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THE LOSS AND REGAIN OF STABILITY DUE TO HUNTING MOVEMENT

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Abstract: *The hunting movement of the railway vehicles is a transversal oscillation of the bodies of the system coupled with a rotation around the vertical axis. The causes are the conicity of the wheels and the fact that they are rigidly mounted on the axle [1]. This oscillation may occur even on a hypothetic perfect track and endangers the running safety. The model which describes this phenomenon is nonlinear and therefore analytical expressions of the amplitudes may not be obtained. In this paper we deduce an approximate analytical expression of the solution by means of the mediation theorem.*

Key words: *Oscillation, hunting, mediation theorem, railway vehicles, amplitude*

INTRODUCTION

The superposition of lateral and rotational vibration of a railway vehicle axle in a straight, uniform motion, caused by a short time perturbation gives rise to a resulting oscillation known as the hunting phenomenon.

The loss of control of this phenomenon by an unstable evolution is important for running safety and for the comfort of the vehicles.

The study of the hunting is possible with the aid of linear and nonlinear models. The linear models may be used to get some information regarding the stability of the straight, uniform motion of the vehicle. The analysis of the amplitudes and stability of the hunting may be carried out only with nonlinear models. The problem is that the nonlinear systems are more difficult to study.

In this paper we build a model of study for the axle which may be solved by means of the mediation theorem in order to obtain approximate analytical solutions.

THE EQUATIONS OF MOTION

The physical model of the structure used to study the hunting phenomenon consists of two wheels with a certain rolling profile, rigidly mounted on an axle whose bearings are fastened by the vehicle with elastic elements. In fig.1 we present this model in a plane co-ordinate systems.

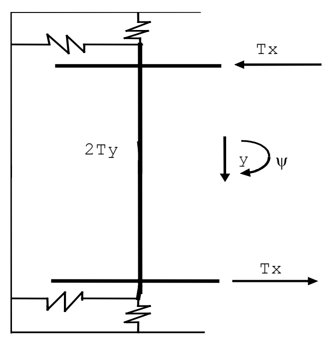

Fig.1: The model consists of two wheels with a certain rolling profile, rigidly mounted on an axle whose bearings are fastened by the vehicle with elastic elements

We shall consider that the transversal sections of the wheels and of the rails are quarters of circles (fig.2).

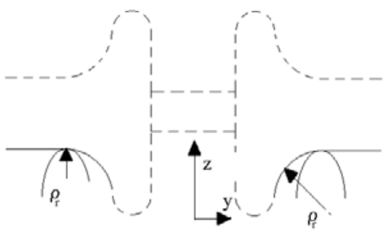

Fig.2: We shall consider that the transversal sections of the wheels and of the rails are quarters of circles

The equations of the profiles are:

$$z_{1w}(y)=\sqrt{r^2-\left(y+\frac{r}{2}\right)^2} \tag{1}$$

$$z_{2w}(y)=\sqrt{r^2-\left(y-\frac{r}{2}\right)^2} \tag{2}$$

The equations describe the left wheel (1) and, respectively, the right wheel (2) where *y* is the transversal displacement of the axle. The parameter *r* is specific to the profile. (the curvature radius).

We shall determine some geometric properties of the contact point by means of expanding their expressions in Taylor series.

The difference between the rolling radiuses of the two wheels is

$$dr = z_{w1} - z_{w2} = \frac{2\sqrt{3}\left(9r^2+8y^2\right)}{27r^2} \tag{3}$$

We denote with γ_i, (i=1,2) the angle of the profile in the contact point on the two wheels. In order to calculate the centering force we use the relation:

$$ct = \left(tg\,\gamma_1 - tg\,\gamma_2\right)/y = \frac{dz_{w1}}{dy_{w1}} - \frac{dz_{w2}}{dy_{w2}} = \\ = \frac{16\sqrt{3}\left(9r^2+16y^2\right)}{81r^3} \tag{4}$$

By means of the equation (3) we calculate the creep and then we determine the friction forces between the wheels and the rails.

