Mechanics Transport Communications

Academic journal

ISSN 1312-3823

volume 12, issue 2, 2014

article № 1099

http://www.mtc-aj.com

CHARACTERISTICS OF ALMOST HERMITIAN MANIFOLDS WITH VANISHING BOCHNER CURVATURE TENSOR

Ognian Kassabov
okassabov@vtu.bg

Todor Kableshkov University of Transport
158 Geo Milev Str., 1574 Sofia,
BULGARIA

Key words: *Almost Hermitian manifolds, Bochner curvature tensor, sectional curvature.*

Abstract: *It is known that a Riemannian manifold of dimension n>3 is conformal flat if and only if its Weil curvature tensor vanishes identically. The Bochner curvature tensor for a Kähler manifold is defined as a formal analogue of the Weil's one. Hence it is important to know its geometric characteristics. In this paper we find such characteristics for the generalization of the Bochner tensor for an arbitrary almost Hermitian manifold.*

1. Introduction.

The Bochner curvature tensor is introduced in local complex coordinates for Kähler manifolds in [2] as a formal algebraic analogue of the Weil tensor. Because of the absence of any geometric base in the definition of the Bochner tensor, some characteristics for it are found. They are related with the curvature tensor and are in accordance with some characteristics of the Weil tensor for Riemannian manifolds.

A generalization *B(R)* of the Bochner tensor was introduced for an arbitrary almost Hermitian manifold after an examination of the LC-tensors on a *2n*-dimensional real vector space in [9]. Here we shall prove geometric characteristics of the tensor *B(R)*, related with some curvatures of the manifold. Namely, we shall prove the following propositions:

Theorem 1. Let M be a *2n*–dimensional (*n>2)* almost Hermitian manifold, such that for every point p in M and for every orthonormal basis *{x,y}* of an antiholomorphic 2-plane in T_pM the following linear relation holds

$$K(x,y)+\mu[S(R)(x,x)+S(R)(y,y)]+\nu[S(R)(Jx,Jx)+S(R)(Jy,Jy)] +\theta[S^*(R)(x,x)+S^*(R)(y,y)]=c(p) \tag{1.1}$$

with $(\mu,\nu,\theta)\neq(0,0,0)$. Then M has vanishing Bochner curvature tensor.

Theorem 2. Let M be a *2n*–dimensional almost Hermitian manifold, *n>3.* The following assertions are equivalent:

a) M has vanishing Bochner curvature tensor;

b) for every point $p\in M$ and for every orthogonal basis *{x,y,z,u}* of a 4-dimensional antiholomorphic subspace of T_pM *R(x,y,z,u)=0* holds;

c) for every point $p\in M$ and for every orthonormal basis *{x,y,z,u}* of a 4-dimensional antiholomorphic subspace of T_pM the following holds

(1.2) $$K(x,y)+K(z,u)=K(x,z)+K(y,u).$$

Theorem 1 is an antiholomorphic analogue of some results in [1], [3]. The converse statement of Theorem 1 is also true and is obvious. Theorem 2 is a generalization of some results in [10] for Kähler manifolds and an analogue of some classical characteristics of the manifolds with vanishing Weil tensor [7], [8]. The equivalence of a) and b) in Theorem 2 is proved in [5].

2. Preliminaries.

Let M be a $2n$-dimensional almost Hermitian manifold with metric tensor g, almost complex structure J and curvature tensor R. As it is well known, R is an LC-tensor, i.e. it has the properties

$$R(x,y,z,u)=-R(y,x,z,u)=-R(x,y,u,z)\,,$$
$$R(x,y,z,u)+R(y,z,x,u)+R(z,x,y,u)=0\,.$$

The sectional curvature of a 2-plane α, spanned by two linearly independent vectors x,y in a tangent space T_pM is defined by

$$K(\alpha,p)=K(x,y)=\frac{R(x,y,y,x)}{g(x,x)g(y,y)-g^2(x,y)}.$$

For every LC-tensor T on T_pM *the Ricci tensor* $S(T)$ and *the* $*$*–Ricci tensor* $S^*(T)$ are defined with

$$S(T)(x,y)=\sum_{i=1}^{2n}T(x,e_i,e_i,y),\qquad S^*(T)(x,y)=\sum_{i=1}^{2n}T(x,e_i,Je_i,Jy),$$

where $\{e_1,\dots,e_{2n}\}$ is an orthonormal basis of T_pM for $p\in M$. The scalar curvatures are respectively:

$$\tau(T)(p)=\sum_{i=1}^{2n}S(T)(e_i,e_i),\qquad \tau^*(T)(p)=\sum_{i=1}^{2n}S^*(T)(e_i,e_i)\ .$$

It is well known that the tensor $S(T)$ is symmetric, and that $S^*(T)$ has the property $S^*(T)(Jx,Jy)=S^*(T)(y,x)$.

