50') (Fig. 4), and enables the impact speed over 15 km/h.

**Fig. 3. The polygon for wagons impact testing**

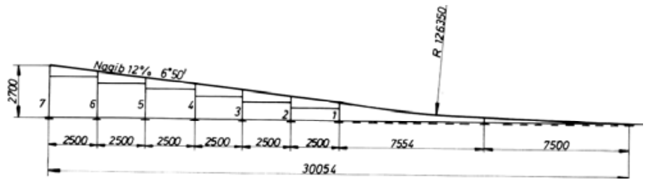



**Fig 4. The slope of polygon for wagons impact testing**

## 5. COMPARATIVE ANALYSIS OF TEORETICAL AND EXPERIMENTAL RESULTS

According to the obtained differential equations, the program "Impact" is made, based on the law of conservation of momentum and energy program "Force" on the basis of which the process of wagons impact is simulated. The results of numerical simulations of wagons impact are compared with the values obtained experimentally (Tables 2, 3 and 4) [11, 12].

**Table 2. The results of impact for wagon Uacns-z**

| Size | Unit | Exp. results | Software “Impact” | Software “Force” |
|---|---|---|---|---|
| $F^o$ | [kN ] | 1280 | 1240 | 1311 |
| $v^o$ | [m/s ] | - | 2,1 | 2,68 |
| $F_u^{max}$ | [kN ] | 2980 | 3020 | 3348 |
| $a_2^{max}$ | [m/s$^2$ ] | +56/-55 | +66/-52 | - |
| τ | [ms ] | 245 | 225 | - |

**Table 3. The results of impact for wagon Tadnss-z**

| Size | Unit | Exp. results | Software "Impact" | Software "Force" |
|---|---|---|---|---|
| $F^o$ | [kN ] | 1502 | 1480 | 1562 |
| $v^o$ | [m/s ] | - | 2,36 | 2.856 |
| $F_u^{max}$ | [kN ] | 3540 | 3220 | 3590 |
| $a_2^{max}$ | [m/s$^2$ ] | +55/-45 | +58/-43 | - |
| $\tau$ | [ms ] | 243 | 223 | - |

**Table 4. The results of impact for wagon Hccrrss-z**

| Size | Unit | Exp. results | | Software "Impact" | |
|---|---|---|---|---|---|
| | | Empty | Full | Empty | Full |
| $F_u^{max}$ | [kN ] | 1142 | 748 | 1161 | 761 |
| $\tau$ | [ms ] | 279 | 298 | 271 | 294 |

## 6. CONCLUSION

By analyzing the results of numerical simulations of wagon impacts presented in the previous chapter, it can be concluded that they are consistent with the results obtained experimentally. In this way, the developed models can be used to determine the dynamic parameters of the impact in the design phase of new wagons. This conclusion is confirmed by the diagrams of theoretical and experimental results which are shown in the figures below.

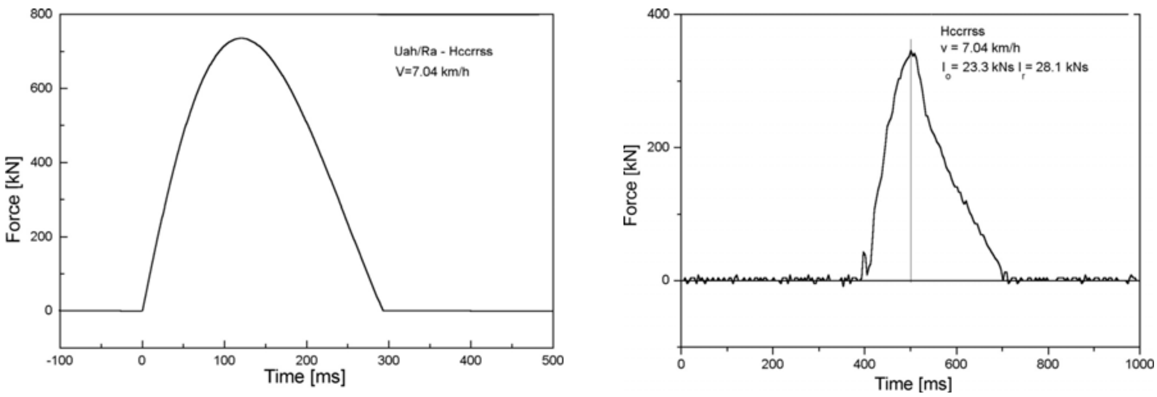



**Fig. 5. Diagrams of change in the total force on the buffers $F_u$ obtained theoretically (left) and force on the buffer $F_o$ obtained experimentally (right)**

Further improvements of developed models can be achieved on the basis of experimental determination of the dynamic stiffness of the buffers, the horizontal stiffness of the wagon, the coefficient of friction between the load and the underframe, etc. Improving the other theoretical models can be accomplished by experimental determination of impulse of loading and unloading of wagons that participate in the impact. Improved experimental research can be accomplished by determining the optimal sampling frequency and quality preparation of the experiment, taking into particular account the cargo that participate in the experiment.

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## ТЕОРЕТИЧЕН И ПРАКИЧЕСКИ АНАЛИЗ НА ДИНАМИЧНОТО ВЪЗДЕЙСТВИЕ ПРИ ВАГОНИТЕ

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***Ключови думи:*** *теоретичен, практически, анализ, вагони, динамично въздействие.*

***Резюме:*** *Съгласно възприетите международни правила, практическото тестване на динамичното въздействие на вагоните е едно от задължителните условия при тяхното сертифициране. По време на това изследване, се наблюдават съществени промени в някои от динамичните параметри, като: деформации, скорост на движение, ускоряване и др. Тези параметри са изключително важни за качествения дизайн и верифициране на вагонната структура. Целта на настоящия доклад е да засегне от теоретична и практическа гл. т. сложни проблеми, отнасящи се до динамичното въздействие на вагоните. В теорията са известни два основни подхода, чрез които може да бъде решен този проблем. Първият е свързан с извеждане на диференциални уравнения за движение на вагоните, а вторият – с прилагане на закона за съхраняване на енергията и количеството на трафика. При използването на математически модели, трябва да се вземат предвид следните ограничения: структурата на вагона, талигата и товарът трябва да бъдат статични; железопътните линии да са хоризонтални; центровете на тежестта на товара и вагоните да се движат паралелно, а пропуските в монтажните елементи на вагоните да се игнорират. Подобен тип теоретични анализи могат да послужат като база за подготовка и извършване на проучвания, които да доведат до постигане на задоволителни резултати. В този смисъл движението на вагоните по време на динамичното въздействие е от значение. В заключение е посочено, че единствено съчетаването на теоретични с практически анализи може да доведе до постигане на надеждни резултати, отчитащи динамичното въздействие при вагоните.*