In this case the equations which describe the motion of the wheel set are:

$$\begin{aligned} &m_0\ddot{y} + \frac{2xQ}{v}y + \left(2c_y + ct\right)y - 2xQy = 0 \\ &Ioz\psi + \frac{2l^2xQ}{v}\psi + 2b^2c_x\psi + 2lxQ\cdot\frac{dr}{r}y = 0 \end{aligned} \tag{5}$$

where v is the forward speed, Q is the load on the wheel, l the distance between the wheels, b the distance between the suspension springs, χ the friction coefficient, m_0 and I_{0z} are the mass and the inertia moment of the axle.

ANALYTICAL APPROXIMATE SOLUTIONS OF THE EQUATIONS OF MOTION USING THE AVERAGING THEOREM

We give an approximate analytical solution of the equations (5) using the averaging theorem. We look for the limit cycles among the periodical solutions of the system (5) following the algorithm proposed by Van der Pol [2].

Looking for this purpose we start with a change of functions and co-ordinates:

$$q_n = A_n(\tau)\cos\theta + B_n(\tau)\sin\theta \quad (n=1,2) \tag{6}$$

where A_n, B_n are unknown functions, ω is a constant that should be determined, $\theta = \omega\cdot\tau$ and

$$(q_1 \quad q_2)^T = (y \quad \psi)^T$$

Supposing that the functions A_n and B_n have a slow variation it follows that

$$A_n\cos\theta + B_n\sin\theta \approx 0 \tag{7}$$

In this case the rate of change of the solutions is:

$$q_n \approx -A_n\omega\sin\theta + B_n\omega\cos\theta \tag{8}$$

The relations (8) may be used together with the equations (5) in order to obtain a system of four first order differential nonlinear equations with the unknown functions A_n and B_n . From the equations (6- 8) and (5) we deduce the system:

$$O = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ -m_0\omega\sin\theta & m_0\omega\cos\theta & 0 & 0 \\ 0 & 0 & -I_{oz}\omega\sin\theta & I_{oz}\omega\cos\theta \end{pmatrix}$$

$$O\cdot\begin{pmatrix}\dot{A}_1\\ \dot{B}_1\\ \dot{A}_2\\ \dot{B}_2\end{pmatrix}=\begin{pmatrix}0\\0\\b_3\\b_4\end{pmatrix} \qquad (9)$$

where:

$$\begin{aligned}
b_3 &= m_0\omega^2(A_1\cos\theta+B_1\sin\theta)+\\
&+\frac{2xQ}{v}\omega(A_1\sin\theta-B_1\cos\theta)-\\
&-c_{ly}(A_1\cos\theta+B_1\sin\theta)-\\
&-c_{ny}(A_1\cos\theta+B_1\sin\theta)^3+\\
&+2xQ(A_2\cos\theta+B_2\sin\theta)\\
b_4 &= I_{oz}\omega^2(A_2\cos\theta+B_2\sin\theta)+\\
&+\frac{2e^2xQ}{v}\omega(A_2\sin\theta-B_2\cos\theta)-\\
&-2e^2cx(A_2\cos\theta+B_2\sin\theta)-\\
&-2exQ\frac{drl}{r}(A_1\cos\theta+B_1\sin\theta)-\\
&-2exQ\frac{drn}{r}(A_1\cos\theta+B_1\sin\theta)^3
\end{aligned} \qquad (10)$$

where

$$\begin{aligned}
c_{ly} &= 16/9\cdot 3^{1/2}/r\cdot Q\\
c_{ny} &= 16/81\cdot 3^{1/2}\cdot 16/r^3\cdot Q\\
dr_l &= 2/27\cdot 3^{1/2}\cdot 9\\
dr_n &= 2/27\cdot 3^{1/2}\cdot 8/r^2
\end{aligned}$$

We apply the averaging theorem associating to the equations (9) a system given by the relations:

$$\begin{aligned}
\overline{\dot{A}}_1 &= -\frac{1}{2\pi\omega}\int_0^{2\pi}\frac{b_3}{m_0}\sin\theta d\theta\\
\overline{\dot{B}}_1 &= +\frac{1}{2\pi\omega}\int_0^{2\pi}\frac{b_3}{m_0}\cos\theta d\theta\\
\overline{\dot{A}}_2 &= -\frac{1}{2\pi\omega}\int_0^{2\pi}\frac{b_4}{I_{oz}}\sin\theta d\theta\\
\overline{\dot{B}}_2 &= \frac{1}{2\pi\omega}\int_0^{2\pi}\frac{b_4}{I_{oz}}\cos\theta d\theta
\end{aligned} \qquad (11)$$