Using the tensors $S(R)$ and $S^*(R)$ we may define also the following curvatures:

$$S(R)(x,x)+S(R)(y,y),\quad S(R)(Jx,Jx)+S(R)(Jy,Jy),\quad S^*(R)(x,x)+S^*(R)(y,y)\,.$$

A k–dimensional subspace E^k of T_pM is said to be *antiholomorphic*, if $E^k\perp JE^k$. Then it is easily seen that $k\leq n$. We say that the vectors $e_1,\dots,e_k$ are *antiholomorphic*, when $\operatorname{span}\{e_1,\dots,e_k\}$ is antiholomorphic.

For convenience the operators φ and ψ, defined with

$$\varphi(Q)(x,y,z,u)=g(x,u)Q(y,z)-g(x,z)Q(y,u)+g(y,z)Q(x,u)-g(y,u)Q(x,z)$$
$$\psi(Q)(x,y,z,u)=g(x,Ju)Q(y,Jz)-g(x,Jz)Q(y,Ju)-2g(x,Jy)Q(z,Ju)$$
$$+g(y,Jz)Q(x,Ju)-g(y,Ju)Q(x,Jz)-2g(z,Ju)Q(x,Jy)$$

for any tensor Q of type (0,2) are introduced. Put also

$$\bar{R}(x,y,z,u)=R(Jx,Jy,Jz,Ju),\ \pi_1=\frac{1}{2}\varphi(g),\ \pi_2=\frac{1}{2}\psi(g).$$

Then the Bochner tensor $B(R)$ for M is defined in [9] with

$$B(R)=R-\left\{\frac{1}{16(n+2)}(\varphi+\psi)(S+3S^*)+\frac{1}{16(n-2)}(3\varphi-\psi)(S-S^*)\right\}(R+\bar{R})$$
$$-\left\{\frac{1}{4(n+1)}\varphi(S^*)+\frac{1}{4(n-1)}\psi(S)\right\}(R-\bar{R})$$
$$+\left\{\frac{\tau(R)+3\tau^*(R)}{16(n+1)(n+2)}(\pi_1+\pi_2)\right\}+\left\{\frac{\tau(R)-\tau^*(R)}{16(n-1)(n-2)}(3\pi_1-\pi_2)\right\}.$$

We will need the following assertions:

Lemma 1. [4] Let T be an LC-tensor on T_pM with the properties:

a) $T(x,Jx,Jx,x)=T(x,y,y,x)=0$ for arbitrary $x,y\in T_pM$, $x\perp y,Jy$;

b) $T(x,y,z,u)=T(Jx,Jy,Jz,Ju)$ for arbitrary $x,y,z,u\in T_pM$.

Then *T=0*.

Lemma 2. [4] Let T be an LC-tensor on T_pM with the properties:

$$T(x,Jx,Jx,x)=T(x,y,y,x)=T(x,Jx,y,x)=0$$

for arbitrary vectors $x,y\in T_pM$, $x\perp y,Jy$. Then *T=0*.

3. Proof of Theorem 1.

Theorem 1 is an immediate consequence of the following two lemmas:

Lemma 3. Under the condition of Theorem 1 the following holds

$$R+\bar{R}=\left\{\frac{1}{8(n+2)}(\varphi+\psi)(S+3S^*)+\frac{1}{8(n-2)}(3\varphi-\psi)(S-S^*)\right\}(R+\bar{R})$$
$$-\left\{\frac{\tau(R)+3\tau^*(R)}{8(n+1)(n+2)}(\pi_1+\pi_2)\right\}-\left\{\frac{\tau(R)-\tau^*(R)}{8(n-1)(n-2)}(3\pi_1-\pi_2)\right\}.$$

Proof: Denote

$$T=R+\bar{R}-\left\{\frac{1}{8(n+2)}(\varphi+\psi)(S+3S^*)+\frac{1}{8(n-2)}(3\varphi-\psi)(S-S^*)\right\}(R+\bar{R})$$
$$+\left\{\frac{\tau(R)+3\tau^*(R)}{8(n+1)(n+2)}(\pi_1+\pi_2)\right\}+\left\{\frac{\tau(R)-\tau^*(R)}{8(n-1)(n-2)}(3\pi_1-\pi_2)\right\}.$$

Let $x,y\in T_pM$, $|x|=|y|=1, x\perp y,Jy$. In (1.1) we replace *x* with $(x+ty)/\sqrt{1+t^2}$ and *y* with $(tJx-y)/\sqrt{1+t^2}$. We transform the result and from the coefficient of t^2 we find

$$H(x)+H(y)-2R(x,Jy,Jx,y)-2R(x,Jx,Jy,y)$$
$$+(\mu+\nu)\{S(R+\bar{R})(x,x)+S(R+\bar{R})(y,y)\}$$
$$+2\theta\{S^*(R)(x,x)+S^*(R)(y,y)\}=2c(p).$$

Replacing in (1.1) $\{x,y\}$ with $\{x,Jy\}$ and $\{Jx,y\}$ we obtain, respectively:

$$K(x,Jy)+\mu[S(R)(x,x)+S(R)(Jy,Jy)]+\nu[S(R)(Jx,Jx)+S(R)(y,y)]$$
$$+\theta[S^*(R)(x,x)+S^*(R)(y,y)]=c(p),$$
$$K(Jx,y)+\mu[S(R)(Jx,Jx)+S(R)(y,y)]+\nu[S(R)(x,x)+S(R)(Jy,Jy)]$$
$$+\theta[S^*(R)(x,x)+S^*(R)(y,y)]=c(p).$$

From the last three equations we derive

$$H(x)+H(y)=2R(x,Jy,Jx,y)+2R(x,Jx,Jy,y)+K(x,Jy)+K(Jx,y),$$

where *H(x)=K(x,Jx).* From the last equality we obtain

(3.1) $$H(x)=\frac{1}{2(n+2)}(S+3S^*)(R+\bar{R})(x,x)-\frac{(\tau+3\tau^*)(R)}{4(n+1)(n+2)}.$$

On the other hand (1.1) implies

(3.2) $$(R+\bar{R})(x,y,y,x)+(\mu+\nu)\{S(R+\bar{R})(x,x)+S(R+\bar{R})(y,y)\}$$
$$+2\theta\{S^*(R)(x,x)+S^*(R)(y,y)\}=2c(p).$$

Since *x* and *y* are arbitrary unit vectors with $x\perp y,Jy$, from (3.2) and (3.1) we find consecutively

$$c(p)=\left\{\frac{\mu+\nu}{n}+\frac{2n+1}{8n(n^2-1)}\right\}\tau(R)+\left\{\frac{\theta}{n}-\frac{3}{8n(n^2-1)}\right\}\tau^*(R),$$

$$(\mu+\nu)S(R+\bar{R})(x,x)+2\theta S^*(R)(x,x)=\frac{1}{n^2-4}\{3S^*(R)(x,x)-\frac{n+1}{2}S(R+\bar{R})(x,x)\}$$

$$+\frac{1}{n}\{(\mu+\nu)\tau(R)+2\theta\tau^*(R)\}-\frac{1}{2n(n^2-4)}\{3\tau^*(R)-(n+1)\tau(R)\}\,.$$

From the last two equalities and (3.2) we derive

$$(R+\bar{R})(x,y,y,x)=\frac{n+1}{2(n^2-4)}\{S(R+\bar{R})(x,x)+S(R+\bar{R})(y,y)\}$$
$$-\frac{3}{n^2-4}\{S^*(R)(x,x)+S^*(R)(y,y)\}$$
$$-\frac{2n^2+3n+4}{4(n^2-1)(n^2-4)}\tau(R)+\frac{9n}{4(n^2-1)(n^2-4)}\tau^*(R)\,,$$

which can be put in the form *T(x,y,y,x)=0*. On the other hand (3.1) shows that *T(x,Jx,Jx,x)=0*. From the definition of the tensor *T* we have *T(x,y,z,u)=T(Jx,Jy,Jz,Ju)*. Now Lemma 3 follows from Lemma 1.