We shall use the denotation:

$$x=(A_1,B_2,A_2,B_2)^T \qquad (12)$$

and we shall write the system (9) as:

$$\dot{x}=\varepsilon\, f(x_1\theta) \qquad (13)$$

In these conditions, the mediate system (11) becomes:

$$\dot{\bar{x}}=\varepsilon\, f(\bar{x})=\frac{\varepsilon}{2\pi}\int_0^{2\pi} f(y_1\theta) \qquad (14)$$

for which we can obtain the stationary solutions.

If $\gamma \ll 1$ the relation between the stationary solution of (14) and the periodical solutions of (13) is given by the averaging theorem [3]:

If $\gamma \ll 1$, f continuous of class r, $r>1$, bordered on a bordered domain, and p_0 is a hyperbolic fixed point for the averaged system, then (13) has a unique periodical solution p_0 of the same stability with p_0.

We notice that the system (14) is autonomous which helps solving the problem. More than that, the information about the periodical solutions of the nonlinear system (5) will be given by the study of the fixed points of the system (11).

The mediate system (11) has four equations with four unknown functions A_n, B_n and an unknown constant T.

Without restricting the generality of the solution, we shall translate the time origin of our co-ordinate system so that $q_1(0) = 0$ and in consequence $B_1 = 0$.

Integrating the system (11) we obtain the equations (15) which give the stationary solutions (fixed points) of the mediate system:

$$\begin{aligned}
&\omega A_1+vB_2=0\\
&4\left(m\omega^2-c_{ly}\right)A_1+8\kappa QA_2-3c_{ny}A_1^{\,3}=0\\
&-\frac{e^2\kappa Q}{v\omega I}A_2+\left(\frac{e^2c_x}{\omega^2 I}-\frac{1}{2}\right)B_2=0\\
&-\kappa Qevdr_lA_1-\frac{3}{4}\kappa Qevdr_lA_1^{\,3}+\left(\frac{1}{2}I\omega^2-\right.\\
&\left.-e^2c_x\right)\cdot vrA_2-e^2\kappa Q\omega rB_2=0
\end{aligned} \qquad (15)$$

Obtaining numerical solutions for the above system we can plot bifurcation diagrams like the one below fig. 3.

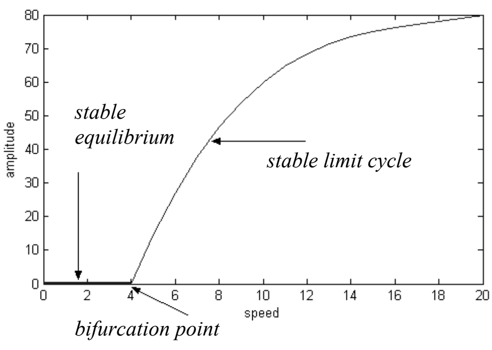

Fig. 3. Above system of diagrams

CONCLUSIONS

The equilibrium point of the wheels is stable until a certain critical speed is reached. Further the movement of the axle is oscillatory. The stable limit cycle is generated softly.

If the circulation speed decreases (under the critical speed) the stability of the static equilibrium point is regained.

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ЗАГУБА И ВЪЗСТАНОВЯВАНЕ НА СТАБИЛНОСТ ПОРАДИ ЛЪКАТУШЕНЕ

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РУМЪНИЯ

Резюме: *Лъкатушенето на железопътните возила е напречно трептене на телата на системата, комбинирано с въртене около вертикалната ос. Причината е в колелата и факта, че те са твърдо монтирани на оста [1]. Това трептене може да възникне дори върху хипотетично перфектен път и застрашава безопасността на движение. Моделът, който описва това явление, е нелинеен и следователно не могат да бъдат получени аналитични изрази за амплитудите. В този доклад извеждаме приблизителен аналитичен израз за решение чрез теорема за посредничество.*

Ключови думи: *трептене, лъкатушене, железопътни возила, амплитуда.*