Lemma 4. Under the assumptions of Theorem 1 the following holds

$$R-\bar{R}=\left\{\frac{1}{2(n-1)}\varphi(S)+\frac{1}{2(n+1)}\psi(S^*)\right\}(R-\bar{R}).$$

Proof: Denote

$$T=R-\bar{R}-\left\{\frac{1}{2(n-1)}\varphi(S)+\frac{1}{2(n+1)}\psi(S^*)\right\}(R-\bar{R}).$$

Hence we have immediately

(3.3) $$T(x,Jx,Jx,x)=0$$

for any vector $x\in T_pM$. On the other hand according to (1.1) for arbitrary unit vectors $x,y\in T_pM$, with $x\perp y,Jy$, we have

$$K(x,y)-K(Jx,Jy)+(\mu-\nu)\{S(R-\bar{R})(x,x)+S(R-\bar{R})(y,y)\}=0\,.$$

Let $\{e_1,\dots,e_{2n}\}$ be an orthonormal basis of T_pM, such that $e_1=x, e_2=Jx$. In the above we put $y=e_i$ $(i>2)$ and we take the sum to obtain

$$(\mu-\nu)S(R-\bar{R})(x,x)=-\frac{1}{2(n-1)}S(R-\bar{R})(x,x),$$

and hence

$$K(x,y)-K(Jx,Jy)=\frac{1}{2(n-1)}\{S(R-\bar{R})(x,x)+S(R-\bar{R})(y,y)\}\,.$$

It is easy to see that this can be written as

(3.4) $$T(x,y,y,x)=0$$

for arbitrary unit vectors $x,y\in T_pM$, with $x\perp y,Jy$. Hence for such vectors x,y and for any real number t it holds

$$T(x+tJy,tJx+y,tJx+y,x+tJy)=0\,,$$

which implies

(3.5) $$T(x,Jx,y,x)+T(x,y,y,Jy)=0\,.$$

Let $z\in T_pM$ be a unit vector, such that $z\perp x,Jx,y,Jy$. From (3.4) we find easily

$$T(x,y,z,x)=0\,.$$

We replace here *(x,y,z)* with *(x+tz,y,tJx-Jz)* and the coefficient of t^1 in the result gives:

$$T(x,Jx,y,x)=T(x,y,Jz,z)+T(x,Jz,y,z).$$

Let $\{e_1,\dots,e_{2n}\}$ be an orthonormal basis of T_pM, such that $e_1=x,e_2=Jx,e_3=y,e_4=Jy$. We put here $z=e_i$ $(i>4)$ and we take the sum on *i*, using (3.5):

(3.6) $$2(n+1)T(x,Jx,y,x)=-3S^*(T)(x,Jy).$$

On the other hand from the definition of the tensor *T* we find

$$S^*(T)(x,y) = \sum_{i=1}^{2n} T(x,e_i,Je_i,Jy) = S^*(R-\bar{R})(x,y)$$

$$-\frac{1}{2(n-1)}\sum_{i=1}^{2n}\{g(x,Jy)S(R-\bar{R})(e_i,Je_i) - g(x,Je_i)S(R-\bar{R})(e_i,Jy)$$
$$+ g(e_i,Je_i)S(R-\bar{R})(x,Jy) - g(e_i,Jy)S(R-\bar{R})(x,Je_i)\}$$
$$-\frac{1}{2(n+1)}\sum_{i=1}^{2n}\{ g(x,y)S^*(R-\bar{R})(e_i,e_i) - g(x,e_i)S^*(R-\bar{R})(e_i,y)$$
$$+g(e_i,e_i)S^*(R-\bar{R})(x,y) - g(e_i,y)S^*(R-\bar{R})(x,e_i)$$
$$+2g(x,Je_i)S^*(R-\bar{R})(Je_i,y) + 2g(Je_i,y)S^*(R-\bar{R})(x,Je_i)\},$$

and hence $S^*(T)(x,y) = 0$. From the last equality, (3.3), (3.4) и (3.6), using Lemma 2, we obtain *T=0*, thus proving Lemma 4.

4. Proof of Theorem 2.

Let *{x,y,z,u}* be an orthonormal antiholomorphic quadruple of vectors in T_pM .

The implication **a) ⇨ b)** is trivial.

b) ⇨c): It follows from *R(x,y+z,y-z,u)=0* that *R(x,y,y,u)=R(x,z,z,u)* . We replace here *(x,u)* with *(x+u,x-u)* and we derive (1.2).

c) ⇨ a): Let *t* be an arbitrary real number. According to (1.2) we may write

$$K\left(\frac{x+ty}{\sqrt{1+t^2}},\frac{tx-y}{\sqrt{1+t^2}}\right) + K(z,u) = K\left(\frac{x+ty}{\sqrt{1+t^2}},z\right) + K\left(\frac{tx-y}{\sqrt{1+t^2}},u\right),$$

which implies

$$H(x) + H(y) = 2R(x,Jy,Jx,y) + 2R(x,Jx,Jy,y) + K(x,Jy) + K(Jx,y).$$

From the last equality we obtain

$$H(x) = \frac{1}{2(n+2)}(S+3S^*)(R+\bar{R})(x,x) - \frac{(\tau+3\tau^*)(R)}{4(n+1)(n+2)}, \tag{4.1}$$

$$\sum_{i=1}^{2n} H(e_i) = \frac{(\tau+3\tau^*)(R)}{2(n+1)}. \tag{4.2}$$

Now let $\{e_1,\dots,e_n,Je_1,\dots,Je_n\}$ be an adapted basis of T_pM, such that $e_1 = x$, $e_2 = y$. In (1.2) we take $(z,u) = (e_i,e_s)$ and $(z,u) = (e_i,Je_s)$ and we take the sum for *s=4,…,n*, $s \neq i$:

$$(2n-5)K(x,y) + S(R)(e_i,e_i) - H(e_i) - K(Jx,e_i) - K(Jy,e_i)$$
$$= (2n-5)K(x,e_i) + S(R)(y,y) - H(y) - K(Jx,y) - K(y,Je_i).$$

Adding all of the above equalities for *i=3,…,n* we have

$$(2n^2-8n+7)K(x,y) + K(Jx,Jy) + (n-2)\{K(Jx,y) + K(x,Jy)\}$$
$$+(n-1)\{H(x) + H(y)\}$$
$$= (n-2)\{S(R)(x,x) + S(R)(y,y)\} + S(R)(Jx,Jx) + S(R)(Jy,Jy) \tag{4.3}$$
$$-\frac{\tau(R)}{2} + \frac{1}{2}\sum_{i=1}^{2n} H(e_i).$$

Here we replace *(x,y)* с *(Jx,Jy)* . From the result and (4.3) we derive

$$K(x,y) - K(Jx,Jy) = \frac{1}{2(n-1)}\{S(R-\bar{R})(x,x) + S(R-\bar{R})(y,y)\},$$

and hence

$$K(x,Jy) - K(Jx,y) = \frac{1}{2(n-1)}\{S(R-\bar{R})(x,x) + S(R-\bar{R})(Jy,Jy)\}.$$

The last three equalities imply

$$(4.4)\qquad \begin{aligned}&(n-2)K(x,y)+K(x,Jy)+\frac{n-1}{2(n-2)}\{H(x)+H(y)\}\\ &=\frac{1}{2}\left\{S\left(R+\frac{1}{n-2}\bar{R}\right)(x,x)+S\left(R+\frac{1}{n-2}\bar{R}\right)(y,y)\right\}\\ &+\frac{1}{4(n-1)(n-2)}\{S(R-\bar{R})(x,x)+S(R-\bar{R})(y,y)\}\\ &+\frac{1}{4(n-1)}\{S(R-\bar{R})(x,x)+S(R-\bar{R})(y,y)\}-\frac{\tau(R)}{2}+\frac{1}{2}\sum_{i=1}^{2n}H(e_i).\end{aligned}$$

Replacing here *y* with *Jy*, and using again (4.4) we obtain

$$K(x,y)-K(x,Jy)=\frac{1}{2(n-1)}S(R-\bar{R})(y,y)\,.$$

From the last two equalities, using (4.1) and (4.2) we derive

$$\begin{aligned}K(x,y)=&\frac{1}{16(n+2)}\{(S+3S^*)(R+\bar{R})(x,x)+(S+3S^*)(R+\bar{R})(y,y)\}\\ &+\frac{3}{16(n-2)}\{(S-S^*)(R+\bar{R})(x,x)+(S-S^*)(R+\bar{R})(y,y)\}\\ &+\frac{1}{4(n-1)}\{S(R-\bar{R})(x,x)+S(R-\bar{R})(y,y)\}\\ &-\frac{\tau(R)+\tau^*(R)}{16(n+1)(n+2)}-\frac{3\big(\tau(R)-\tau^*(R)\big)}{16(n-1)(n-2)}.\end{aligned}$$

Now applying Theorem 1 we obtain Theorem 2.

5. Corollaries.

An almost Hermitian manifold *M* is said to be *of pointwise constant antiholomorphic sectional curvature* ν, if for every point $p\in M$ the curvature of every antiholomorphic 2-plane α in T_pM is $K(\alpha,p)=\nu(p)$.

Using Theorem 1 we can derive the following characterizations of the almost Hermitian manifolds with pointwise constant antiholomorphic sectional curvature:

Theorem 3. [6] Let *M* be a *2n*–dimensional almost Hermitian manifold, *n>2.* Then *M* is of pointwise constant antiholomorphic sectional curvature ν if and only if it has vanishing Bochner curvature tensor and the tensor $2(n+1)S(R)-3S^*(R+\bar{R})$ is proportional to the metric tensor. The function ν has the form

$$\nu=\frac{(2n+1)\tau(R)-3\tau^*(R)}{8n(n^2-1)}.$$

Theorem 4. [6] A *2n*–dimensional almost Hermitian manifold *M*, *n>2* is of pointwise constant antiholomorphic sectional curvature ν if and only if its curvature tensor has the form

$$R=\frac{1}{2(n+1)}\,\psi\big(S^*(R)\big)+\nu\,\pi_1-\frac{\tau^*(R)+2(n+1)\nu}{2(n+1)(2n+1)}\pi_2.$$

Acknowledges. The author wants to thank the reviewer for his remarks that improved the article.

References

[1] Г. Станилов: *Докторска дисертация*. София, 1977.

[2] S. Bochner: *Curvature and Betti numbers II*. Ann. of Math. **50**(1949), 77-93.

[3] G. Ganchev: *Characteristic of some classes of almost Hermitian manifolds.* Serdica Math. J., **4**(1978), 19-23.

[4] G. Ganchev: *Almost Hermitian manifolds similar to the complex space forms*. C. R. Acad. Bulgare Sci., **32**(1979), 1179-1182.

[5] G. Ganchev: *Conformal type and Bochner tensors for almost Hermitian manifolds*. C. R. Acad. Bulgare Sci., **34**(1981), 1065-1068.

[6] G. Ganchev: *On Bochner curvature tensors in almost Hermitian manifolds.* Pliska Stud. Math. Bulgar., **9**(1987), 33-43.

[7] R. S. Kulkarni: *Curvature structure and conformal transformations.* Bull. Amer. Math. Soc., **75**(1969), 91-94.

[8] J. A. Schouten: *Ricci-calculus.* Springer-Verlag, Berlin and New York, 1954.

[9] Fr. Tricerri, L. Vanhecke: *Curvature tensors on almost Hermitian manifolds*. Trans. Amer. Math. Soc., **267**(1981), 365-398.

[10] L. Vanhecke, K. Yano: *Almost Hermitian manifolds and the Bochner curvature tensor.* Kodai Math. Sem. Rep., **29**(1977), 10-21.

ХАРАКТЕРИСТИКИ НА ПОЧТИ ЕРМИТОВИ МНОГООБРАЗИЯ С НУЛЕВ ТЕНЗОР НА БОХНЕР

Огнян Касабов
okassabov@vtu.bg

ВТУ"Тодор Каблешков", кат. „Математика и информатика"
София 1574, ул. „Гео Милев" 158
БЪЛГАРИЯ

Ключови думи: *Почти Ермитови многообразия, тензор на Бохнер, секционна кривина.*

Резюме. *Както е известно, едно Риманово многообразие с размерност n>3 е конформно плоско тогава и само тогава, когато е с нулев тензор на Вайл. Тензорът на Бохнер за Келерово многообразие е дефиниран като формален аналог на този на Вайл. Затова е важно да знаем негови геометрични характеристики. Тук намираме такива за обобщението на тензора на Бохнер за произволно почти Ермитово многообразие.*